BASED THEOREM OF GENERALIZED TRIGONOMETRIC FUNCTIONS

E. Konopatsky

***Summary.* In work is formulated and proved the based theorem of generalized trigonometric functions. Also consequences the offered theorem are investigated.**

***Keywords:* generalized trigonometric functions, basic theorem plane BN-calculus theorems consequences.**

Formulation of the problem. While studies use UGA-halnenyh trigonometric functions to determine the plane using angular and radial parameterization [1] in the BN-found numerous theorem, which can significantly simplify radial and angular alignment parameters in simplex plane to go from one to another parameterization .

Analysis of recent research. Tryhonometry-term generalized functions CSSR was introduced in [2]. Properties honometrychnyh three generalized functions was studied in [3]. In [4, 5] The author was offered the use of generalized trigonometric functions to determine the plane curves and their use for parameterization transition from one to another. Also the author in [6] The features parameterization plane in BN-many.

The wording of Article purposes. Formulate and prove the basic theorem of generalized trigonometric functions.

Main part. Formulates the basic theorem of generalized trigonometric functions.

Theorem. If arbitrary triangle point M defined by three angles φ, τ and ϕ (Fig. 1), the fair value is the following:

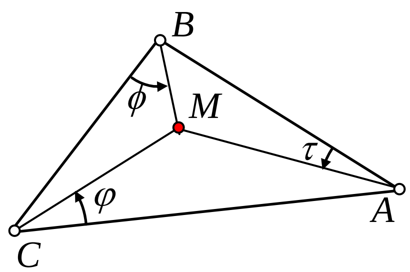

Fig. 1. Determination of the triangle.

$$\sin_{\tau}(\alpha-\tau)\cdot\sin_{\phi}(\beta-\phi)\cdot\sin_{\varphi}(\gamma-\varphi)=1. \qquad (1)$$

where $\angle CAB=\alpha$, $\angle ABC=\beta$ i $\angle BCA=\gamma$.

Evidence. Determine using generalized sine length segments AM, BM and CM :

$$\begin{aligned} |AM| &= b\sin_{(\varphi+\alpha-\tau)}\varphi = c\sin_{(\beta-\phi+\tau)}(\beta-\phi), \\ |BM| &= c\sin_{(\tau+\beta-\phi)}\tau = a\sin_{(\gamma-\varphi+\phi)}(\gamma-\varphi), \\ |CM| &= b\sin_{(\alpha-\tau+\varphi)}(\alpha-\tau) = a\sin_{(\phi+\gamma-\varphi)}\phi. \end{aligned} \qquad (2)$$

For greater clarity, proceed to standard trigonometric functions:

$$b\sin(\beta-\phi+\tau)\sin\varphi = c\sin(\varphi+\alpha-\tau)\sin(\beta-\phi),$$
$$c\sin(\gamma-\varphi+\phi)\sin\tau = a\sin(\tau+\beta-\phi)\sin(\gamma-\varphi), \qquad (3)$$
$$a\sin(\alpha-\tau+\varphi)\sin\phi = b\sin(\phi+\gamma-\varphi)\sin(\alpha-\tau).$$

Multiply together the left and right respectively chystyny ditch-nian (3) and then get simplifications:

$$\frac{\sin(\alpha-\tau)}{\sin\tau}\cdot\frac{\sin(\beta-\phi)}{\sin\phi}\cdot\frac{\sin(\gamma-\varphi)}{\sin\varphi}=1. \qquad (4)$$

Turning to obtain generalized sine expression (1). The theorem is proved.

Similar words be, it is, what is your name, my name is, there is, this is, that is, she is, he is, what is up, where is. You can also prove the proposed theorem by using Ceva's Theorem. To determine this value $u = BCM_A$, $v = CAM_B$ and $w = ABM_C$ on the sides of the triangle ABC (Fig. 2) using generalized sine.

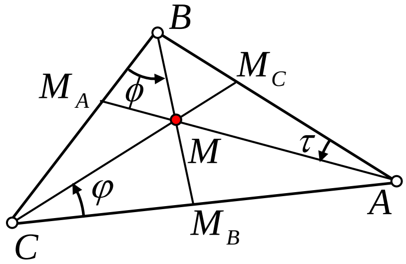

Fig. 2. Proof using Ceva's Theorem.

$$u = BCM_A = \sin_\beta\gamma\cdot\sin_{(\alpha-\tau)}\tau,$$
$$v = CAM_B = \sin_\gamma\alpha\cdot\sin_{(\beta-\phi)}\phi, \qquad (5)$$
$$w = ABM_C = \sin_\alpha\beta\cdot\sin_{(\gamma-\varphi)}\varphi.$$

According to Ceva's Theorem $u\cdot v\cdot w=1$. Then we have:

$$\sin_\beta\gamma\cdot\sin_{(\alpha-\tau)}\tau\cdot\sin_\gamma\alpha\cdot\sin_{(\beta-\phi)}\phi\cdot\sin_\alpha\beta\cdot\sin_{(\gamma-\varphi)}\varphi=1. \qquad (6)$$

Consider one of the properties of generalized sinuses, which was proved in [3]:

$$\sin_\beta\gamma\cdot\sin_\gamma\alpha\cdot\sin_\alpha\beta=1. \qquad (7)$$

Substituting relations (7th ratio (6) after some transformations we obtain the equation (1). Theorem bring-no.

Using the theorem Chevy plane can be determined ABC by using generalized sinuses. As we know from [7], point the plane defined by the ABC following equation point:

$$M=(A-C)\frac{1}{1+u+uv}+(B-C)\frac{u}{1+v+uv}+C, \qquad (8)$$

Substitute values u and v the ratio (5) after some transformations we obtain the equation in terms M of plane ABC:

$$M=(A-C)\frac{1}{1+\sin_\beta\gamma\cdot\sin_{(\alpha-\tau)}\tau\left[1+\sin_\gamma\alpha\cdot\sin_{(\beta-\phi)}\phi\right]}+$$
$$+(B-C)\frac{\sin_\beta\gamma\cdot\sin_{(\alpha-\tau)}\tau}{1+\sin_\gamma\alpha\cdot\sin_{(\beta-\phi)}\phi\left[1+\sin_\beta\gamma\cdot\sin_{(\alpha-\tau)}\tau\right]}+C. \qquad (9)$$

Next, consider some of the implications of the proposed theorem.

Corollary 1. Is fair value is contrary to (1):

$$\sin_{(\alpha-\tau)}\tau\cdot\sin_{(\beta-\phi)}\phi\cdot\sin_{(\gamma-\varphi)}\varphi=1. \qquad (10)$$

Corollary 2. Given the properties of generalized trigonometric functions [3], expression (1) can be represented using generalized cosine:

$$\cos_{\tau}\alpha\cdot\cos_{\phi}\beta\cdot\cos_{\varphi}\gamma=1. \qquad (11)$$

Corollary 3. If through any point M that belongs to the plane of the triangle ABC (Fig. 3), to direct parallel to the sides of the triangle that form on the sides of the triangle points P_A, Q_A, P_B, Q_B, P_C і Q_C, then fair value is the following:

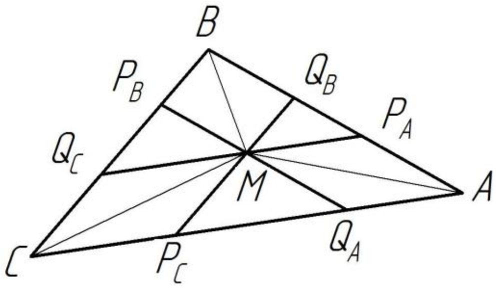

Рис. 3. Визначення точки M паралельними прямими.

$$P_AQ_AA\cdot P_BQ_BB\cdot P_CQ_CC=1, \qquad (12)$$

where $P_AQ_AA=\dfrac{P_AA}{AQ_A}$, $P_BQ_BB=\dfrac{P_BB}{BQ_B}$, $P_CQ_CC=\dfrac{P_CC}{CQ_C}$ – simple ratio of three points with a straight fracture [2] according to points A, B і C.

As shown in Fig. 1, direct parallel to the sides of the triangle formed by three parallelograms: P_AQ_AAM, P_BQ_BBM і P_CQ_CCM, and three triangle P_CQ_AM, P_AQ_BM і P_BQ_CM, are similar to each other and similar to the original triangle ABC. Taking this into account, the ratio of (10) can be represented as follows:

$$AMP_A\cdot BMP_B\cdot CMP_C=1. \qquad (13)$$

Value (12) and (13) are equivalent to the relations (1) and (11) only submitted without three of generalized functions honometrychnyh.

Corollary 4. Using equation (1) can move away from excessive parameterization and eliminate any of the corners φ, τ and ϕ defining point M in the plane ABC. For example, we define the angle φ and τ corners and using. Given the value (1), we obtain:

$$\frac{\left(\sin_{(\gamma-\varphi)}\varphi\cdot\sin_{(\alpha-\tau)}\tau+\cos\beta\right)^2}{\sin^2\beta}=\frac{1-\sin^2\phi}{\sin^2\phi},$$

$$\sin\phi=\pm\frac{\sin\beta}{\sqrt{1+2\cos\beta\cdot\sin_{(\gamma-\varphi)}\varphi\cdot\sin_{(\alpha-\tau)}\tau+\sin^2{}_{(\gamma-\varphi)}\varphi\cdot\sin^2{}_{(\alpha-\tau)}\tau}}. \qquad (14)$$

Similarly, you can determine the angles φ і τ.

Corollary 5. For a right triangle ABC with angle $\gamma=\dfrac{\pi}{2}$ fair value is the following:

$$\frac{\sin(\alpha-\tau)}{\sin\tau}\cdot\frac{\cos(\alpha+\phi)}{\sin\phi}\cdot tg\varphi=1. \qquad (15)$$

Similarly, you can get value for cases when other corners of the triangle are equal $\frac{\pi}{2}$.

Corollary 6. To equilateral triangle ABC expression (1) takes the following form:

$$\frac{\sqrt{3}\cos\tau-\sin\tau}{2\sin\tau}\cdot\frac{\sqrt{3}\cos\phi-\sin\phi}{2\sin\phi}\cdot\frac{\sqrt{3}\cos\varphi-\sin\varphi}{2\sin\varphi}=1. \qquad (16)$$

Conclusions. The article is formulated and proved the basic theorem of generalized trigonometric functions, and are the consequences of this theorem, which can significantly simplify radial and angular alignment parameters in simplex plane to go from one to another parameterization.

Literature

1. *Конопацький Є.В.* Бікутова та бірадіальна параметризації в площині загального положення / Є.В. Конопацький // Прикладна геометрія та інженерна графіка: міжвідомчий науково-технічний збірник. – Вип. 90. – К.: КНУБА, 2012. – С.156-160.
2. Геометричний сенс узагальнених тригонометричних функцій. / І.Г. Балюба, В.М. Верещага, Є.В. Конопацький [та ін.] / Праці Таврійського державного агротехнологічного університету. – Вип. 4. Прикладна геометрія та інженерна графіка. – Т. 55. – Мелітополь: ТДАТУ, 2012. – С.42-47.
3. *Конопацький Є.В.* Дослідження властивостей узагальнених тригонометричних функцій / Є.В. Конопацький // Праці Таврійського державного агротехнологічного університету. – Вип. 4. Прикладна геометрія та інженерна графіка. – Т. 56. – Мелітополь: ТДАТУ, 2013. – С. 263-267.
4. *Конопацький Є.В.* Використання узагальнених тригонометричних функцій для визначення плоских кривих / Є.В.Конопацький , І.Г. Балюба, В.М. Верещага // Праці Таврійського державного агротехнологічного університету. – Вип. 4. Прикладна геометрія та інженерна графіка. – Т. 57. – Мелітополь: ТДАТУ, 2013. – С.119-124.
5. *Конопацький Є.В.* Конструювання системи спеціальних плоских кривих типу «синусоїда» методом узагальнених тригонометричних функцій / Є.В. Конопацький / Сборник научных трудов SWorld. – Вып. 3. – Том 12. – Иваново: МАРКОВА АД, 2013. – С.76-80.

6. *Konopatskiy E.* Parameterization planes in BN-calculation / E. Konopatskiy / International Conference «Technical sciences: modern issued and development prospects»: Conference Proceedings. – Sheffield: Scope Academic House, 2013. – Vol. 10. – P. 183-187.
7. *Балюба И.Г.* Конструктивная геометрия многообразий в точечном исчислении: дисс. д-ра техн. наук: 05.01.01 / Балюба Иван Григорьевич. – Макеевка: МИСИ, 1995. – 227 с.