TURN OF IMAGES ON SURFACES WHICH ARE REFERRED TO SPACE ISOMETRIC WEBS

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***Summary.* It is considered variants of turn of the image on a surface which is referred to an isometric web. The analytical model of turn of the web on surfaces, and also image turn in relation to a web is created. Examples of images on a surface of an orb before turn and after turn are directed.**

***Keywords:* dimensional isometric grid, flat image, internal equation, rotate the grid image rotation.**

Formulation of the problem. Orthogonal coordinate grid lines on the surface, particularly on the plane, which divides it into small squares is infinite, is called isometric. This particular case is a grid mesh Cartesian coordinate system formed by the intersection of two families of coordinate straight lines. Isometric grid can be spatial, ie, the surface can be attributed to isometric coordinates. However, not all surfaces can be so described. In [1] the construction of surfaces of revolution referred to isometric grid coordinate lines. Based on mathematical correspondence between isometric grid cells on the surface and the Cartesian plane can be conformally flat display surface image [2]. In the article the ability to rotate the image on the surface.

Analysis of recent research.. In work [2] shows the transition from a rectangular grid polar coordinate system to the appropriate isometric grid. Showing straight lines and curves in this isometric grid and construction of these patterns are considered in the work [4]. Conformal mappings of plane figures (inscriptions) on isometric spatial mesh cone and balls shown at work [2].

Formulation of Article purposes. Develop analytical model rotate the image on the surface, referred to isometric grid coordinate lines.

Main part. Flat Cartesian coordinate system is isometric. Its parametric equations are of the form:

$$X = u; \quad Y = v, \tag{1}$$

where u, v – independent variables.

First fundamental form of isothermal mesh characterized in that it includes a component $du^2 + dv^2$, which can be multiplied by a certain factor dependent variables u and v. For the grid (1) this ratio is unity, ie the first fundamental form looks $dS^2 = du^2 + dv^2$.

If dependent variables *u* and *v* link together through a third variable, for example, *t*, then two equations $u=u(t)$ i $v=v(t)$, called internal line to describe an isometric grid. For example, ask the internal equation in the form:

$$u = at + u_0; \qquad v = b\sin t + v_0, \tag{2}$$

where *a, b* – constant, which influence the sinewave;

u_0, v_0 – constant, which influence on location of sinewave.

When substituting (2) in (1) we get sine wave (Fig. 1,a).

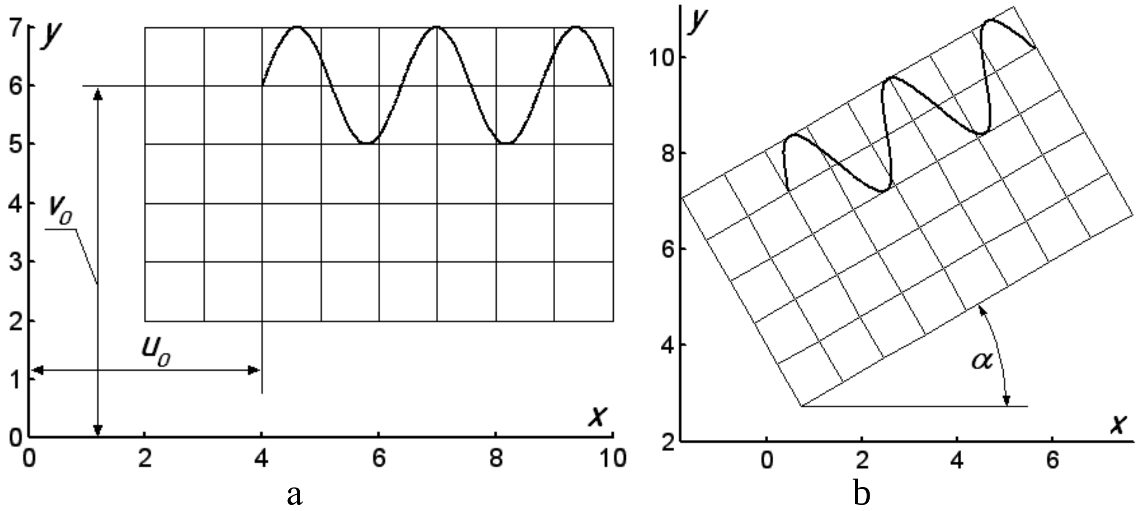


Fig.1. Flat Cartesian coordinate system with a sine wave, attributed to the isometric grid: a) without rotation; b) at an angle of turn α.

Cartesian grid will return to the isometric angle α. We use well-known formula of rotation, which *u*-lines and *v*- line will be returned to the angle α. After this turn Cartesian isometric grid (1) written:

$$\begin{aligned} X &= u\cos\alpha - v\sin\alpha; \\ Y &= u\sin\alpha + v\cos\alpha. \end{aligned} \tag{3}$$

When substituted internal sine wave equation (Equation second grid (3) we get the image shown in Fig. 1b, ie isometric grid and built against her will be returned to the sine wave angle α. With $\alpha=0$ Equation (3) are transformed into the equation (1).

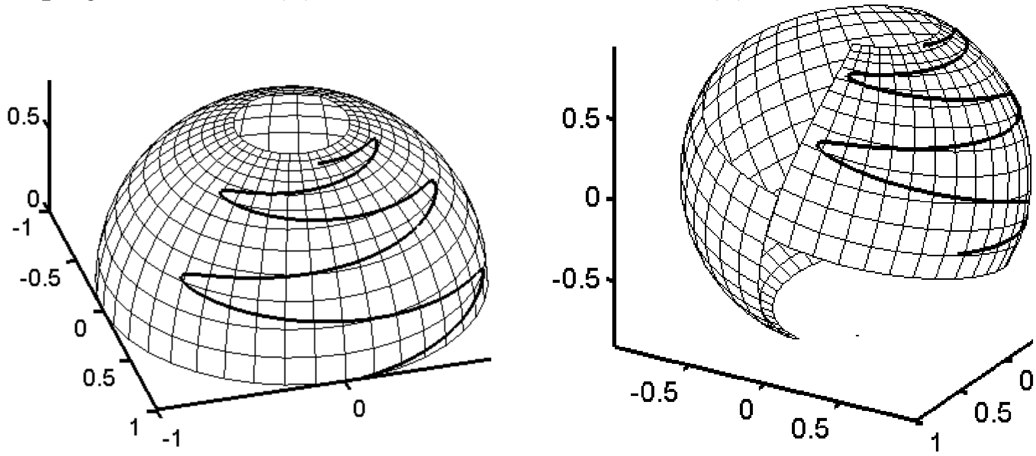
Consider the isometric grid on the surface of a sphere of radius. Its parametric equation in this case, the first fundamental form and have the form [2]:

$$\begin{aligned} X &= \operatorname{sech} u \cos v; \\ Y &= \operatorname{sech} u \sin v; \qquad dS^2 = \operatorname{sech}^2 u\left(du^2 + dv^2\right). \\ Z &= \tanh u; \end{aligned} \tag{4}$$

Constituents let's turn coordinate lines *u* and *v* at angle α on formulas (3). In this case, the equation balls (4) and its first fundamental form shall Appearance:

$$X = \mathrm{sech}(u\cos\alpha - v\sin\alpha)\cos(u\sin\alpha + v\cos\alpha);$$
$$Y = \mathrm{sech}(u\cos\alpha - v\sin\alpha)\sin(u\sin\alpha + v\cos\alpha); \qquad (5)$$
$$Z = \tanh(u\cos\alpha - v\sin\alpha).$$
$$dS^2 = \mathrm{sech}^2(u\cos\alpha - v\sin\alpha)(du^2 + dv^2). \qquad (6)$$

With $\alpha=0$ Equation (5) and the first fundamental form (6) are converted into the corresponding expression given in (4). When substituted internal sine wave equations (2) bullet equation (4) we get conformal mapping sine wave on the surface of the ball (Fig. 2a). Fig. 2, b built to display sine wave (2) on the surface of the ball (5) in $\alpha=15^0$.



a b

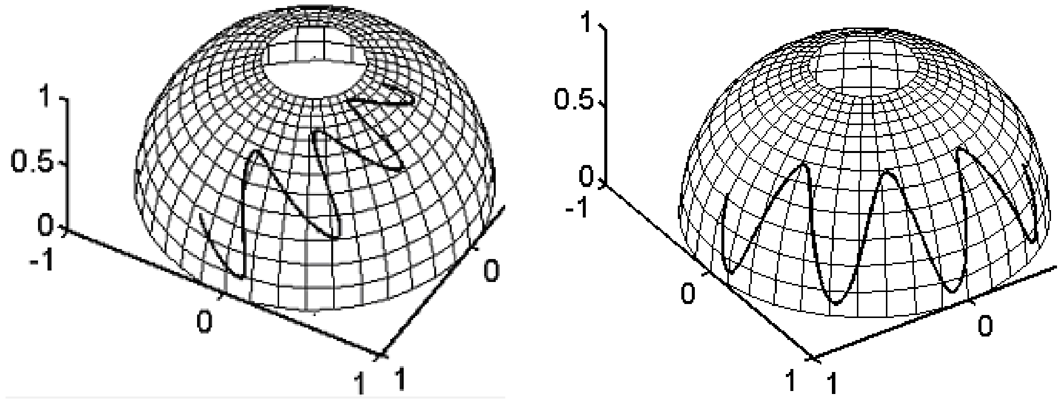
Fig. 2. isometric grid balls (5) of referred to it sinusoid (2):
a) without rotation ($\alpha=0$); б) with rotation by $\alpha=\pi/24$.

As shown in Fig. 2, would net when turning at an angle α deformed: its coordinate line, were flat parallels and meridians are transformed in the space. But it remains an isometric grid, as evidenced by the first fundamental form (6).

In addition to turning the grid relative to the surface, you can rotate the image itself towards the net. We write the internal sine wave equation (2) to turn it into an angle β:

$$u = (at + u_0)\cos\beta - (b\sin t + v_0)\sin\beta;$$
$$v = (at + u_0)\sin\beta + (b\sin t + v_0)\cos\beta. \qquad (7)$$

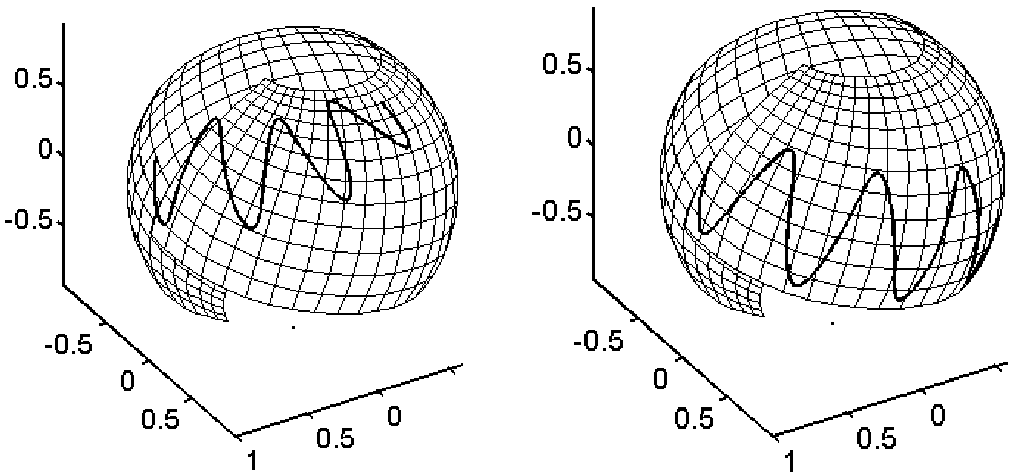
If the internal sine wave equation (7) into the equation nets (4) or (5), it will be turned towards her at an angle β. Sketch. 3 built spherical isometric grid equations (5) at $\alpha = 0$, i.e. no rotation, and marked sinusoid (7) with a different angle β.



a b

Fig. 3 Isometric grid balls (5) at α = 0 with the image on it sinusoid(7):

a) $\beta=\pi/3;$ b) $\beta=\pi/2$.



a b

Fig. 4. isometric grid balls (5) at $\alpha=\pi/24$ with the image on it sinusoid(7):

a) $\beta=\pi/3;$ b) $\beta=\pi/2$.

Fig. 4 made a double turn: net turned at an angle $\alpha=\pi/24$ and sine wave at different angles relative to mesh.

Consider a spherical isometric grid, which turns flat Cartesian grid using inversion. Its parametric equations before and after the turn angle α to be written:

$$X=\frac{u}{u^2+v^2+1}; \qquad X=\frac{u\cos\alpha - v\sin\alpha}{u^2+v^2+1};$$

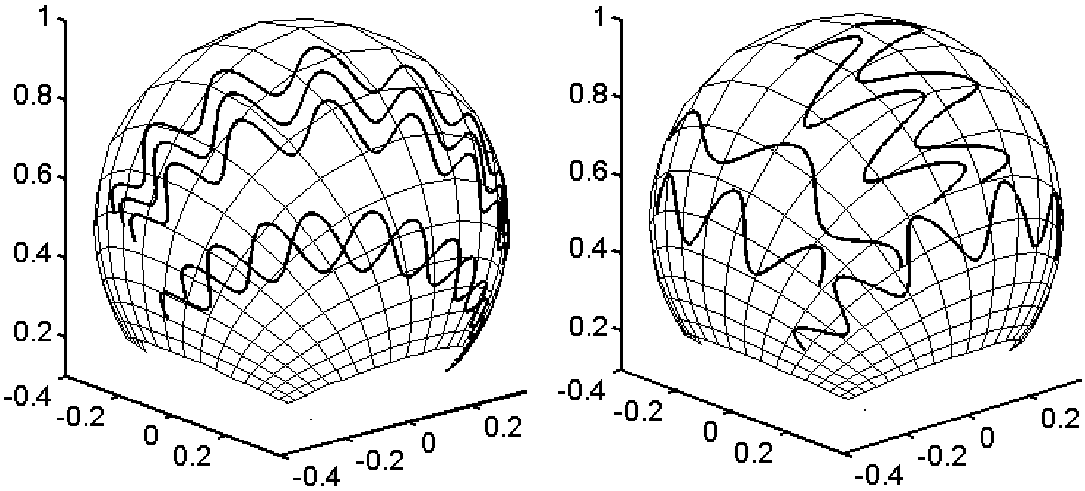
$$Y=\frac{v}{u^2+v^2+1}; \qquad Y=\frac{u\sin\alpha + v\cos\alpha}{u^2+v^2+1}; \qquad (8)$$

$$Z=\frac{1}{u^2+v^2+1}. \qquad Z=\frac{1}{u^2+v^2+1}.$$

First fundamental form surfaces (8) does not depend on the angle of rotation α and has the form:

$$dS^2 = \frac{du^2 + dv^2}{\left(u^2 + v^2 + 1\right)^2} . \quad (9)$$

Independence isometric grid (8) on the angle α means that does not deform when rotated at a given angle α: when you turn it slides on the balls around its vertical axis without deformation. So when you turn the grid does not make sense to use angle α, and image rotation can be achieved through the internal angle β his equations. Fig. 5 built sinusoid (7) with different angles of rotation β relative to the grid (8).

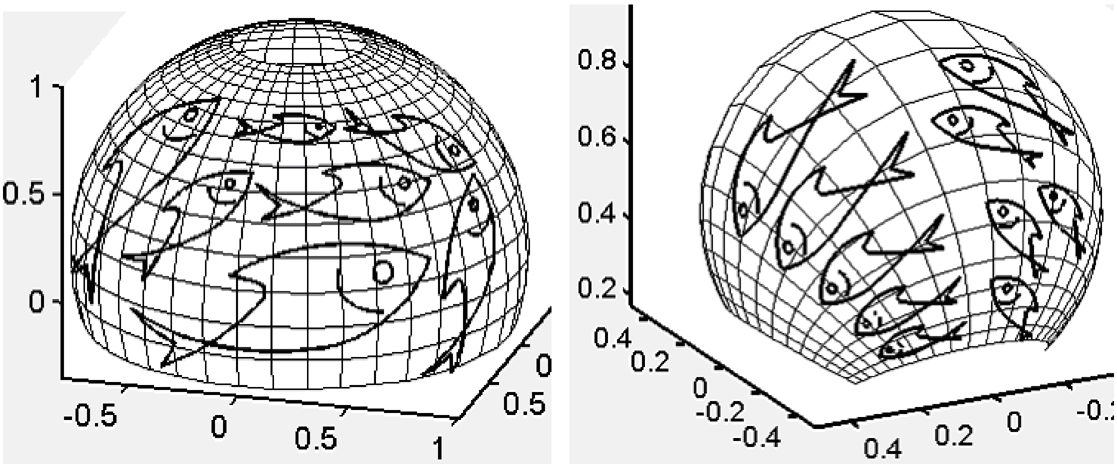


а б

Fig. 5. Isometric grid balls (8) showing her sinusoids (7) of varying amplitude and pitch:

a) $\beta=\pi/3$; b) $\beta=0$ і $\beta=\pi/2$.

Rotate isometric grid, and lines with respect to the grid can be used to rotate the image on the surface, which consists of a combination of individual lines. For example, you can put on the ball of fish image formed by conjugated arcs of circles, the location and size are known [5]:



a b

Fig. 6. Images of fish on the surface of the world:
a) ball grid attributed to isothermal (4);
b) attributed to ball grid isothermal (8).

Fig. 6 built on the surface fish balls, attributed to various isometric grids. Image wondered Each position on the grid angle relative thereto and scale factor which sets the image size.

Conclusions. The surface of the balls can be attributed to various isometric grids. The mesh can be rotated on the surface at a specific angle. There grid, which in turn then deformed, being isometric. Other grid at turn not deformed. Analytical they can be distinguished by the first fundamental form of the surface: in the first case it includes the angle of rotation, and the second - is not included. In both types of nets can be conformally flat image display, and they too can be rotated at a given angle on the surface of the ball but did not have respect to the surface and into the net.

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