SIMULATION OF A FLAT CURVE TO SPECIFIED LAW CURVATURE

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Summary. **Determination of polynomial segment is offered after the set two points and first derivatives and curvatures in these points.**

Keywords: **a polynomial segment, curvature, first and second derivatives.**

Formulation of the problem. In planning of contour of machines and aggregates that work in a movable environment (airplanes, cars and others like that) important is a task to contour on the set law of change of curvature. It is necessary to have an analytical vehicle of decision of this task.

Analysis of the recent researches. In works [2-6] the interactive methods of planning of contours are offered with the set form and curvature, but they do not give an opportunity to envisage results at the beginning of planning.

The wording of the purposes of the article. The aim of the article is a construction of analytical vehicle of design of curvilinear contour on the beforehand set law changes of curvature, that are important for planning of contours of airplanes, cars, ships, and others like that.

Main part. Will consider a next task:

Point row set on a plane :

$$\Delta : Xi,\ Yi,\ i=0,1,\ \dots,\ n,$$

and also in every point the set curvature K_i.

As known from [1], curvature for the curve of y=f (x) is determined by a formula:

$$K^2 = \frac{y''^2}{(1+y'^2)^3}\,. \tag{1}$$

As evidently from a formula (1) in an order to set curvature in the set point, it is necessary, at least, to set the first derivative of y_i'. At the set first derivative of y_i' and the size of flexon is determined curvature of K_i

$$y_i''^2 = K_i^2\,(1+y_i'^2)^3 \quad \text{або} \quad \pm y_i'' = \pm K(1+y_i'^2)^3. \tag{2}$$

The formulation of task:

A point row is set with the first and second derivatives in them:

$$\Delta : x_i,\ y_i,\ y_i',\ y_i'',\ i=0,\ 1,\ \dots,\ n.$$

Will design a curve that is built from the joined segments of polynomials on the areas of i÷ (i+1). Id est, it is necessary to find such polynomial curve that on the area of i÷ (i+1) passes through the points of i, (i+1) and has in these points the derivatives of y are set $y_i', y_i'', y_{i+1}', y_{i+1}''$.

First will prove a next theorem.

In order that a polynomial *segment* passed through two set points 0 and 1 and had the set first and second derivatives in these points, it is necessary, that a polynomial was not less 5th degree.

Such polynomial is determined by the system of 6-и of linear equalizations :

$y(x_0) = y_0;$
$y'(x_0) = y_0';$
$y''(x_0) = y_0'';$
$y(x_1) = y_1;$
$y'(x_1) = y_1';$
$y''(x_1) = y_1''.$

The system from 6-и of linear equalizations gets untied and has a decision only then, if amount of unknown not less than and not more than 6. A polynomial of y=f (*x*) has 6 coefficients only at 5th degrees.

A theorem is well-proven.

Thus will search the polynomial of 5th degree.

$$y = a + bx + cx^2 + dx^3 + ex^4 + fx^5. \tag{3}$$

Derivatives will equal :

$$y' = b + 2cx + 3dx^2 + 4ex^3 + 5fx^4 \; ; \tag{4}$$

$$y'' = 2c + 6dx + 12ex^2 + 20fx^3 \; . \tag{5}$$

Will put in (3), (4), (5) value of coordinates of points 0 (x_0, y_0) and 1 (x_1, y_1) and also derivatives in these points of y_0' , y_0'', y_1' , y_1'' . Will have the next system from 6-и of linear equalizations

$$\begin{bmatrix} 1 & x_0 & x_0^2 & x_0^3 & x_0^4 & x_0^5 \\ 0 & 1 & 2x_0 & 3x_0^2 & 4x_0^3 & 5x_0^4 \\ 0 & 0 & 2 & 6x_0 & 12x_0^2 & 25x_0^3 \\ 1 & x_1 & x_1^2 & x_1^3 & x_1^4 & x_1^5 \\ 0 & 1 & 2x_1 & 3x_1^2 & 4x_1^3 & 5x_1^4 \\ 0 & 0 & 2 & 6x_1 & 12x_1^2 & 25x_1^3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} y_0 \\ y_0' \\ y_0'' \\ y_1 \\ y_1' \\ y_1'' \end{bmatrix}. \tag{6}$$

The decision of this system will give the coefficients of a to us, b, c, d, e, f, that will define a polynomial (3) that satisfies to the put task.

Application to the polynomial not very comfortably, in addition it is necessary to decide the system of 6-и of linear equalizations.

Therefore for the decision of this task will take the segment of curve of Bezier of 5th degree [2].

$$r = r_0(1-t)^5 + 5r_1(1-t)^4 t + 10r_2(1-t)^3 t^2 + \\ 10r_3(1-t)^2 t^3 5r_4(1-t)t^4 + r_5 t^5 \; . \tag{7}$$

Will reconstruct (7) in a formula:

$$r = a + bt + ct^2 + dt^3 + et^4 + ft^5. \tag{8}$$

In the point of $r_0(x_0, y_0)$ t=0. Will get:

$$a = r_0; \quad b = 5(r_1 - r_0); \tag{9}$$
$$c = 10(r_0 - 2r_1 + r_2); \quad d = 10(r_3 + r_1 - r_0 - 3r_2); \tag{10}$$
$$e = 5(r_0 - 4r_1 + 6r_2 - 4r_3 - r_4)\ ; \tag{11}$$
$$f = (-r_0 + 5r_1 - 10r_2 + 10r_3 + 5r_4 + r_5)\ ; \tag{12}$$
$$r' = b = 5(r_1 - r_0); \tag{13}$$
$$r'' = 2c = 20(r_0 - 2r_1 + r_2). \tag{14}$$

At set r_0' and r_0'' from (13) and (14) will have:

$$r_1 = r_0 + \frac{r_0'}{5}\ ; \tag{15}$$
$$r_2 = 2r_1 - r_0 + \frac{r_0''}{20}\ . \tag{16}$$

In the point of r_5 (t=1) will have next equalizations:

$$r_4 = r_5 - \frac{r_5'}{5}\ ; \tag{17}$$
$$r_3 = r_5 + 2r_4 + \frac{r_5''}{20}. \tag{18}$$

Thus formulas (13) - (18) fully determine the segment of Bezier of 5th degree that passes through two points of r 0 and r 5 with the set first and second derivatives in them.

Conclusions. In the article the got result of the analytical planning of contour is on the set law of change of curvature. Further researches will be conducted in relation to development of methods of management the form of designed curvilinear contour.

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