# GEOMETRIC MODEL OF THE RULED SURFACE TILLAGER WORKING BODIES OF VARIABLE CURVATURE 

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#### Abstract

Summary. The paper deals with geometric model of a ruled surface, based on the assumption deployable, which allows to build the surface, the curvature of which can vary from zero to a certain value.


Keywords: geometry, surfaces, a model, a curvature, cultivation.
Formulation of the problem. Cultivating working bodies, especially of the plowshare-weeding type, change soil chunk during the work. Thus to meet the agro technological demands, such as the degree of plant residues serifs, soil crushing, surface of working body have to be certain curvature. If the surface curvature is equal to zero, the surface will be development, and the soil chunk will experience simple bending deformation. When the surface curvature is nonzero, plastic deformation will occur in the chunk, that will contribute ground crushing. Therefore, during the constructing of tillage plowshare-weeding working tools there is a need of geometrical surface models, which allow you to build a surface, which curvature can vary from zero to some value.

Analysis of recent research. In papers [2,6] technique of surfaces construction according to their spherical reflection, which for reamer surface looks like a line, but for non reamer - an area. The method needs at designing development surface to know the form spherical reflection of line, and for non reamer surface - spherical shape and area of the image. The necessity to know these quantities considerably reduces suitability of methods, as to build surfaces it is necessary to have reliable data on line of spherical image of development surface, or the area for non reamer. In the work [5] the construction of especially the development surfaces is provided, and in the work [7] the construction of surface curves along two guide rails is proposed, which are the trajectories of soil movement. The disadvantage in this case is the construction of surface curvature of which is uncertain.

Formulation of article purposes. The geometric a model is considered, which allows the design the ruled surface with curvature, which varies from zero to a specific value.

Main part. Among ruled surfaces reamer surface occupy a special place. These surfaces are deployed on a plane without folds. This provision ensured them differential-parametric properties:

[^0]- Gaussian curvature surface is equal to zero;
- normal to the surface does not change the direction while moving along the generating.

These properties of the surface make the working tools more technological in manufacturing, because they are less warp and have low traction resistance [1].

To develop the model surface, let us define the Cartesian coordinate system $O x y z$, so that the $O_{x}$ is directed oppositely movement of the working body, $O_{z}$ is perpendicular to the horizontal plane $O x y$. Then the oy will lie in a horizontal plane and will be perpendicular to the longitudinal vertical plane Oxz.

We set in the coordinate system the guide curve (Fig. 1) as:
$m$ :

$$
\begin{align*}
& x=x(u) \\
& y=y(u)  \tag{1}\\
& z=z(u)
\end{align*}
$$

where $u$ - any setting.


Fig. 1. Shame of building linear surface.

The generating (Fig. 1), which has with a guide curve (1) point of incidence $A\left(x_{A}, y_{A}, z_{A}\right)$, vector $\vec{l}$ is represented as follows: $\vec{l}\{a, b, c\}$, where $\mathrm{a}, \mathrm{b}, \mathrm{c}$ - parameters of the position vector generators.

In general, these parameters can have any value, and at the moment they are identified. Vector position in space can be set in two ways. If it will be generating angles to coordinate axes, then the coordinates $\vec{l}$ are as follows:

$$
\begin{equation*}
\vec{l}\left\{\frac{a}{c}, \frac{b}{c}, 1\right\}, \text { or for short } \vec{l}\{k, r, 1\} \tag{2}
\end{equation*}
$$

where $k=\operatorname{tg} \gamma, r=\operatorname{tg} \beta$.
The corners $\gamma$ are $\beta$ angles of generating inclination to the axis $O x$ on the plane $O x y$ and $O x z$ respectively [4].

Vector tangent $\vec{\tau}$ to the guide curve (1) is determined with coordinates:

$$
\begin{equation*}
\vec{\tau}\left\{x^{\prime}, y^{\prime}, z^{\prime}\right\} \tag{3}
\end{equation*}
$$

where $x^{\prime}, y^{\prime}, z^{\prime}$ - the first derivatives of functions (1) for the parameter $u$.
Normal for designing surface is defined as the vector product of vectors generating (2) and the tangent (3):

$$
\begin{equation*}
\vec{n}=\vec{l} \cdot \vec{\tau} \tag{4}
\end{equation*}
$$

We write the equation (4) coordinate form through individual orty $\vec{i}$, $\vec{j}, \vec{k}$, which directed along axes $O x, O y, O z$ :

$$
\left|\begin{array}{lll}
\vec{i} & \vec{j} & \vec{k}  \tag{5}\\
1 & k & r \\
x^{\prime} & y^{\prime} & z^{\prime}
\end{array}\right|=\vec{i}\left(k z^{\prime}-r y^{\prime}\right)-\vec{j}\left(z^{\prime}-r x^{\prime}\right)+\vec{k}\left(y^{\prime}-k x^{\prime}\right) \text {. }
$$

According to [3] set the normal vector to the surface in the form:

$$
\begin{equation*}
\vec{n}\{p, q-1\}, \tag{6}
\end{equation*}
$$

where $p$ and $q$ - the partial derivatives of the surface: $z=z(x, y)$ :

$$
p=\frac{\partial z}{\partial x}, q=\frac{\partial z}{\partial y} .
$$

To obtain coordinate of normal vector (6) divide the right side of the equation (5) $-\left(y^{\prime}-k x^{\prime}\right)$, as a result we obtain the following equation:

$$
-i \frac{k z^{\prime}-r y^{\prime}}{y^{\prime}-k x^{\prime}}+j \frac{z^{\prime}-r x^{\prime}}{y^{\prime}-k x^{\prime}}+\vec{k} .
$$

From here we will have dimension of partial derivatives:

$$
\begin{equation*}
p=-\frac{k z^{\prime}-r y^{\prime}}{y^{\prime}-k x^{\prime}}, \quad q=\frac{z^{\prime}-r x^{\prime}}{y^{\prime}-k x^{\prime}} . \tag{7}
\end{equation*}
$$

Reamer surfaces are one parameter, which are defined by the equation:

$$
F(x, y, z, a)=0,
$$

where $a$ - the surface option that distinguishes one specific surface of the whole set.

Condition reamer are expressed through $p$ and $q$, is as follows [3]:

$$
\begin{equation*}
a p^{\prime}+b q^{\prime}=0, \tag{8}
\end{equation*}
$$

where - the first derivatives of equation (7)
Differentiating $p$ and $q$ to the parameter, we have:

$$
\begin{gathered}
p^{\prime}=\frac{-1}{\left(a y^{\prime}-b x^{\prime}\right)^{2}}\left[\left(a y^{\prime}-b x^{\prime}\right)\left(b^{\prime} z^{\prime}+b z^{\prime \prime}-c^{\prime} y^{\prime}-c y^{\prime \prime}\right)-\left(b z^{\prime}-c y^{\prime}\right)\left(a^{\prime} y^{\prime}+a y^{\prime \prime}-b^{\prime} x^{\prime}-b x^{\prime \prime}\right)\right] \\
q^{\prime}=\frac{1}{\left(a y^{\prime}-b x^{\prime}\right)^{2}}\left[\left(a y^{\prime}-b x^{\prime}\right)\left(a z^{\prime}+a z^{\prime \prime}-c^{\prime} x^{\prime}-c x^{\prime \prime}\right)-\left(a z^{\prime}-c x^{\prime}\right)\left(a^{\prime} y^{\prime}+a y^{\prime \prime}-b^{\prime} x^{\prime}-b x^{\prime \prime}\right)\right]
\end{gathered}
$$

Substituting the obtained expressions in the condition reamereans (8), have differentive equation of position of generatrix:

$$
\begin{equation*}
c^{\prime}\left(a y^{\prime}-b x^{\prime}\right)+c\left(b^{\prime} x^{\prime}-a^{\prime} y^{\prime}\right)=\left(a b^{\prime}-a^{\prime} b\right) \cdot z^{\prime} \tag{9}
\end{equation*}
$$

With the known functions $a, b, x, y, z$, for example, we get the function $c$ and will have comprehensive information on the status of generating.

If you need to design surface, which curvature is nonzero, you need to right side (8) to enter the value $\lambda$ that will be analogous to curvature of the surface:

$$
a p^{\prime}+b q^{\prime}=\lambda
$$

Substituting the values and conditions reamereans (8), after conversion in this case we obtain a differential equation:

$$
\begin{equation*}
c^{\prime}\left(a y^{\prime}-b x^{\prime}\right)+c\left(b^{\prime} x^{\prime}-a^{\prime} y^{\prime}\right)-z^{\prime} \cdot\left(a b^{\prime}-a^{\prime} b\right)=\lambda \cdot\left(a y^{\prime}-b x^{\prime}\right) \tag{10}
\end{equation*}
$$

If as a parameter $u$ you take a coordinate $x$, the equation (9) and (10) will be simplificated

$$
\begin{gather*}
c^{\prime}\left(a y^{\prime}-b\right)+c\left(b^{\prime}-a^{\prime} y^{\prime}\right)=\left(a b^{\prime}-a^{\prime} b\right) \cdot z^{\prime}  \tag{11}\\
c^{\prime}\left(a y^{\prime}-b\right)+c\left(b^{\prime}-a^{\prime} y^{\prime}\right)-z^{\prime} \cdot\left(a b^{\prime}-a^{\prime} b\right)=\lambda \cdot\left(a y^{\prime}-b\right)
\end{gather*}
$$

These equations combine two functions tangents projections generatrix angles of inclination to the horizontal Oxy and longitudinally vertical plane $O y z$ of projection. Thus, if the function of slope angle tangent to the horizontal plane $O x y$ of projections $k=\operatorname{tg} \gamma(x)$ is given, (11) we obtain the function slope angle tangent generating to the longitudinally vertical plane $O y z$ of projection $r=\operatorname{tg} \beta(x)$. In this case $a=1$, as $k=\frac{b}{a}, r=\frac{c}{a}$, a $y=y(x), z=z(x)$.

In this case, the equation (11) will have the following form:

$$
\begin{equation*}
r^{\prime}\left(y^{\prime}-k\right)+r\left(k^{\prime}-y^{\prime}\right)=\left(k^{\prime}-k\right) \cdot z^{\prime} \tag{12}
\end{equation*}
$$

If the guiding curve is convex relative to the axis $O z$, then as a parameter $u$ necessary take the coordinate $z$. Then the equation of the guide will look like:

$$
m: \quad x=x(z), \quad y=y(z)
$$

This function of the inclination generating angle will be also function coordinates $z$ :

$$
k=F_{1}[\operatorname{tg} \gamma(z)], \quad r=F_{2}[\operatorname{tg} \beta(z)]
$$

Frame surface of the body we write the equation of the lines, that pass through the incidence point $A\left(x_{A}, y_{A}, z_{A}\right)$ in a given as direction:

- on a horizontal plane projections $O_{x y}$ :

$$
y=k\left(x-x_{A}\right)+y_{A}
$$

- on longitudinal vertical plane projections $O_{x z}$ :

$$
x=r\left(z-z_{A}\right)+x_{A}
$$

- on transverse vertical plane projections $O_{y z}$ :

$$
y=t\left(z-z_{A}\right)+y_{A}
$$

Where $x_{A}, y_{A}, z_{A}$-coordinates of incidence $A\left(x_{A}, y_{A}, z_{A}\right)$;
$t=F_{3}[\operatorname{tg} \delta(z)]$ function generating slope angle on the transverse vertical plane projections $O_{y z}$.

Knowing the generating inclination angles $\gamma$ and $\beta$, spherical trigonometry formulas obtain the following formula for the angle of generating inclination on the transverse vertical plane of projections $O_{y z}$ :

$$
\delta=\arccos (\cos \gamma \cdot \cos \beta)
$$

Conclusions. The developed model allows geometric design linear surface as reamereans, which Gaussian curvature is equal to zero, and the surface, Gaussian curvature which is different from zero. Thus, the model allows you to design working tools that have an effect on the ground, from a simple bending to plastic deformation.

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