THE GEOMETRIC MODEL OF OSCILLATORY PROCESSES IN A TWO-DIMENSIONAL GRID OF MATHEMATICAL PENDULUM

V. Eremeev, V. Kuzminov

Summary. **The geometric model for the study of oscillatory phenomena in two-dimensional lattice on the basis of mathematical pendulums of different mass with arbitrary initial and boundary conditions is proposed. A program for the calculation and visualization of vibrations of individual nodes is developed.**

Keywords: **oscillation, pendulum, geometric model, resonance phenomena.**

Formulation of the problem. Nature of oscillatory phenomena is studied quite well [4]. They accompany all mechanical constructions, where there is a parts rotation of machines, engines, aircraft and ships, and so on. Different parts of the mechanical system or the whole system can come into resonance with forcing power. The phenomenon of resonance can cause destruction of machines, buildings, bridges and other structures. Therefore, the study of oscillatory processes in complex load conditions has great practical interest.

Analysis of recent research. Analytical solutions of the vibrations of many simple models are obtained: pendulum, case an harmonic oscillations, vibrations with one and two degrees of freedom at presence of friction or external forces and so on. Using numerical methods [1] allows us to solve more complex problems with many degrees of freedom. In the paper [2] mathematical model of oscillations in a two-dimensional system with fixed limits is proposed. This work is its extension. It is dedicated to the mechanical vibrations modeling of two-dimensional rectangular grid with random initial and boundary conditions.

Formulation of article purposes. Development of a model for the analysis of two-dimensional rectangular grid fluctuations at arbitrary initial and boundary conditions.

Main part. Let us consider oscillating mechanical system, which is a flat uniform rectangular grid $\{M_{i,j}(ih_g, jh_v) : i = \overline{0, m+1}, j = \overline{0, n+1}\}$ (Figure 1). In grid nodes $\{M_{i,j}(ih_g, jh_v) : i = \overline{1, m}, j = \overline{1, n}\}$ are masses balls $\{m_{i,j} : i = \overline{1, m}, j = \overline{1, n}\}$. Between each pair of points $\{M_{i-1,j}, M_{i,j}\}$, i = 1 ... m + 1, j = 1 ... n is the connection (spring), which prevents deformation of compression / tensile stiffness of; between each pair of points $\{M_{i,j-1}, M_{i,j}\}$, i = 1 ... m, j = 1 ... n + 1 is a connection (spring) with stiffness.

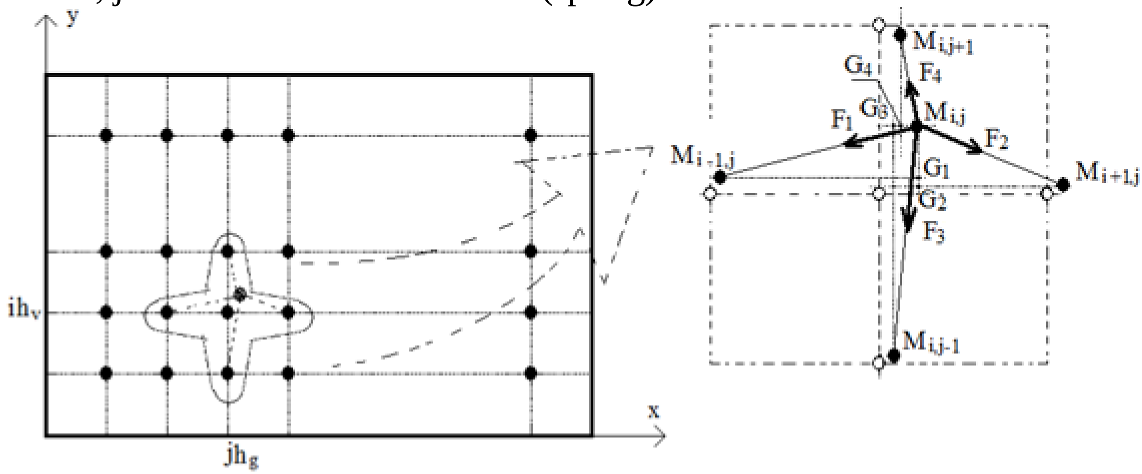

Fig.1. Arrangement of balls in the two dimensional lattice

Limit grid point range $Gr = \{M_{i,j} : i \in \{0, m+1\}, j = \overline{1,n} \vee i = \overline{1,m}, j \in \{0, n+1\}\}$ with predetermined frequencies and amplitudes. During each ball oscillation the force of friction is proportional to the speed of the ball, that is $\overline{F_{mpi,j}} = -\mu \overline{v_{i,j}}$.

As coordinates and projection speed of each ball we assume *{(j* h_g *+* $x_{i,j}$ *(t), i* h_v *+* y_i*, j (t)): i = 1 ... m, j = 1 ... n)}, {(*$v_{xi,j}(t)$*,* $v_{yi,j}(t)$*): i = 1 ... m, j = 1 ... n)}*. Initial rejection of all balls *{(*$x_{i,j}(0) = x_{i,j0}$*,* $y_{i,j}(0) = y_i, j0$*): i = 1 ... m, j = 1 ... n)}*, and their initial velocity *{(*$v_{xi,j}(0) = v_{xi,j0}$*,* $v_{yi,j}(0) = = v_{yi,j0}$*): i = 1 ... m, j = 1 ... n)}* are known. It is necessary according to given accuracy to get balls in each interval of time $t \in [0; \tau]$ laws shift coordinates of the corresponding point of balance and speed, that is the function *{(*$x_{i,j}(t), y_{i,j}(t), v_{xi,j}(t), v_{yi,j}(t)$*): i=1…m, j=1…n}*.

We denote the forces acting on some balls (*ij*) of the links which connect this ball with other balls *(i-1, j), (i + 1, j), (i, j-1), (i, j + 1)* according $F_{1i,j}$, $F_{2i,j}$, $F_{3i,j}$, $F_{4i,j}$. Their meaning:

$$F_{1i,j} = k^g_{i,j}(s^{(-1,0)}_{i,j} - h_g), \quad F_{2i,j} = k^g_{i+1,j}(s^{(+1,0)}_{i,j} - h_g),$$

$$F_{3i,j} = k^v_{i,j}(s^{(0,-1)}_{i,j} - h_v), \quad F_{4i,j} = k^v_{i,j+1}(s^{(0,+1)}_{i,j} - h_v),$$

where

$$s^{(-1,0)}_{i,j} = \sqrt{(x_{i-1,j} - x_{i,j} - h_g)^2 + (y_{i-1,j} - y_{i,j})^2},$$

$$s^{(+1,0)}_{i,j} = \sqrt{(x_{i+1,j} - x_{i,j} + h_g)^2 + (y_{i+1,j} - y_{i,j})^2}, \qquad (1)$$

$$s^{(0,-1)}_{i,j} = \sqrt{(x_{i,j-1} - x_{i,j})^2 + (y_{i,j-1} - y_{i,j} - h_v)^2},$$

$$s_{i,j}^{(0,+1)} = \sqrt{(x_{i,j+1} - x_{i,j})^2 + (y_{i+1,j} - y_{i,j} + h_v)^2}.$$

Instantaneous length corresponding relations (Pythagoras theorem for triangles in $M_{i-1j}G_1M_{ij}$, $M_{i+1j}G_2M_{ij}$, $M_{ij-1}G_3M_{ij}$, $M_{ij+1}G_4M_{ij}$, *Figure 1)*. Projections of these forces on the coordinates *OX* and *OY* are equal.

$$F_{1xi,j} = \mathrm{F}_{1i,j}\frac{G_1M_{i-1,j}}{M_{i-1,j}M_{i,j}} = F_{1i,j}\frac{x_{i-1,j} - x_{i,j} - h_g}{s_{i,j}^{(-1,0)}} = k_{i,j}^g(s_{i,j}^{(-1,0)} - h_g)\frac{x_{i-1,j} - x_{i,j} - h_g}{s_{i,j}^{(-1,0)}},$$

$$F_{2xi,j} = F_{2i,j}\frac{x_{i+1,j} - x_{i,j} + h_g}{s_{i,j}^{(+1,0)}} = k_{i+1,j}^g(s_{i,j}^{(+1,0)} - h_g)\frac{x_{i+1,j} - x_{i,j} + h_g}{s_{i,j}^{(+1,0)}},$$

$$F_{3xi,j} = k_{i,j}^v(s_{i,j}^{(0,-1)} - h_v)\frac{x_{i,j-1} - x_{i,j}}{s_{i,j}^{(0,-1)}}, F_{4xi,j} = k_{i,j+1}^v(s_{i,j}^{(0,+1)} - h_v)\frac{x_{i,j+1} - x_{i,j}}{s_{i,j}^{(0,+1)}},$$

$$F_{1yi,j} = k_{i,j}^g(s_{i,j}^{(-1,0)} - h_g)\frac{y_{i-1,j} - y_{i,j}}{s_{i,j}^{(-1,0)}}, F_{2yi,j} = k_{i+1,j}^g(s_{i,j}^{(+1,0)} - h_g)\frac{y_{i+1,j} - y_{i,j}}{s_{i,j}^{(+1,0)}},$$

$$F_{3yi,j} = k_{i,j}^v(s_{i,j}^{(0,-1)} - h_v)\frac{y_{i,j-1} - y_{i,j} - h_v}{s_{i,j}^{(0,-1)}}, F_{4yi,j} = k_{i,j+1}^v(s_{i,j}^{(0,+1)} - h_v)\frac{y_{i,j+1} - y_{i,j} + h_v}{s_{i,j}^{(0,+1)}}.$$

In addition, the dissipation power acts on each bead from the environment (friction) which the projection axis coordinate the following:

$$F_{mpxi,j} = -\mu v_{xi,j},\ F_{mpyi,j} = -\mu v_{yi,j}.$$

Newton's second law of motion for ball can be written as:

$$m_{i,j}\dot{v}_{xi,j} = F_{1xi,j} + F_{2xi,j} + F_{3xi,j} + F_{4xi,j} + F_{mpxi,j},$$

$$m_{i,j}\dot{v}_{yi,j} = F_{1yi,j} + F_{2yi,j} + F_{3yi,j} + F_{4yi,j} + F_{mpyi,j},$$

where the point over the letter denotes the single differentiation according change of time.

Algorithm for solving systems of differential equations is considered in the papers [1,3]. For the study oscillatory processes is established program on the algorithmic language Delphi [4]. As an example we will consider oscillatory process in the lattice of nodes 11x11. The calculations assumed that if on the boundaries x = 0 and x = 10, there are forces of sinusoidal nature, which amplitude equal to 0.1. The dependence of the oscillation amplitude node in the fifth row and fifth column are shown in Fig. 2 and 3.

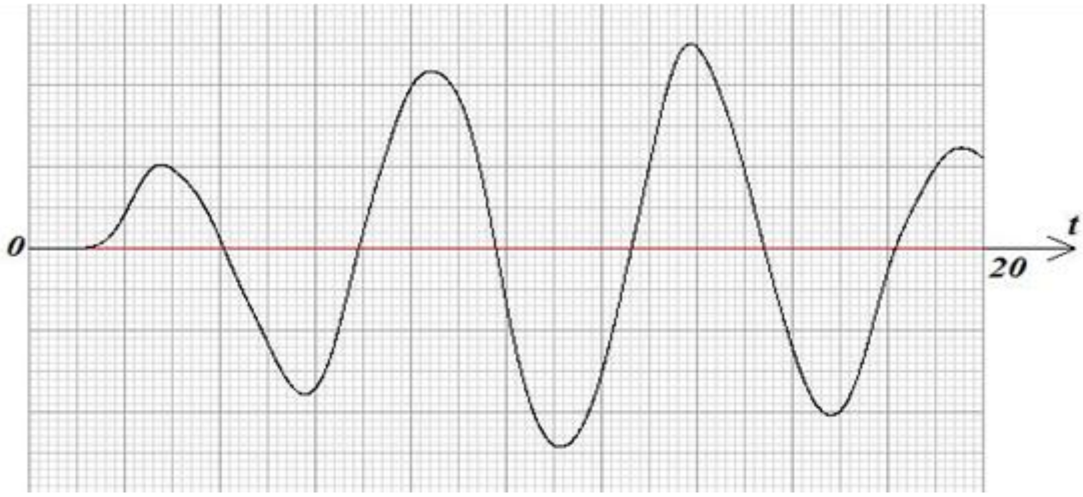

Fig. 2. Dependence of the fluctuations node amplitude in the fifth row and fifth column (oscillation frequency on the brink 0.2).

Fig. 2 shows that the nature node fluctuations differ from harmonious - amplitude and frequency change over time, the amplitude in some cases almost three times exceeds the amplitude, which was set on the edge.

Numerous experiments have shown that oscillatory process in the system to a great extent depends on the frequency. Fig. 3 shows the time dependence of the amplitude of the same node with little change in frequency from 0.2 to 0.25.

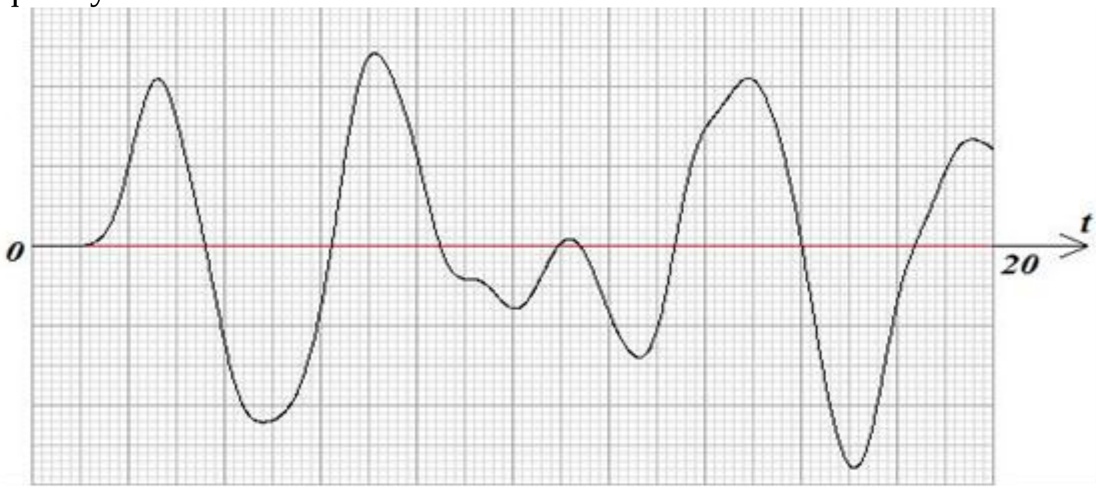

Fig. 3. The dependence of the oscillation amplitude node in the fifth row and fifth column (oscillation frequency on the brink 0.25).

If the oscillation frequency, which is set on the edge, approaches the natural frequency of the system, there are resonant phenomena - amplitude in some nodes unlimited increases.

Conclusion. Geometric model is developed for the analysis of relaxation phenomena during mechanical vibrations of two-dimensional uniform rectangular grid of varying hardness and with different masses concentrations in the grid at arbitrary initial and boundary conditions.

Literature

1. *Єремєєв В.С.* Фазовий портрет коливальних процесів математичних маятників у двовимірній гратці / В.С.Єремєєв, В.В.Кузьминов // Праці ТДАТУ. Геометричне моделювання і інформаційні технології проектування. Мелітополь. Випуск 4. – Т. 54. - 2012. С. 48-57.
2. *Савельев И. В.* Курс общей физики, том I. Механика, колебания и волны, молекулярная фізика. – М.:Наука, 1970. – 504с.
3. Понтрягин Л.С. Обыкновенные дифференциальные уравнения / Л.С. Понтрягин. – М.:Наука, 1974 – 329 с.
4. *Фленов М.Е.* Библия Delphi / М.Е. Фленов– СПб.: БХВ-Петербург, 2004. – 880 с.
5. Явление резонанса. [Електронний ресурс] Режим доступу: http://xreferat.ru/102/498-1-yavlenie-rezonansa.html.