THE THIRD DEGREE POLYNOMIAL SPLINE WITH THE OPERATING POINTS INCIDENTAL TO A CURVE

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***Summary*. The paper proposes a method for calculating the cubic curve segment with control points incident the curve, and is an example of a test.**

***Keywords:* polynomial in the form of Lagrange, polynomial curve segment, control point, the point that "the incident to the curve".**

Formulation of the problem. For modeling smooth curves, surfaces and bodies in the construction engineering units widely used is way of presenting vector-parametric curves in the form Ferguson and Bernstein-Bezier or with help of rational vector-parametric functions [1]. And when there is a need to work only with the points on the curve, these methods are not entirely comfortable for use, as the frame mid-point in the shape of Bernstein-Bezier doesn’t lie on primary given point and in the final frame is not in the form of the original curve. There are times when it is not at all easy to work with curves in the Ferguson’s form, because of the need to calculate derivatives in nodal points. This is especially concerns free-form deformation method (the so-called FFD - Free Form Deformation [2]).

Analysis of recent research. Quite a large class of lines can be constructed on set points. These lines are called point-set. This broken line and different splines, cubic spline, spline in the form of Eremite, spline based on the polynomial Lagrange, in the form of Newton and others. In general, the problem reduces to constructing of interpolation curve. In the papers [1-3] the development of existing algorithms for constructing smooth contours is amply described. The article contains algorithm construction segments cubic curve with control points, that are incident (belonging) curve using polynomial interpolation by Lagrange, which is an improvement of the existing machine geometric modeling.

The wording of the purposes of the article. You can offer presentation vector-parametric curve in this form, at which point frame will belong to the primary point series and lie on the initial curve, and conduct research of properties such filing. For the study of this method we apply Lagrange formula for polynomial interpolation.

Main part. Let us consider polynomial curve of n-th degree with control points, which belong to the curve.

Take the $N + 1$ points: $0(x_0,y_0)$, $1(x_1,y_1),\ldots,$ $N(x_N,y_N)$. To obtain polynomial formulas apply for Lagrange polynomial interpolation [3] in dependence $y = y\ (x)$.

Assign the parameter $u = (x\text{-}x_0)/(x_N\text{-}x_0)$ (Fig. 1). get:

$$\begin{aligned} y &= y_0 \frac{(u-u_1)(u-u_2)\ldots(u-u_i)\ldots(u-u_N)}{(u_0-u_1)(u_0-u_2)\ldots(u_0-u_i)\ldots(u_0-u_N)} + \\ &+ y_1 \frac{(u-u_0)(u-u_2)\ldots(u-u_i)\ldots(u-u_N)}{(u_1-u_0)(u_1-u_2)\ldots(u_1-u_i)\ldots(u_1-u_N)} + \\ &+ \ldots + y_N \frac{(u-u_0)(u-u_1)\ldots(u-u_{(N-1)})}{(u_N-u_0)(u_N-u_1)\ldots(u_N-u_{(N-1)})} = \\ &= \sum_{i=0}^{N} [y_i \prod_{\substack{j=0 \\ j\neq i}}^{N} \frac{(u-u_j)}{(u_i-u_j)}]. \end{aligned} \tag{1}$$

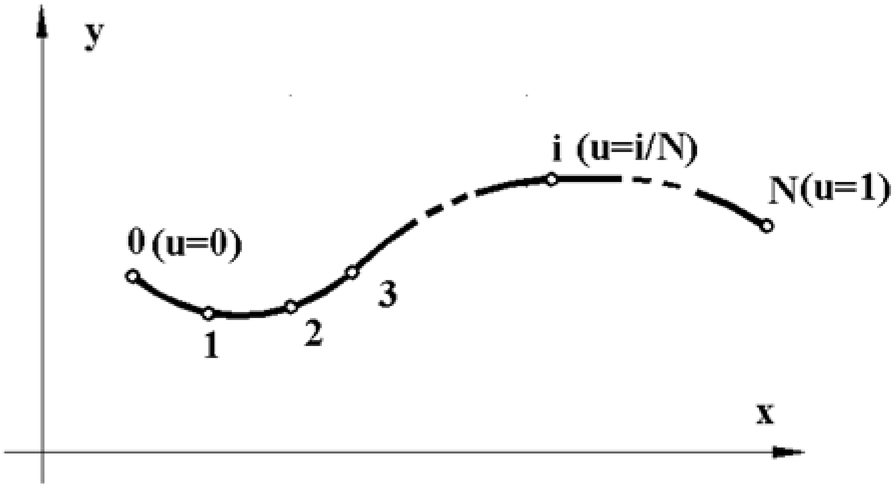


Fig. 1. The segment polynomial curve of n-th degree

If you take the uniform arrangement of points, then $u_0 = 0$, $u_N = 1$, $u_i = i/N$, and the formula (1) will look like:

$$y = \sum_{i=0}^{N} [y_i \prod_{\substack{j=0 \\ j\neq i}}^{N} \frac{(u - j/N)}{(i/N - j/N)}] = \sum_{i=0}^{N} [y_i \prod_{\substack{j=0 \\ j\neq i}}^{N} \frac{(Nu - j)}{(i - j)}]. \tag{2}$$

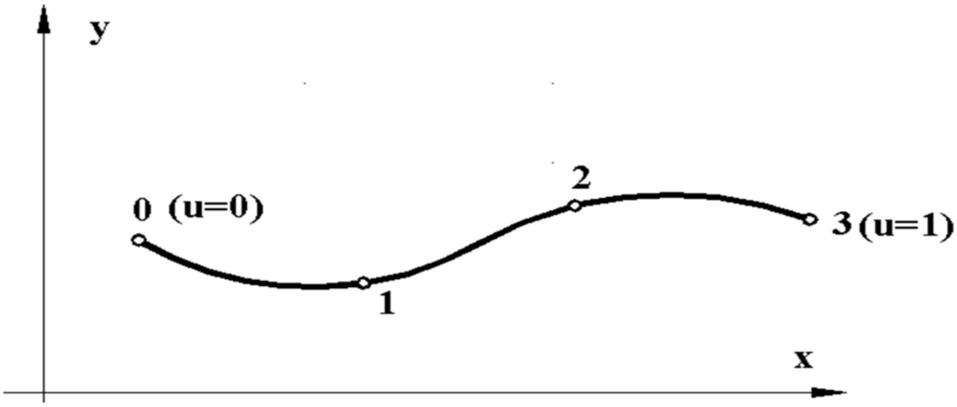
Thus formula (1) and (2) determine the polynomial curve for pointsб which belong to the curve.

Let us consider polynomial curve of the third degree with control points, which belong to the curve.

Take four points *0 (x0, y0), 1 (x1, y1), m2 (x2, y2), 3 (x3, y3)*. To obtain cubic formula we apply (1).

We record segment polynomial curve:

$$y = y_0 \frac{(u-u_1)(u-u_2)(u-u_3)}{(u_0-u_1)(u_0-u_2)(u_0-u_3)} + y_1 \frac{(u-u_0)(u-u_2)(u-u_3)}{(u_1-u_0)(u_1-u_2)(u_1-u_3)} +$$
$$+y_2 \frac{(u-u_0)(u-u_1)(u-u_3)}{(u_2-u_0)(u_2-u_1)(u_2-u_3)} + y_3 \frac{(u-u_0)(u-u_1)(u-u_2)}{(u_3-u_0)(u_3-u_1)(u_3-u_2)}. \quad (3)$$



Substituting specific parameter value *u*. Assign at points parameter value of *u: u = 0, u = 1/3, u = 2/3, u = 1*, which will match evenly arrangement of points. Get:

$$y = y_0 \frac{(u-1/3)(u-2/3)(u-1)}{(0-1/3)(0-2/3)(0-1)} + y_1 \frac{(u-0)(u-2/3)(u-1)}{(1/3-0)(1/3-2/3)(1/3-1)} +$$
$$+y_2 \frac{(u-0)(u-1/3)(u-1)}{(2/3-0)(2/3-1/3)(2/3-1)} + y_3 \frac{(u-0)(u-1/3)(u-2/3)}{(1-0)(1-1/3)(1-2/3)} =$$
$$= \frac{9}{2}[-y_0(u^3 - 2u^2 + \frac{11}{9}u - \frac{2}{9}) + 3y_1(u^3 - \frac{5}{3}u^2 + \frac{2}{3}u) -$$
$$-3y_2(u^3 - \frac{4}{3}u^2 + \frac{1}{3}u) + y_3(u^3 - u^2 + \frac{2}{9}) = \quad (4)$$
$$= y_0 + (-\frac{11}{2}y_0 + 9y_1 - \frac{9}{2}y_2 + y_3)u + (9y_0 - \frac{45}{2}y_1 + 18y_2 - \frac{9}{2}y_3)u^2 +$$
$$+(-\frac{9}{2}y_0 + \frac{27}{2}y_1 - \frac{27}{2}y_2 + \frac{9}{2}y_3)u^3.$$

Also curve (4) after conversion coefficients can be written also in the following way:

$$y = \frac{9}{2}[y_0(1-u)(\frac{2}{3}-u)(\frac{1}{3}-u) + 3y_1(1-u)(\frac{2}{3}-u)u + 3y_2(1-u)(u-\frac{1}{3})u +$$
$$+y_3(u-\frac{2}{3})(u-\frac{1}{3})u]. \quad (5)$$

Thus all four points lie on the curve in the range $0 \le u \le 1$. In this case, assigned specifically parameter value u at each point, which corresponds to evenly location.

Rewrite (3) in matrix form:

$$y = [y_0 \quad y_1 \quad y_2 \quad y_3] \begin{bmatrix} \alpha_0(u) \\ \alpha_1(u) \\ \alpha_2(u) \\ \alpha_3(u) \end{bmatrix}, \tag{6}$$

where

$$\alpha_0(u) = \frac{(u-u_1)(u-u_2)(u-u_3)}{(u_0-u_1)(u_0-u_2)(u_0-u_3)},$$

$$\alpha_1(u) = \frac{(u-u_0)(u-u_2)(u-u_3)}{(u_1-u_0)(u_1-u_2)(u_1-u_3)},$$

$$\alpha_2(u) = \frac{(u-u_0)(u-u_1)(u-u_3)}{(u_2-u_0)(u_2-u_1)(u_2-u_3)},$$

$$\alpha_3(u) = \frac{(u-u_0)(u-u_1)(u-u_2)}{(u_3-u_0)(u_3-u_1)(u_3-u_2)},$$

$$u_i = \frac{x - x_0}{x_3 - x_0}.$$

The test example is shown in Fig. 3. The program is implemented with help language AutoLisp.

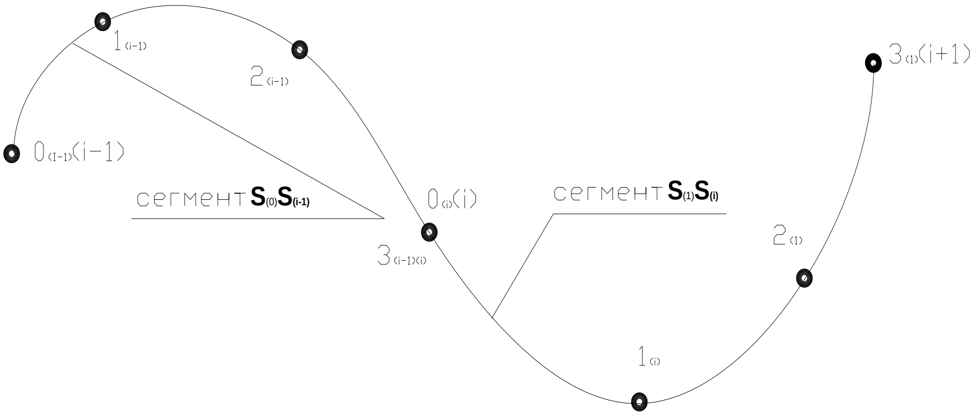


Fig. 3. Test example of cubic curve segments with control points that incident curve

Based on these curves can be constructed splines with help of control points, that incident curve.

Conclusions. Splines of the third degree with control points incidental curve, let you freely correct the shape of the curve on software user demand. Also control point belongs to curve, which greatly simplifies the process of construction. Existing shortcomings of existing splines types (including cubic) and great industry need in new types of smooth curves requires further research spline types, which are discussed in the article.

Literature

1. *Якунин В.И.* Геометрические основы автоматизированного проектирования технических поверхностей. / В.И. Якунин – М.: Маи, 1080.–86 с.
2. *Alan Watt*. 3D Computer Graphics. Third Edition/ Addison-Wesley, 2000.-570 p.
3. *Завьялов Ю.С.* Методы сплайн - функций./ Ю.С. Завьялов - М.:Наука, 1980. – 246 с.