GEOMETRIC DESIGN BASICS OF ONE-DIMENSIONAL CONTOURS BY *k* PRESCRIBED POINTS IN BN-CALCULATION

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***Summary.* In work proposed the geometric design principles of one-dimensional convex contours through the k prescribed points in the BN-calculation, which are used for geometric modeling of the real surface of thin-walled shells of technical forms.**

***Keywords*: convex bypass, bypass arc tangent, BN-calculus, point equation.**

Problem formulation. During transport, installation and operation of thin-walled shells on of engineering structures affected by objective and subjective factors, changing its initial geometric shape. To account for imperfections in the geometrical shape of the strength and sustainability of a shell, you need an analytic description of its actual surface.

From the geometric point of view, the model of the real surface of the thin shell of engineering structure - a closed segment of the surface, which is formed by arcs of convex bypass of the first order of smoothness. This leads to the need to develop the foundations for the creation of geometric algorithms for constructing one-dimensional convex bypass by *k* preassigned points.

Analysis of recent research. Research in the field of design convex bypasses have been the subject of many studies, for example, [1-4], in which the cases have been considered design bypass to the order of smoothness and higher than the first. However, all these studies were conducted in the framework of the mathematical tools, which were used to solve specific practical problems. In our case, to simulate the actual surface of thin-walled shells of technical forms rolling simplex method was used, which was developed on the basis of the mathematical apparatus of the BN-calculus [1]. Therefore, to solve this problem it is necessary to develop algorithms for constructing convex bypasses of the first-order smoothness is within BN-calculus. It should be noted that the use of lines of higher orders smoothness is not required to address this particular problem and only leads to unnecessary complication of the calculation algorithm.

Geometric design principles of one-dimensional and two-dimensional bypasses in the BN-calculus were developed in [1]. But in this study were not considered particularly task tangent and accordingly, the formation of arcs bypass on the first and last sections.

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The wording of the purposes of the article. To develop the one-dimensional geometric bases of designing of translation with help of k preassigned points in the BN-calculus, taking into account features of arcs formation in the bypass of the first and last sections.

Main part. Let there be given k of n-dimensional space points (Fig. 1): $A_1, A_2, A_3, ..., A_i, ..., A_k$. Required through these points hold a curved line. This curve line in this case will be the one-dimensional convex bypass smoothness of the first order. The main requirements that apply to the bypasses, is that it is not allow unforeseen oscillations.

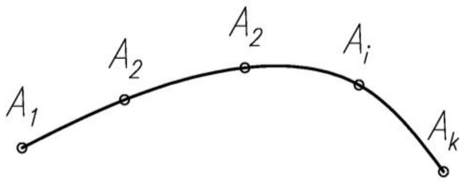

Fig. 1. The convex bypass, built bv *k* points.

Let us note that the constructed bypass, in general, will consist of arcs of double curvature. We select what properties should have an arc:

1. It should not contain kinks. In other words, the bypass arc must have unique tangent on the interval [0,1] at each point. Analytically, this is achieved by the fact that the point equation of the arc has defined functions on the interval, which at least once are differentiable.

2. The arc should not have self-intersection points (node points).

3 Bypass algorithm should have flexibility in relation to the dimension of the space. That is, it must retain all its properties as for three-dimensional and for two-dimensional space. This requirement makes it more stringent restrictions on the bypass arch of double curvature. Arc bypass must be such that the flat version of it had no inflection points (Fig. 2a) or in the case (Fig. 2b) when the directions of the tangents $A_1 B_1$ and $A_2 S_2$ not allows to avoid the inflection point, then this point must not be greater than one.

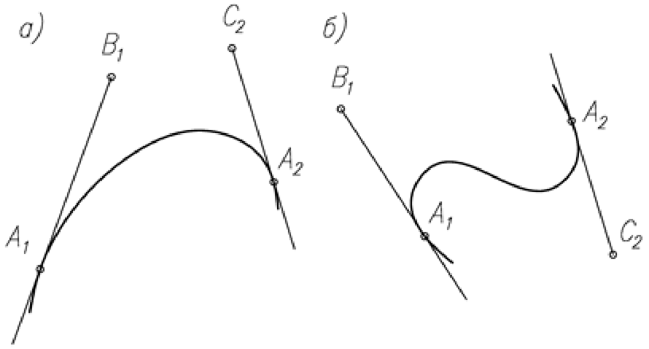

Fig. 2. Bypass arcs at different directions of tangents.

Professor Balyuba in [1] studied the arc bypass will certainly have the properties of 2 and 3, if the algorithm is applied to the following condition:

$$|A_1A_2| = |A_1B_1| + |A_2C_2|. \tag{1}$$

This condition allows you to fulfill the requirement of the arc, which is not infrequently applied to the bypass in the construction of technical surfaces forms. Encountered in practice lines often include straight sections. From another property, which must comply with perfect arc of bypass:

4. Bypass have to work with the same parameters as in the case when the bypass arc degenerates into a straight line segment.

An arc bypass can have these properties obtained on the basis of three curves relations [1]. We use to build a simple arc bypass of the third order double curvature:

$$M = A_i\bar{t}^3 + 3B_i\bar{t}^2t + 3C_{i+1}\bar{t}t^2 + A_{i+1}t^3, \tag{2}$$

where A_i, A_{i+1} - the beginning and end of the bypass arc;

A_iB_i- the tangent to the arc at the point;

$A_{i+1}C_{i+1}-$ the tangent to the arc at the point;

$0 \le t \le 1-$ parameter point of the equation;

$\bar{t} = 1 - t-$ addition to the parameter unit.

To form the bypass through the k points A_i, with the help of the arc (2.2), is required to determine the point B_i and C_{i+1} so that the arc (2.2) on [0,1] did not have singular points. The second requirement for selection C_{i+1} C_{i+1}, is the fact that the selection doesn't create the tangent of unforeseen inflection points on the arc A_i, A_{i+1}. In other words, the bypass of the given points must to be convex.

Selecting tangents at given points will be divided into two stages:

1. Choose the tangents at interior points of bypass.

2. Choose the tangent at the starting point A_1 and end point A_k.

Getting the first stage of the solution. Given points A_i, A_{i+1}, A_{i+2},, is required to determine the points B_{i+1}, C_{i+1}, fixing tangents of adjacent bypass arcs (fig. 3). In [1] it was noted that two adjacent arc-pass will be convex if the tangent $A_{i+1}B_{i+1}$ will be parallel A_iA_{i+2}. Such direct and determine with the points B'_{i+1} and C'_{i+1}. Naturally, $B'_{i+1}, C'_{i+1}, A_{i+1}$ are collinear and to define a sufficiently direct it is enough to use two points: A_{i+1} and one point of B'_{i+1} or C'_{i+1}. We define B'_{i+1} and C'_{i+1} to create a symmetry point equations of +bypass. Points B'_{i+1} and C'_{i+1} we define from the two

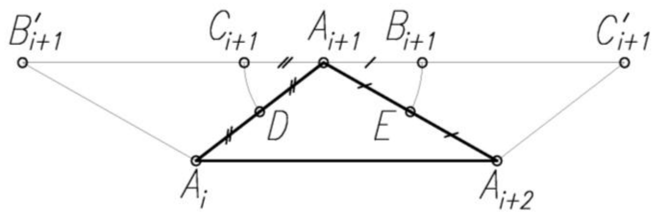

Fig. 3. Construction of the tangent at point A_{i+1}.

parallelograms $A_i A_{i+2} C'_{i+1} A_{i+1}$ and $A_{i+2} A_i B'_{i+1} A_{i+1}$. Using the point equation of parallel translation, we get:

$$B'_{i+1} = A_i + A_{i+1} - A_{i+2}. \tag{3}$$

$$C'_{i+1} = A_{i+1} + A_{i+2} - A_i. \tag{4}$$

Points B'_{i+1} and C'_{i+1} define a tangent at the point A_{i+1} but point equation of the bypass arc (2) assumes presence on the tangent points B_i and C_{i+1}, forming bypass arcs view (Fig. 4).

B'_{i+1} and C'_{i+1} could be taken as the formative points. However, these points are known to give rise on the arc unwanted, a singular point of the first kind if the tangents lie in one plane, and admissible, but also not very necessary, double curvature, when tangents are crossed. You may find that the best option of the arc $A_i A_{i+1}$ is the chord $A_i A_{i+1}$. We find the best algorithm for finding points B_i and C_{i+1}, providing the desired contour of the arc at all variants of the task $A_i A_{i+1}$:

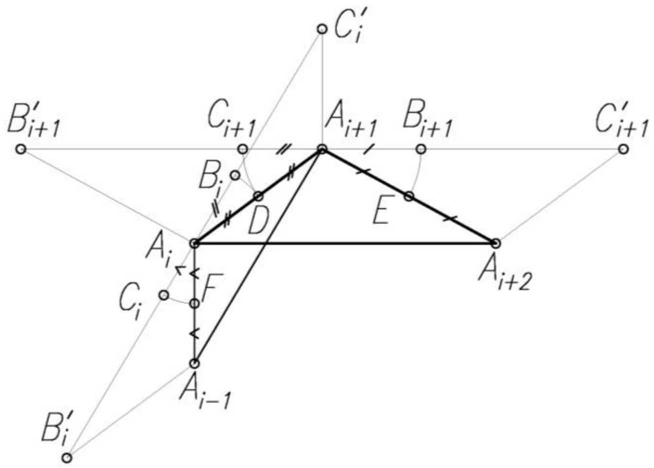

Fig. 4. Selection of tangents at interior points of bypass.

1. The points B_i, C_{i+1} coincide when the arc is converted into a chord.

2. In all other cases, segments of tangents $A_i B_i$ and $A_{i+1} C_{i+1}$ cannot be crossed, which will not allow appear a special point on the arc bypass.

This can be achieved if the points B_{i+1} and C_{i+1} will belong to the line and satisfies the conditions:

$$|A_{i+1} B_{i+1}| = \frac{|A_{i+1} A_{i+2}|}{2}; \qquad |A_{i+1} C_{i+1}| = \frac{|A_i A_{i+1}|}{2}. \tag{5}$$

Let us determine the required points:

$$\frac{|A_{i+1} B_{i+1}|}{|A_{i+1} C'_{i+1}|} = \frac{|A_{i+1} A_{i+2}|}{2|A_i A_{i+2}|} = \frac{A_{i+1} B_{i+1}}{A_{i+1} C'_{i+1}} \Rightarrow B_{i+1} = \left(C'_{i+1} - A_{i+1}\right) \frac{|A_{i+1} A_{i+2}|}{2|A_i A_{i+2}|} + A_{i+1}.$$

Further, taking into account (4), we obtain:

$$B_{i+1} = \left(A_{i+2} - A_i\right) \frac{|A_{i+1} A_{i+2}|}{2|A_i A_{i+2}|} + A_{i+1}. \tag{6}$$

Similarly, taking into account (3), determine:

$$C_{i+1} = (A_i - A_{i+2})\frac{|A_i A_{i+1}|}{2|A_i A_{i+2}|} + A_{i+1}. \qquad (7)$$

The second step is to determine the tangents at the extremities of the bypass. There are various ways of defining the tangent.

In this paper, we propose to use for the construction of bypass the bypass arc of the2nd order.

$$M_1 = A_1\bar{t}^2 + 2C_2 t\bar{t} + A_2 t^2. \qquad (8)$$

$$M_k = A_{k-1}\bar{t}^2 + 2B_{k-1} t\bar{t} + A_k t^2. \qquad (9)$$

In this case, the tangent at point A_1 is set by C_2 and tangent at point A_k - by B_{k-1}.

On the basis of equations (6,7) define the point C_2 and B_{k-1}:

$$C_2 = (A_1 - A_3)\frac{|A_1 A_2|}{2|A_1 A_3|} + A_2. \qquad (10)$$

$$B_{k-1} = (A_k - A_{k-2})\frac{|A_{k-1} A_k|}{2|A_{k-2} A_k|} + A_{k-1}. \qquad (11)$$

When you create algorithm of bypass constructing, it should be noted that the points C_2 and B_{k-1} are determined by the equations (6.7) with the corresponding value of *i*.

Conclusions. In the paper were identified the tangents and, with their help, was obtained point equation arcs bypass 2nd and 3rd order taking into account the characteristics of the formation of arcs on the first and last sections of bypass, which is the geometric basis for the creation of algorithms of constructing convex contours of the first-order smoothness that in turn, allows them to be used to model the real surface of the thin-walled shells of engineering structures in the BN-calculus.

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