RESEARCH BY CURVATURES OF PHASE TRAJECTORIES OF VIBRATIONS OF POINT ON THE REVOLVED CIRCUMFERENCE

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***Summary.* The method of determination of critical values of parameters of differential equalization of vibrations of point is brought around to a circle that runs around about vertical axis. A method is based on the concept of perverted of phase trajectories and takes into account the change of sign of their curvature along trajectories.**

***Keywords:* a phase trajectory, critical values of parameter, oscillation of point on a circle, perverted of curve, curvature of curve.**

Formulation of the problem. The study of pendulum vibrations at quality level it is expedient to carry out the method of phase trajectories. The traditional analysis of pendulum vibrations is folded [1,2] from determination of the special points, that answer position of equilibrium of the oscillating system, construction of phase portraits of the system with the values of managing parameter within the limits of the special points, and also determination of separatrices, that pass through the special points by means of equalization of integral of energy of the system, when kinetic energy equals a zero. But for practical introductions the engineering methods of calculation of critical values of managing parameter of vibrations are needed especially [6,7], taking into account of that can improve the construction of the oscillating system, or prevent it to the emergency state. It specifies on actuality of select theme of researches.

Analysis of recent research. Bases of search of critical values of parameters of phase trajectories (theories of bifurcations) are stopped up A.Puankare and O. Lyapunov, then these researches were developed O.Andronov and students [1,2]. In works [3,4] the review of various methods of research of phase trajectories is brought around to a quality level, from where a conclusion follows about insufficient development of grapho-analytical methods of search of critical values of parameters of phase trajectories for engineering practice. The graphic methods of search of critical values of parameters of phase trajectories on the basis of the isocline field it was expedient to complement and such that are based on graphic character of perverted of phase trajectory, and that is determined by totality of values of her curvature along this trajectory.

Formulation of article purposes. Development of method of determination of critical values of parameters of differential equalization of vibrations of point is on a circle that runs around about vertical axis. A method is based on the concept of perverted of phase trajectories and takes into account the change of sign of their curvature along trajectories.

Main part. Will consider that motion of point on a phase trajectory comes true in limits, that is determined by the borders of change of parameter of t time, and "turn" to the right or to the left at movement set by different signs at the values of curvature of this trajectory.

At the search of critical values of parameters of phase trajectories main will be a question of determination curvatures of phase trajectories, for that in general case not well-known descriptions by analytical formulas. Because the coordinates of points on a phase trajectory are calculated by mainly numeral methods after the decision of differential equalization. Id est it is necessary to define the curvature of phase trajectory, set by the great number of N of points (x_i, y_i) when i=2.M- 1. A managing parameter farther marks as p.

Will choose on a phase curve three nearby points (x_{i-1}, y_{i-1}) (x_i, y_i) and (x_{i+1}, y_{i+1}). For the calculation of curvature in a point (x_i, y_i) will find the radius of circle r_1, that passes through data three points:

$$r_i = \frac{\sqrt{B^2 + C^2 - AD}}{A}, \tag{1}$$

where

$$A = x_{i-1}y_i + x_i y_{i+1} + x_{i+1}y_{i-1} - x_{i+1}y_i - x_i y_{i-1} - x_{i-1}y_{i+1};$$

$$B = (y_{i+1}^2 y_i - x_{i+1}^2 y_{i-1} + x_{i+1}^2 y_i - y_{i+1}^2 y_{i-1} - y_i^2 y_{i+1} + y_i^2 y_{i-1} +$$
$$+ x_i^2 y_{i-1} - x_t^2 y_{i+1} - y_{i-1}^2 y_i + y_{i-1}^2 y_{i+1} - x_{i-1}^2 y_i + x_{i-1}^2 y_{i+1})/2;$$

$$C = (y_{i+1}^2 x_{i-1} - y_{i+1}^2 x_i + x_{i+1}^2 x_{i-1} - x_{i+1}^2 x_i + y_i^2 x_{i+1} - y_i^2 x_{i-1} +$$
$$+ x_i^2 x_{i+1} - x_t^2 x_{i-1} + y_{i-1}^2 x_i - y_{i-1}^2 x_{i+1} + x_{i-1}^2 x_i - x_{i-1}^2 x_{i+1})/2;$$

$$D = y_{i+1}^2 x_i y_{i-1} - y_{i+1}^2 x_{i-1} y_i - x_{i+1}^2 x_{i-1} y_i + x_{i+1}^2 x_i y_{i-1} +$$
$$+ y_i^2 x_{i-1} y_{i+1} - y_i^2 x_{i+1} y_{i-1} - x_i^2 x_{i+1} y_{i-1} + x_t^2 x_{i-1} y_{i+1} -$$
$$- y_{i-1}^2 x_{i+1} y_i - y_{i-1}^2 x_i y_{i+1} - x_{i-1}^2 x_i y_{i+1} + x_{i-1}^2 x_{i+1} y_i.$$

Then a value of curvature in a point (x_i, y_i) will be k=1/r_1. Condition A= 0 will determine a zero curvature (when points are located on a line).

The close calculation of curvature by means of radius of the circle conducted through three points is explained by that it was succeeded to calculate the numeral method of decision of differential equalization of vibrations of the system only the value of function and her derivative. And for a calculation [5] curvatures of flat curve that is set by equalization of x=x (t), y=y (t), it is necessary to know yet and second derivative:

$$k = \pm \frac{y''x' - x''y'}{\left(x'^2 + y'^2\right)^{3/2}} .$$

In addition, for flat curves there is possibility to distinguish direction of rotation of tangent of line at along movement crooked, that is why a sign is added curvature depending on direction of this rotation.

After determination of curvatures for all points on a phase trajectory build the cobbed-linear chart of function of *k (t)* curvatures for the defined value of parameter of p . Determine the area of subchart (for the initial value of p) by means of one of numeral methods (for example, method Simpson). Producing the marked actions in the loop for other values of parameter of *p*, obsessed close chart of function of *S (p)*, that will consist of linear segments. In obedience to the above-mentioned, critical values will be determined by means of vertical constituents of cobbed-linear chart of function of *S (p)*.

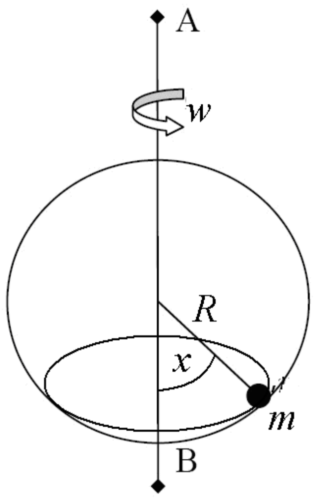


Fig. 1 Scheme of a circulating-shake system.

For an example, will investigate oscillation of point of mass of m, located on the circle of radius of R, that evenly runs around about vertical axis of AB (fig.1). Here x (t) is a corner of rejection of point from a vertical line, ω is a corner of turn of circle about axis of AB. Differential equalizations have on the coordinate of x (t) [1,2] kind:

$$\frac{d}{dt}x(t) = y(t); \qquad \frac{d}{dt}y(t) = \omega\left(\cos(x(t)) - \lambda\right)\sin(x(t)) , \qquad (2)$$

where $\lambda = \dfrac{g}{R\,\omega^2}$.

In works [1] (p.28) and [2] (p.129) it is shown that the phase portraits of the circulating system with description (2) it is expedient to build for such three variants: a) $1 < \lambda < 0$ (fig.2); b) $\lambda = 0$ (fig.3); c) $0 < \lambda < 1$ (fig.4). In these cases they will be will differ at quality level.

From equalizations (2) it is possible to calculate the special points that answer position of equilibrium : a)y_1=0; x_1=0; b) y_2=0; x_2=π; c) y_3=0; x_3=λ. .

The integral of energy of the system (2) looks like :

$$\frac{y^2(t)}{2} - \Omega^2\left(\frac{\sin^2\left(x(t)\right)}{2} + \lambda\cos\left(x(t)\right)\right) = C . \qquad (3)$$

Equalization of separatrix can be got from expression for the integral of energy of the system (3), where the constant of C is determined from a that condition, that separatrix passes through the special point. Thus kinetic energy equals a zero, in fact position of equilibrium will be realized here. From here get equalization two separatrix:

$$\omega^2 = \Omega^2\left[\sin^2\left(x(t)\right) + 2\lambda\left(\cos\left(x(t)\right)+1\right)\right]; \qquad (4)$$
$$\omega^2 = \Omega^2\left[\sin^2\left(x(t)\right) + 2\lambda\left(\cos\left(x(t)\right)-1\right)\right].$$

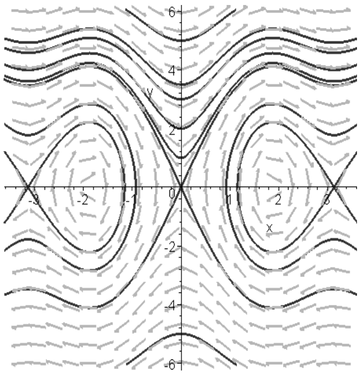


Рис. 2. $-1<\lambda<0$.

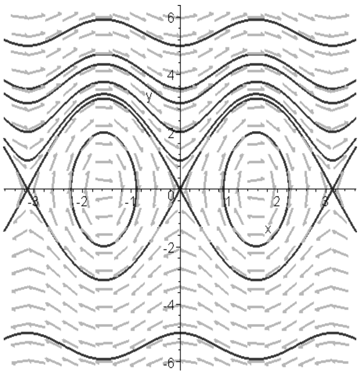


Рис. 3. $\lambda=0$.

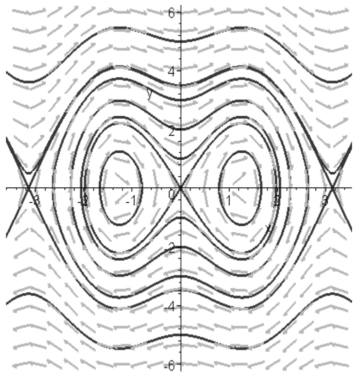


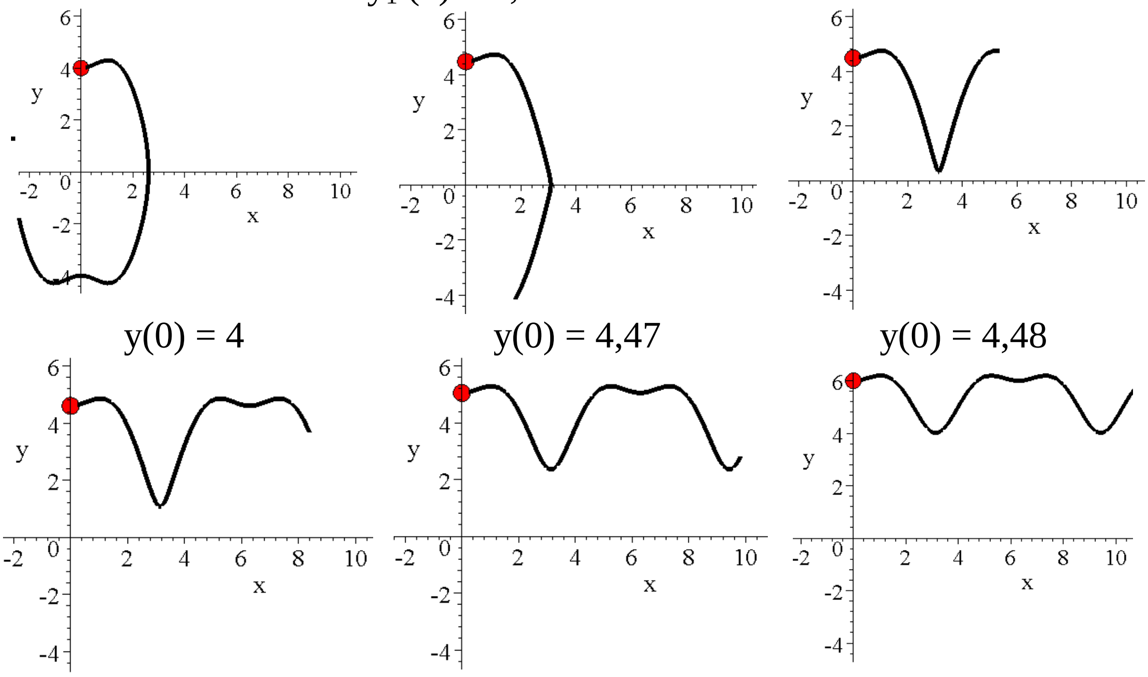
Рис. 4. $0<\lambda<1$.

On fig. 4 on condition of 0 < λ < 1, presented both separatrix: external separatrix, that passes through points (π, 0) and (-π, 0) and surrounds internal separatrix, that passes through a point (0,0) and has a form of eight. On rice.3, when λ = 0, separatrix meet. On fig.2, when - 1 < λ < 0, the same situation will be realized, that and on rice. 4, but an image is moved along a horizontal axis θ on π units.

Thus, the quality change of phase portrait of the system takes place in transition a parameter λ through a zero. Id est there was bifurcation. It is possible also to show that in transition a parameter λ through a value λ = ±1 another change of phase portrait takes place. It can be educed most expediently, making the program of construction of animation images of phase portrait depending on the change of managing parameter.

Unlike the brought higher classic analysis over [1,2] shake process by facilities of phase portraits, in-process an analysis offers marked on the basis of perverted of phase trajectories taking into account the change of sign of their curvature along trajectories.

Will decide the system of differential equalizations (2) numeral a method Runge-kut with the initial conditions of x (0) and in y(0) depending on a value λ. For test calculations it was made the language of Maple program of decision of the system of equalizations (2) and construction of phase portrait for a value λ = 0,5 with initial conditions x (0) = 0 and 3,5 < y (0) < 6. On fig. 5 the animation shots of change are represented phase to the portrait, from that y_1 (0) = 4,48. For this confirmation on fig. 6 the got chart over of function of S (p) is brought, from that follows, that critical will be a close value of y_1 (0) =4,48.



y(0) = 4 y(0) = 4,47 y(0) = 4,48

y(0) = 4,6 y(0) = 5,0 y(0) = 6,0

Fig. 5. Animation shots of change phase to the portrait depending on y (0).

On figures a circle is designate a point that answers initial conditions. Thus a step between points on a phase curve was elected Δ= 0,1; amount of points of M=1000.

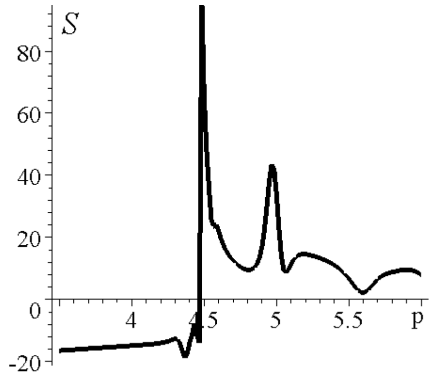


Fig.6.Scheme of function S(p); p= y (0).

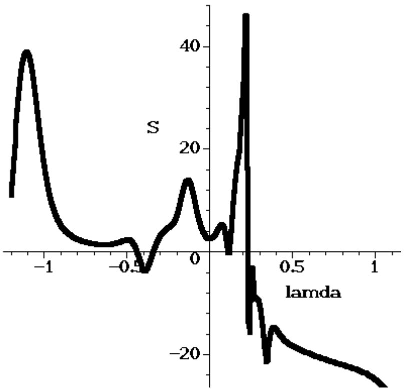
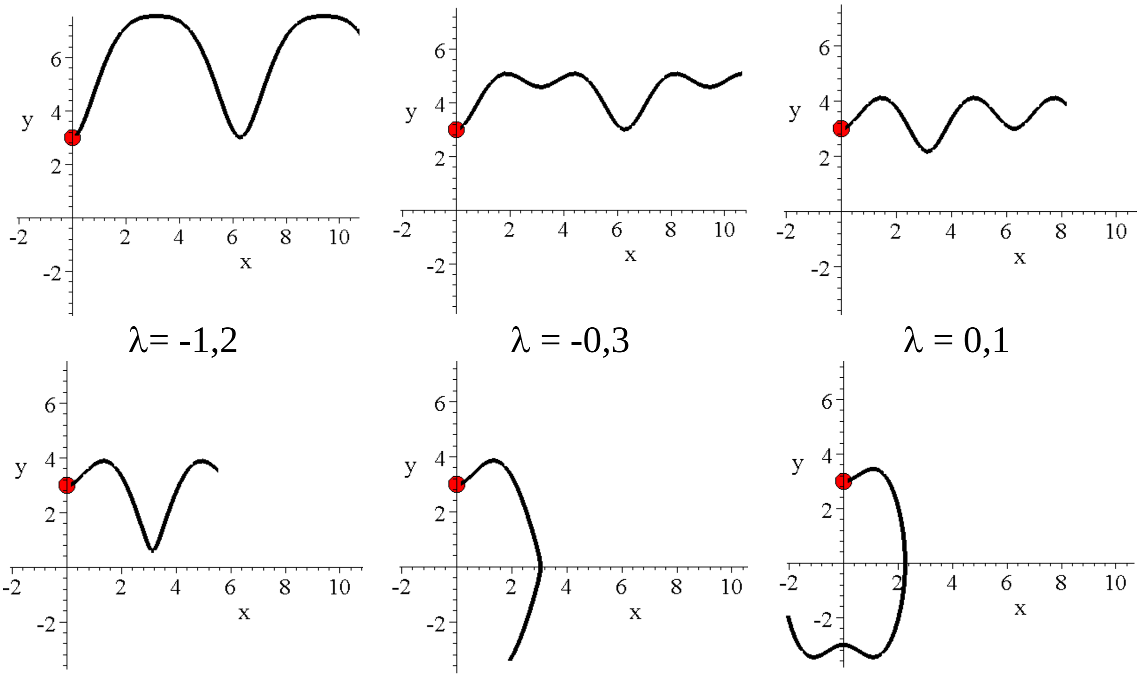


Fig.7. Scheme of function S(λ).

More interesting is a case of analysis of vibrations on condition of in-out parameter λ: $0<\lambda<6$ and initial conditions of x(0) = 0 and y(0) = 3. On fig. 7 a chart over of corresponding function of S (λ) is brought with the found critical values λ_1 =-0,3, λ_2 =0,1 and λ_3=0,22. On fig. 8 shots over of change are brought corresponding phase to the portrait depending on values λ.

Giving arbitrary permanent C in the integral of conservation of energy different values, obsessed great number of phase spaces that in totality will form phase space of the conservative system of the second order, that and it was considered in this work.



λ= -1,2 λ = -0,3 λ = 0,1

λ = 0,21 λ= 0,22 λ= 0,45

Fig. 6. Animation shots of change phase to the portrait depending on λ.

Conclusions. For determination of critical values of parameter of p family of phase trajectories it is necessary to find coordinates on wasp of abscissas of vertical constituents on the chart of *S=S (p)* dependences on the parameter of p area of subscheme of cobbed-linear function of curvature of *k (t)*.

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