

GEODESIC WINDING OF CORD OF TIRE TAKING INTO ACCOUNT HER FORM ON-LOADING

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Summary. A method over of geometrical design of geodesic trajectories is in-process brought on a surface that approaches the surface of tire taking into account her geometrical form on-loading.

Keywords: a geodesic trajectory, motor-car tire, process of winding of cord, periodicity of location of geodesic.

Formulation of the problem. Making of the reinforced materials a winding method is one of perspective technologies of strengthening of wares. The special value she has at making of motor-car tires, when winding comes true by a metallic (sometimes kevlar) cord [1]. For realization in practice of winding it is necessary to know the law of distribution of coils on framework. The geodesic trajectories of winding are mostly used. Provided in this case, in the certain understanding, maximal durability of good. In addition, for providing of reliability and longevity of tires it is necessary to take into account the dynamics of the mutual moving of coils of cord in points their mutual crossing as a reaction on the change of loading on a tire. The marked specifies on actuality of theme of work.

Analysis of recent research. One of technologies of making [1] motor-car tires consists in that the reticulated cylindrical shell is transformed like as torus surface by tires. The process of making of motor-car tire is based on the operation of transformation of shell-purveyance in eventual good. After transformation the law of change of corners of reinforcement is described by the so-called "bus geometry". Modern technologies envisage winding of cylindrical shell-purveyance after geodesic trajectories (fig.1). In-process [2] a method over of calculation of geodesic trajectory of cord tire is brought taking into account the size of pantograph corners between contiguous cords (fig.2). But in-process [2] mainly empiric dependences are used, and the construction of geodesic trajectories of cord is not based on the decision of the system of differential equalizations.

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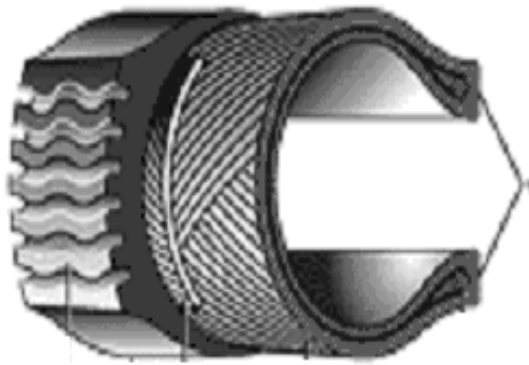


Fig. 1. Location cord on the surface of the tire.

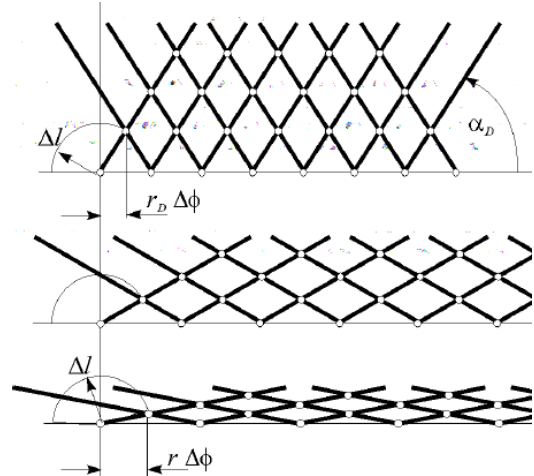


Fig.2. Pantograph model at points of entry.

Formulation of article purposes. To work out the method of geometrical design of geodesic trajectories on the surface of rotation, that approaches the surface of tire taking into account her form on-loading. Basic part. Logically to consider that in the process of exploitation of motor-car tire the corners of pantograph model must insignificantly change at the change of loading. It must decrease the risk of breakage of tire "from middle" due to the mutual moving of cords.

Will consider the surface of rotation, that reminds the surface of motor-car tire approximately. Conditionally will name her a like tire surface equalization of that it is suggested to choose in a kind

$$X = \left[\left(R - \cos^w(u) \right) \cos(v), \left(R - \cos^w(u) \right) \sin(v), u - \frac{\pi}{2} \right]. \quad (1)$$

Here u is a parameter along the meridian of surface, v is a parameter along the parallel of surface, R is a radius of base circle.

The feature of description (1) consists in the presence of parameter of w , that regulates the form of chiropody surface depending on her state - free or loaded. Yes, in the free state of value of parameter of w must be elected within the limits of $w = 1.4$, and in the loaded state within the limits of $w = 12.16$. All sizes are in conditional units.

The presence of parameter of w will allow in the future to watch the change of pantograph of corner between winding cords in case of change of loading on a tire.

At the stowage of the system of differential equalizations it is necessary to calculate for description of geodesic on a surface tire [3,4] coefficients of the second quadratic form and other accompanying formulas.

$$\begin{aligned}
E &= \frac{\cos^2(u) + w^2 \cos^{2w}(u) - w^2 \cos^{2(w+1)}(u)}{\cos^2(u)}; \\
F &= 0; \quad G = \left(R - \cos^w(u)\right)^2; \\
D &= \left(R^2 \cos^2(u) - 2R \cos^{(2+w)}(u) + \cos^{2(1+w)}(u) + R^2 w^2 \cos^{2w}(u) - \right. \\
&\quad \left. - 2Rw^2 \cos^{3w}(u) + w^2 \cos^{4w}(u) - R^2 w^2 \cos^{2(1+w)}(u) + \right. \\
&\quad \left. + 2Rw^2 \cos^{(3w+2)}(u) - w^2 \cos^{2(2w+1)}(u) \right) / \cos^2(u); \\
E_u &= 2w^2 \sin(u)(1 - w + w \cos^2(u)) \cos^{(2w-3)}(u); \\
E_v &= 0; \quad F_u = 0; \quad F_v = 0; \\
G_u &= \frac{2w \sin(u)(R \cos^w(u) - \cos^{2w}(u))}{\cos(u)}; \quad G_v = 0.
\end{aligned}$$

As a result obsessed system of differential equalizations for description of geodesic on a chiropody surface in a kind:

$$\begin{aligned}
\left(\frac{d^2}{dt^2} u(t) \right) + \frac{GE_u}{2D} \left(\frac{d}{dt} u(t) \right)^2 - \frac{GG_u}{2D} \left(\frac{d}{dt} v(t) \right)^2 &= 0; \quad (2) \\
\left(\frac{d^2}{dt^2} v(t) \right) + \frac{EG_u}{D} \left(\frac{d}{dt} u(t) \right) \left(\frac{d}{dt} v(t) \right) &= 0.
\end{aligned}$$

The system of equalizations (2) was untied by a method Runge-kut. Thus a test variant was calculated with initial conditions $u_0 = \pi/2$ and $v_0 = \pi$ and by the limits of change of parameters $\pi/40 < u < \pi - \pi/40$ and $0 < v < 2\pi$; $R=3$. The amount of points on a geodesic line equals 1500. The corners of "exit" of geodesic changed only from an initial point.

On fig. 3 results over of calculations of geodesic trajectory are brought no-load at $w = 4$ (on the left) and with loading at $w = 16$ (on the right) for $u'_0 = 0,43$. It was succeeded to find an initial $u'_0 = 0,43$ self, the method of "animation" iterations that provides acceptable periodicity of geodesic trajectories on a surface regardless of presence or absence of loading.

he considered geodesic winding of cord it follows to consider the first step design of technology, in fact the geometrical form of tire on-loading and in the free state must change not "globally" - id est simultaneously on all surface of tire, but "locally" - only in the zone of contact of tire with the surface of road.

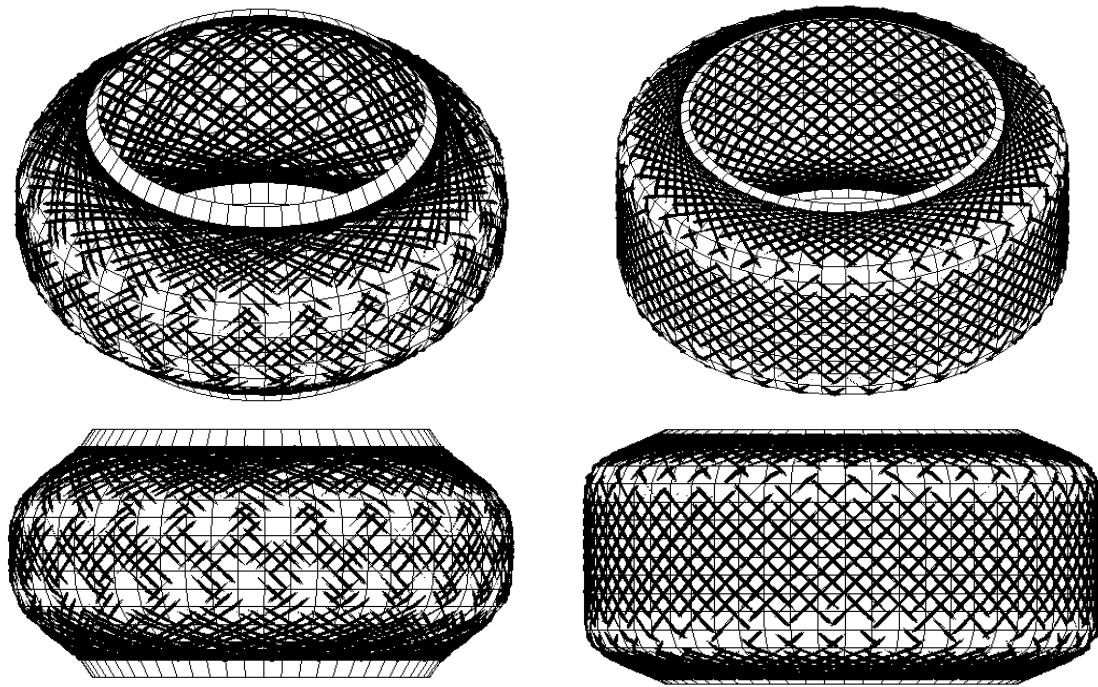


Fig. 3. Chiropody surface for a parameter $u'_0 = 0,43$.

The next step of design will be taking into account of change of geometrical form of tire only in the zone of contact with the surface of road. For this purpose in equalization (1) the constant of w will substitute by a function

$$w = 16 - 12H(v - \beta), \quad (3)$$

what must regulate the form of chiropody surface only in the zone of contact with a road depending on the state of loading of tire. In a formula (3) β is a central corner of arc of contact, H is a function of Heavyside.

Walking away from the geometrical form of surface of rotation causes complication of decision of this task. At the stowage of the system of differential equalizations it is as (2) necessary to take into account already unzero values F , E_v , F_u and F_v . A stowage and decision of the system of differential equalizations are carried out in the environment of mathematical package of Maple.

Will point the type of the system of differential equalizations

$$\begin{aligned}
& -2D \left(\frac{d^2}{dt^2} u(t) \right) - GE_u \left(\frac{d}{dt} u(t) \right)^2 + 2FF_u \left(\frac{d}{dt} u(t) \right)^2 - FE_v \left(\frac{d}{dt} u(t) \right)^2 - \\
& - 2GE_v \left(\frac{d}{dt} u(t) \right) \left(\frac{d}{dt} v(t) \right) + 2FG_u \left(\frac{d}{dt} u(t) \right) \left(\frac{d}{dt} v(t) \right) - \\
& - 2GF_v \left(\frac{d}{dt} v(t) \right)^2 + GG_u \left(\frac{d}{dt} v(t) \right)^2 + FG_v \left(\frac{d}{dt} v(t) \right)^2 = 0 ;
\end{aligned} \tag{4}$$

$$\begin{aligned}
& -2D \left(\frac{d^2}{dt^2} v(t) \right) - EE_u \left(\frac{d}{dt} u(t) \right)^2 + 2EF_u \left(\frac{d}{dt} u(t) \right)^2 - FE_v \left(\frac{d}{dt} u(t) \right)^2 - \\
& - 2EG_u \left(\frac{d}{dt} u(t) \right) \left(\frac{d}{dt} v(t) \right) + 2FE_v \left(\frac{d}{dt} u(t) \right) \left(\frac{d}{dt} v(t) \right) - \\
& - EG_v \left(\frac{d}{dt} v(t) \right)^2 + 2FF_v \left(\frac{d}{dt} v(t) \right)^2 - FG_u \left(\frac{d}{dt} v(t) \right)^2 = 0 .
\end{aligned}$$

At a decision, the left parts of equalizations (4) must be divided into D. The coefficients of the second quadratic form and other accompanying formulas have a too bulky kind and that is why not led here.

On fig. 4 an offer geometrical model of chiropody surface is represented with the zone of contact with the surface of road. On fig. 5 a geometrical model over of chiropody surface is brought with a few coils of geodesic winding at $\beta=0,8$. Notedly, that geodesic winding takes into account the geometrical form of bulge of chiropody surface. On fig. 6 it is represented 1500-torns of geodesic winding of chiropody surface for a parameter $u'_0 = 0,43$.

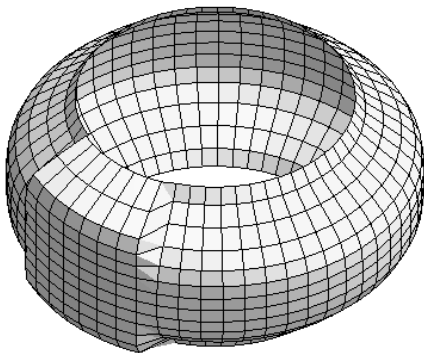


Fig. 4. Chiropody surface with the zone of contact with a road.

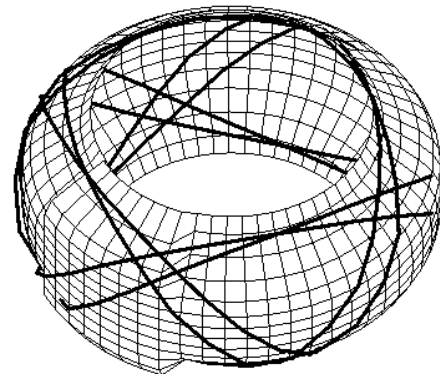


Fig. 5. Geodesic winding chiropody surfaces.

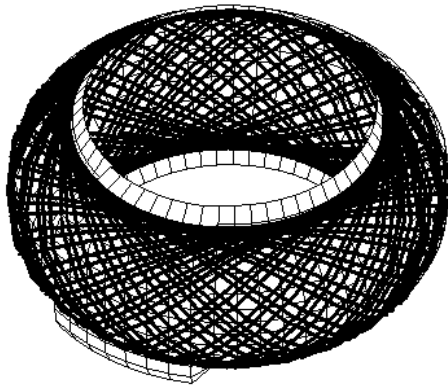


Fig. 6. Geodesic winding
chirpody surfaces.

Conclusions. The brought method over of construction of geodesic trajectories on a surface that approaches the surface of motor-car tire taking into account her form on-loading allows in the first approaching to analyse the location of family of geodesic with an aim in further to analyse time-histories of pantograph corners.

Literature

1. *Иванов А. М.* Основы конструкции автомобиля [Учебник для ВУЗов.] / А.М. Иванов, А.Н. Солнцев, В.В. Гаевский. – М.: ООО «За рулём», 2005, - 140 с.
2. *Koutny F.* Geometry and mechanics of pneumatic tires / F. Koutny. – Zlin: CZE –2007, – 142 p.
3. *Погорелов А. И.* Дифференциальная геометрия. / А.И. Погорелов – М.: Наука, 1974, - 176 с.
4. *Голованов Н.Н.* Геометрическое моделирование. / Н.Н. Голованов. – М.: Издательство Физико-математической литературы, 2002. – 472 с.
5. *Смирнов, В.И.* Курс высшей математики. / В.И.Смирнов. - 6-е изд., перераб. – М. : Наука, – 1974. – 336 с.
6. *Жукова Н.И.* Геодезические линии на поверхностях: учеб. пособие / Н.И. Жукова, А.В.Багаев. – Н. Новгород: Издательство Нижегородского госуниверситета, – 2008. – 54 с.