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FORMATION OF THE PART CUBIC B-SPLINE TO GIVEN CONDITIONS

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This article describes the control's problem of laws of change of curvature along the flat cubic B-spline. The control of form is carried out through the polygon settings.

Keywords: cubic B-spline, polygon, control polygon, tangent, radius of curvature, basic triangle.

Statement of the problem. One of the important tasks of geometric modeling is the formation of complex surfaces defined by discrete bar frame. Elements of the frame, on which the surface is formed, may be planar and spatial one-dimensional contours which are given by point line. An example is the dynamic surface, the purpose of which is interaction with the environment. Requirements of linear elements of dynamic surface – monotonous change of curvature along the curve and ensure second order smoothness of the contour [1].

In most modern packages, geometric modeling, linear elements of a surface model is formed by interpolation of the original point number of cubic B-spline. In cubic-spline automatically provides second order smoothness of the contour and with the help of the polygon that defines it, allows you to control the appearance of oscillations along the curve. Adjustment of regularities of change of curvature along the B-spline is possible in manual mode and can be effective when interpolating a small number of nodes. When assigning the nodal points in fixed positions, tangent and curvature values monitoring the change in curvature becomes much more complicated. At the moment we are not aware of ways that allows in automatic mode to provide monotonous change of curvature along the cubic B-spline.

Development of a tool that will allow you to control the regularity of change of curvature along the cubic B-spline will provide an opportunity to design surfaces effectively with desired functional properties.

Analysis of recent researches and publications. Methods of formation and properties the cubic B-spline considered in [1]. The point that is called the spline control, and the segments connecting the control points and control polygon. B-splines are generated by fitting points. Changing the parameters of the control polygon can affect the presence of convexity-concavity B-spline, but to control the regularity of change of curvature along a curve is difficult or impossible.

Setting of B-spline through points is discrete in nature and provides the flexibility to control its form. Approach to the management of a B-spline through the control points close to the formation of the discretely presented curve based on the input point number [2]. The problem of formation of geometric images that defined an ordered set of points can be solved variable discrete geometric modeling (WDGM). The main principles and directions are formulated in WDGM [2].

The method of controlling the shape of the spline through the points that it set proposed in [3]. The outline is based on the Bezier spline of the second order, which section is defined by three control points. The monotonous change of curvature along the curve and second order smoothness of the contour is ensured by the settings of the base of the triangle (BT) whose vertices are the points describing the plot of Bezier curve.

The objectives of the article. The purpose of this article is to study the possibility of providing a monotonic change of curvature along the cubic B-spline control parameters of the control polygon.

The main part. Consider the problem of formation of the parcel is flat the cubic B-spline along which the radii of curvature monotonically increase. One area cubic B-spline is defined by four control points P0, P1, P2 and P3 (Fig. 1).

Tangents to a spline that passes through points P0 and P3, and the chord P0P3 determine BT. The monotony of change of curvature along the B-spline is possible to provide, if the shape BT corresponds to the condition [3]: $a_1 > b_1; a_2 > b_2$. (1)

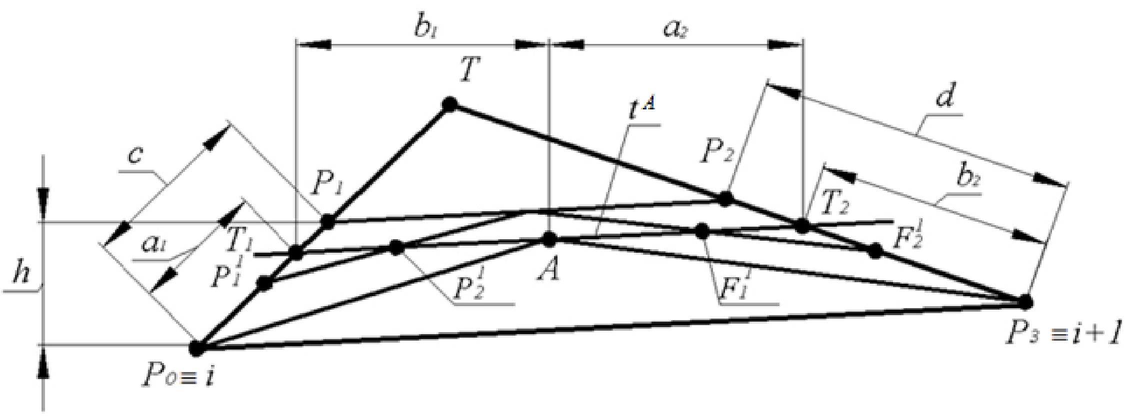

Fig. 1

If assign point *A* on B-spline and to determine the position of the tangent t^A, passing through it, we get BT *P0T1A* та *P3T2A*, each of which specifies a corresponding part of the original B-spline. Received BT will call BT thickening. The monotony of change of curvature along the B-spline ensures compliance form BT thickening (1). The same conditions (1) must provide for BT thickening that define as many small-plot In the spline. The position of the point And In-splain assigned so that the value of the parameter $u=\frac{1}{2}$, and the tangent t^A was parallel to the *P0P3.* In this case, the link P1P2 control polygon is uniquely determined and is parallel to the chord BT *P0P3* [1], and the tangent t^A is at the distance $\frac{3}{4}h$ from the chord *P0P3*, where *h* – is the distance from the chord *P0P3 to* the link*P1P2* of polygon *P0P1P2P3*. When you assign a point *A*, which corresponds to $u=\frac{1}{2}$ obtaine control polygons $P_0P_1^1P_2^1A$ та $AF_1^1F_2^1P_3$, each of which defines the shape of the corresponding plot B-spline [1]. The links P_2^1A and AF_1^1 of the newly formed polygons are tangent t^A, links $P_1^1P_2^1$ and $F_1^1F_2^1$ parallel to the basics BT основам БТ thickening *P0T1A* and *P3T2A*.

To control the position of the tangent t^A is possible due to the movement of the link *P1P2* of polygon *P0P1P2P3* and changing the distance *h*. The range of the position of the link is limited by the condition (1) when in its limit position, one of BT thickening becomes isosceles. The limit position of the link *P1P2* correspond to a limiting position the tangent t^A, at which the task of providing a monotonous change of curvature has a solution [3]. At this stage, to develop a method of adjusting the position of the tangent is assumed in interactive mode.

Define how the position of the tangent t^A determines the radii of curvature at points *P0* and *P3*, limiting the area of the spline.

For the curve given by parametric equations, radius of curvature is determined by the formula

$$R_i=\frac{|r'(u)|^3}{|r'(u)\times r''(u)|},$$

where *r(u)* – vector-function of the curve.

Expressing the first and second derivative of a spline using the radius-vectors of points [1] derived the formula to determine the radii of curvature at the extreme points of the plot P0 and P3 (Ri and Ri+1) :

$$R_i=\frac{3|r_1-r_0|^3}{2|(r_1-r_0)\times(r_2-r_1)|};$$

$$R_{i+1}=\frac{3|r_3-r_2|^3}{2|(r_2-r_1)\times(r_3-r_2)|}.$$

Expressing the radius-vectors of the parameters of the control polygon we get

$$R_i = \frac{3c^3}{4S_1}, \quad R_{i+1} = \frac{3d^3}{4S_2},$$

where *c, d* – are the lengths of the sides of the control polygon (Fig.1); *S1* та *S2* – squares of triangles *P0P1P2* and *P1P2P3.*

Conclusions. Proposed method of control the patterns of change of curvature along the flat cubic B-spline through the parameters of the polygon, being set. The developed scheme allows to determine the parameters of the control polygon, in which the task of forming a plot B-spline with monotonic variation of the curvature has a solution. Possible under the terms of the problem parameters the control polygon determines the range of radii of curvature at the points bounding the area cubic B-spline. The definition of the mentioned ranges provides an opportunity to reconcile the values of the radii of curvature at the junction of sections B-spline and form a perimeter of the second order of smoothness. A task for future research is to monitor the differential-geometric characteristics along cubic b-splines consisting of an arbitrary number of plots. Development of a method of formation cubic B-spline with control of the regularity of changes of curvature will provide an effective tool for modelling complex surfaces with desired functional qualities.

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