STUDY OF OSCILLATIONS OF A SPRING PENDULUM TRUCK BY THE EXAMPLE OF HELICOPTER SUSPENSION MODEL

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The determining of the non-chaotic trajectory of cargo vibration on external sling rope in helicopter under the account of the elastic properties of the cable is examined.

Keywords: **the helicopter rope suspension, the elastic properties of the cable, Lagrangian, Lagrange equation of 2-nd kind.**

Formulation of the problem. Transportation of cargo on external sling rope helicopter has proven its effectiveness in assembly and rescue operations. Delivery of water to a helicopter fighting forest fires is an effective method to eliminate them. But uncontrolled swinging load in the longitudinal-transverse direction can cause an emergency situation. Fight with vibrations achieved a decrease airspeed or implementation of horizontal or vertical accelerations of the helicopter, which greatly depends on the skill of the pilot. [1] Therefore, the actual work will be related to the study of the traffic load suspension point to quickly extinguish fluctuations.

Analysis of recent research and publications. There are many works devoted to the dynamics of the movement of foreign helicopter rope suspension (see. For the thesis [1]). The ultimate goal of such research is to build hardware that can assist the pilot in case of emergencies. To do this, compiled and researched various mathematical models describing the process fluctuations rope suspension. For understanding the physics of the process and identify the main factors often considered a simplified model of the phenomenon, which should always check with more accurate models and field tests. In [2] it is believed that fluctuations rope suspension occurs in the plane. In models of [1.2] load on the helicopter rope suspension seen as spherical pendulum floating-point suspension. But an adequate description of the process must take into account fluctuations and elastic properties of the rope suspension helicopters.

The wording of article purposes. Nehaotychnoyi develop a method for determining the trajectory of the cargo on external sling rope helicopter provided taking into account the elastic properties of the rope.

Main part. As the helicopter rope suspension are increasingly using modern synthetic materials such as «Kevlar» or «Dyneema», which far exceed the strength of steel products and thus much easier. However, synthetic materials are flexible and this property should be considered in the calculations. Note that some fiber elongation can reach 1% -3% of the length rope.

Consider oscillation system "helicopter-load" when moving suspension point and load occur in the same plane. Assume the following assumptions [2]: weightless rope suspension is resilient in the longitudinal direction and the same in the transverse direction, the hinge at the point of suspension load is ideal aerodynamic damping of oscillations is absent, the load is spherical radius which is considerably less than the length of rope, wire rope attached to the load at its center of mass.

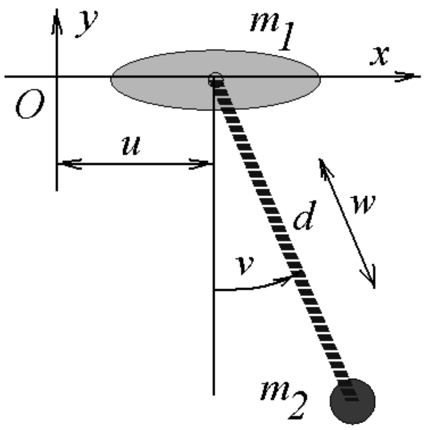

Fig. 1. Scheme helicopter rope suspension

The scheme oscillatory system (in literature "pendulum trolley") is shown in Fig. 1 where *m1* - mass of the helicopter, *m2* - mass of the load, *d* - the length of the cable, which is equal stiffness *k*. In addition, as generalized coordinates selected: *u (t)* - horizontal displacement helicopter, *v (t)* - the cable angle from the vertical, and *w (t)* - elastic extension cable.

To study the dynamic characteristics of helicopter external suspension was prepared and resolved relative to the coordinate system of generalized Lagrange equations of the second kind. For this used [3] Lagrangian $L = K - P$, where the formula for kinetic and potential energy are:

$$K := \frac{1}{2}(m1 + m2)\left(\frac{d}{dt}\mathrm{u}(t)\right)^2 + \frac{1}{2}m2\left(\left(\frac{d}{dt}\mathrm{w}(t)\right)^2 + \mathrm{w}(t)^2\left(\frac{d}{dt}\mathrm{v}(t)\right)^2 + 2\left(\frac{d}{dt}\mathrm{u}(t)\right)\left(\left(\frac{d}{dt}\mathrm{w}(t)\right)\sin(\mathrm{v}(t)) + \mathrm{w}(t)\left(\frac{d}{dt}\mathrm{v}(t)\right)\cos(\mathrm{v}(t))\right)\right)$$

$$P := -m2\, g\, \mathrm{w}(t)\cos(\mathrm{v}(t)) + \frac{1}{2}k(\mathrm{w}(t) - d)^2$$
.

The system of Lagrange equations of the second kind is:

$$(m1+m2)\left(\frac{d^2}{dt^2}\mathrm{u}(t)\right)+\frac{1}{2}m2\left(2\left(\frac{d^2}{dt^2}\mathrm{w}(t)\right)\sin(\mathrm{v}(t))\right.$$
$$+4\left(\frac{d}{dt}\mathrm{w}(t)\right)\cos(\mathrm{v}(t))\left(\frac{d}{dt}\mathrm{v}(t)\right)+2\,\mathrm{w}(t)\left(\frac{d^2}{dt^2}\mathrm{v}(t)\right)\cos(\mathrm{v}(t))$$
$$\left.-2\,\mathrm{w}(t)\left(\frac{d}{dt}\mathrm{v}(t)\right)^2\sin(\mathrm{v}(t))\right)=0$$

$$\frac{1}{2}m2\left(4\,\mathrm{w}(t)\left(\frac{d}{dt}\mathrm{v}(t)\right)\left(\frac{d}{dt}\mathrm{w}(t)\right)+2\,\mathrm{w}(t)^2\left(\frac{d^2}{dt^2}\mathrm{v}(t)\right)+2\left(\frac{d^2}{dt^2}\mathrm{u}(t)\right)\mathrm{w}(t)\cos(\mathrm{v}(t))\right.$$
$$\left.+2\left(\frac{d}{dt}\mathrm{u}(t)\right)\left(\frac{d}{dt}\mathrm{w}(t)\right)\cos(\mathrm{v}(t))-2\left(\frac{d}{dt}\mathrm{u}(t)\right)\mathrm{w}(t)\sin(\mathrm{v}(t))\left(\frac{d}{dt}\mathrm{v}(t)\right)\right)$$
$$-m2\left(\frac{d}{dt}\mathrm{u}(t)\right)\left(\left(\frac{d}{dt}\mathrm{w}(t)\right)\cos(\mathrm{v}(t))-\mathrm{w}(t)\left(\frac{d}{dt}\mathrm{v}(t)\right)\sin(\mathrm{v}(t))\right)+m2\,g\,\mathrm{w}(t)\sin(\mathrm{v}(t))=0$$

$$\frac{1}{2}m2\left(2\left(\frac{d^2}{dt^2}\mathrm{w}(t)\right)+2\left(\frac{d^2}{dt^2}\mathrm{u}(t)\right)\sin(\mathrm{v}(t))+2\left(\frac{d}{dt}\mathrm{u}(t)\right)\cos(\mathrm{v}(t))\left(\frac{d}{dt}\mathrm{v}(t)\right)\right)$$
$$-\frac{1}{2}m2\left(2\,\mathrm{w}(t)\left(\frac{d}{dt}\mathrm{v}(t)\right)^2+2\left(\frac{d}{dt}\mathrm{u}(t)\right)\cos(\mathrm{v}(t))\left(\frac{d}{dt}\mathrm{v}(t)\right)\right)-m2\,g\cos(\mathrm{v}(t))+k\,(\mathrm{w}(t)-d)=0$$

Untie this system will equations numerically [4,5] with Runge-Kutta with initial conditions *u (0) = u0, u` (0) = Du0, v (0) = v0, v` (0) = Dv0* and subject to determine the values of elongation rate Dv0 rope depending on other parameters constant circuit. For definiteness choose the setting (all in arbitrary units): *m1 = 6,5`103* - mass of the helicopter; *m2 = 103* - bulk cargo*; k = 8`104* - stiffness rope; *d = 30* - rope length, *g = 9,81*.

In the process of calculation should take into account the speed *Dv*0 extension cables, which provide value nehaotychnu trajectory of movement of goods. Will untie the system of equations numerical method of Runge-Kutta conditions: *u0 = 0; u`0 = 0; v0 = 0,01; v`0 = 0; w0 = 30*. As a result, image building close integral curve in phase space *{v, Dv, t}*, which will depend on the specific meaning of "control" option *Dv0*. At random values *Dv0* phase space *{v, Dv, t}* formed "confused" integral curve, the projection of which on the phase plane *{v, Dv}* will also be "confusing" phase trajectories (Fig. 2a) that lead to chaotic motion circuit elements suspension. If you change the values of "control" option *Dv0* has changed and the nature of the phase trajectory. When the critical value *Dv0 = 0* the trajectory will change to qualitative level - turn into "a natural" curve (Fig. 2b). Fig. 3 are derived graphics functions *u (t), v (t)* and *w (t).*

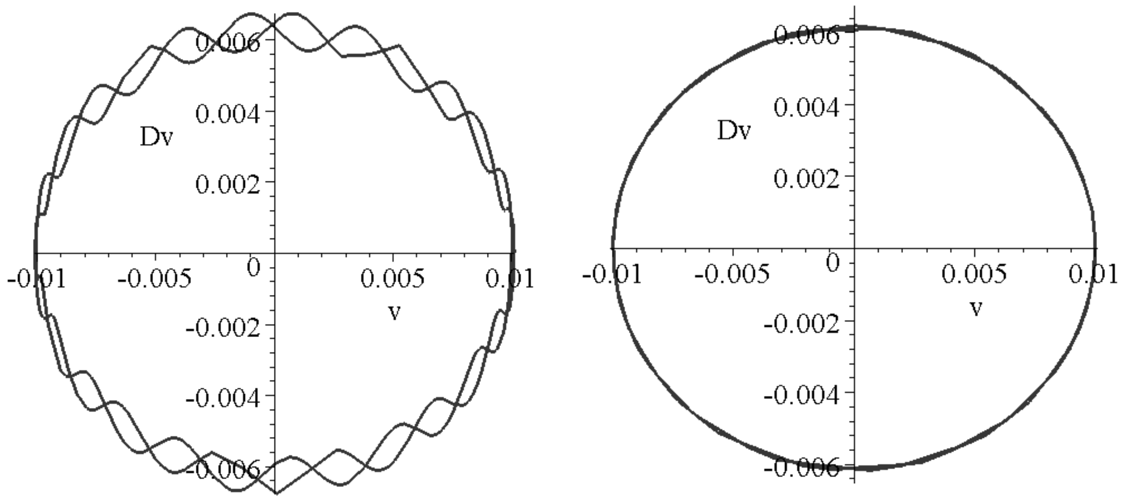

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Fig. 2. Phase trajectory for:

a) random value Dv0; b) the calculated value Dv0 = 0

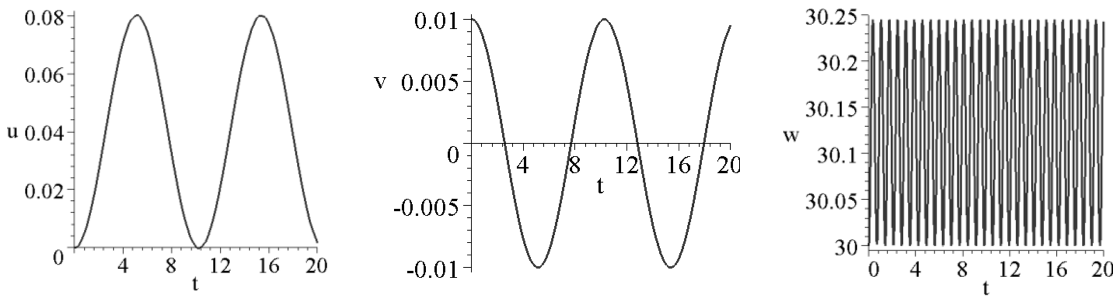

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Fig. 3. The graphics functions: a) u (t); b) v (t); a) w (t)

Thus, in this example, taking into account the importance Dv0 = 0 in the process of solving the Lagrange equations of the second kind allows calculate approximately generalized coordinates u (t), v (t), and w (t), which provide movements nehaotychni time load on the suspension . Ensure this by using Fig. 4, which shows the trajectory of movement of the center point load.

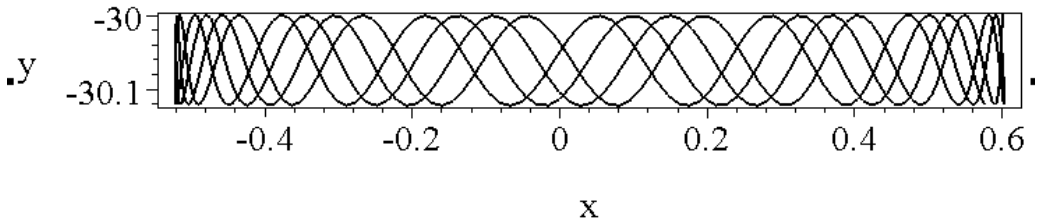

Fig. 4. The trajectory of movement of the center point load

Conclusions. Reproduced a way to evaluate fluctuations of external parameters suspension helicopter. Further studies will be associated with determining the limits of parameters to ensure the necessary movements suspension scheme.

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