MODELING OF OSCILLATION OF PENDULUM UNDER THE TROLLEY AS MECHANICAL MODEL OF LIQUID IN VESSEL

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Summary. A method for determining the non-chaotic trace of oscillations of a pendulum under the truck, combined with a spring, which brings the wave profile of fluid as a result of longitudinal vibrations of the container is examined.

Keywords: **fluid wave profile, the pendulum under the truck, Faraday waves, mechanical vibrations analogue liquid.**

Formulation of the problem. Research of liquid fluctuations in the moving containers are important for solving many problems [1-3]. One example is the problem of horizontal movements of the liquid in the tank with the emergence of Faraday waves [1]. These longitudinal oscillations to study using their mechanical analogue - the pendulum under the trolley (Fig. 1, taken from [1]).

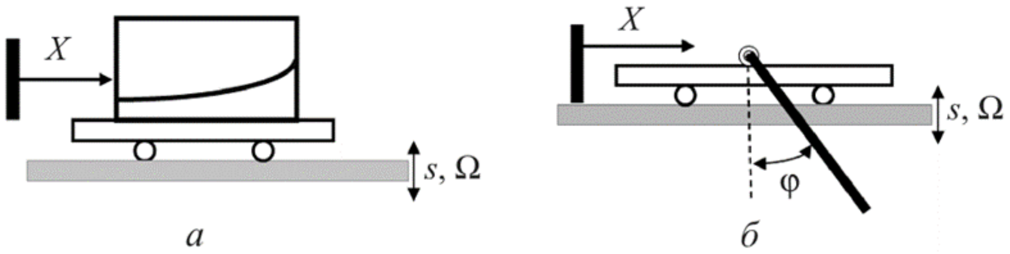


Fig. 1. Experiment with Faraday waves in moving containers (a);

and their mechanical analogue - pendulum under the trolley (b)

The use of "pendulum" analogy to study hydrodynamic effects of fluctuations of the liquid in containers can simplify the course of calculations and visualized the results of calculations.

Analysis of recent research and publications. In article [2] discovered the dynamic pressure tanker on the roadway during braking considering moving fluid that is relevant, for example, to operate fire engines. In work [3] investigated the dynamic model of tanker capacity allocation based on compartments. Used an analogy between the motion of a pendulum under the trolley and related fluctuations in fluid containers in case of parametric perturbations standing waves. Also considered resonant oscillation of the pendulum at the lower equilibrium fluctuations under vertical suspension point.

These purely analytical researches should be supplemented with graphics illustrate the results of the pendulum in the trolley.

Formulation of article purposes. Develop a method for determining of non-chaotic trace pendulum under the trolley, combined with a spring that has a closer form of wave profile liquid that results from the longitudinal oscillations capacity.

Main part. We assume that the parameters of hydrodynamic waves in the mobile fluid containers can be associated with the following parameters pendulum trolley, combined with spring: m1 - mass of the trolley; m2 - mass of the pendulum; k - stiffness of spring. d - the length of the pendulum (Fig. 2).

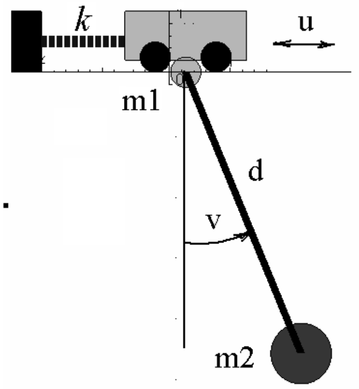


Fig. 2. Scheme of the pendulumin under the trolley

From a geometrical point of view to solve this problem it is necessary to determine a combination of these parameters to provide education non-chaotic track of the movement of a pendulum (also non-chaotic traces of "nodes" should be deleted). Then consider shall be found close profile of a wave of liquid, which was the result of longitudinal oscillations capacity (Fig. 1 a).

The dynamics of pendulum oscillation under the trolley we will describe as a system of differential equations of the second kind Lagrange:

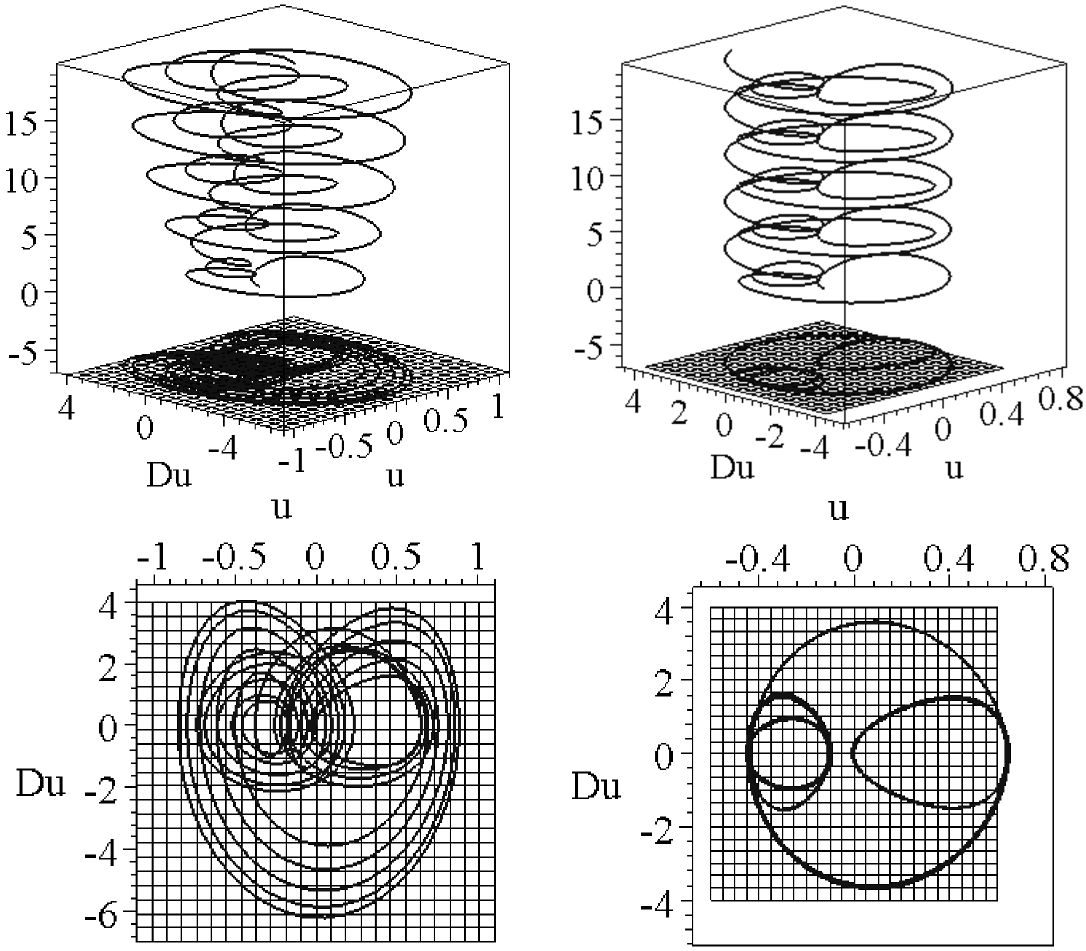
$$m1\left(\frac{d^2}{dt^2}\mathrm{u}(t)\right)+m2\left(\frac{d^2}{dt^2}\mathrm{u}(t)\right)+\frac{1}{2}m2\,d\left(\frac{d^2}{dt^2}\mathrm{v}(t)\right)\cos(\mathrm{v}(t))$$
$$-\frac{1}{2}m2\,d\left(\frac{d}{dt}\mathrm{v}(t)\right)^2\sin(\mathrm{v}(t))+k\,\mathrm{u}(t)=0\,, \qquad (1)$$
$$\frac{1}{6}m2\,d\left(3\left(\frac{d^2}{dt^2}\mathrm{u}(t)\right)\cos(\mathrm{v}(t))+2\,d\left(\frac{d^2}{dt^2}\mathrm{v}(t)\right)+3\,g\,\sin(\mathrm{v}(t))\right)=0\,.$$

It uses syntax of Maple language and selected: u (t) - the horizontal position of the trolley at time t; v (t) - the angle deviation of the pendulum from the vertical; g = 9,81.

Untie the system of equations (1), we numerically using Runge-Kutt method with initial conditions u(0) = u0, u′(0) = Du0, v(0) = v0, v′(0) = Dv0.

Here is an example of calculating of the non-chaotic trajectory of the pendulum provided calculations stiffness k constants depending on other system parameters. For definiteness choose the settings (all in arbitrary units): $m_1 = 1$; $m_2 = 20$; $d = 5$. The initial conditions are chosen: $u(0) = 0$, $u'(0) = 0$, $v(0) = \pi/3$, $v'(0) = 0$.

Numerically solve the system of equations (1) and construct an approximate image integral curve in phase space {U, Du, t}. Images consist of a plurality of segments connecting successive points obtained from the approximate solution of systems of equations. This will depend on the particular illustrate the importance of "managing" the parameter k. At random values k in the phase space {u, Du, t} formed "confused" integral curve, the projection of which on the phase plane {u, Du} will also be "confusing" phase trajectory (Fig. 3, a), leading to chaotic pendulum movements in trolley. If you change the values of "managing" the parameter k has changed and the nature of the phase trajectory.



a б

Fig. 3. Integral curves and phase trajectories for:

a) random value k; b) the calculated value k = 323,5

When the critical value k = 323,5 change it at a qualitative level - will become a "focused" curve (Fig. 3, B). In the phase plane {u, Du} if true optical effect "pointing to sharpen" the confusion phase trajectories (OHP will focus [4, 5]). Considering the value of k = 323,5 in the process of solving the system of equations (1) can calculate the coordinates in the coordinate system {x, y} (Fig. 4a), which are disposed on non-chaotic trace the trajectory of the pendulum. If you selected "input" parameters, there are several values of k, which cause non-chaotic fluctuations of the track of pendulum (Fig. 4, b-d).

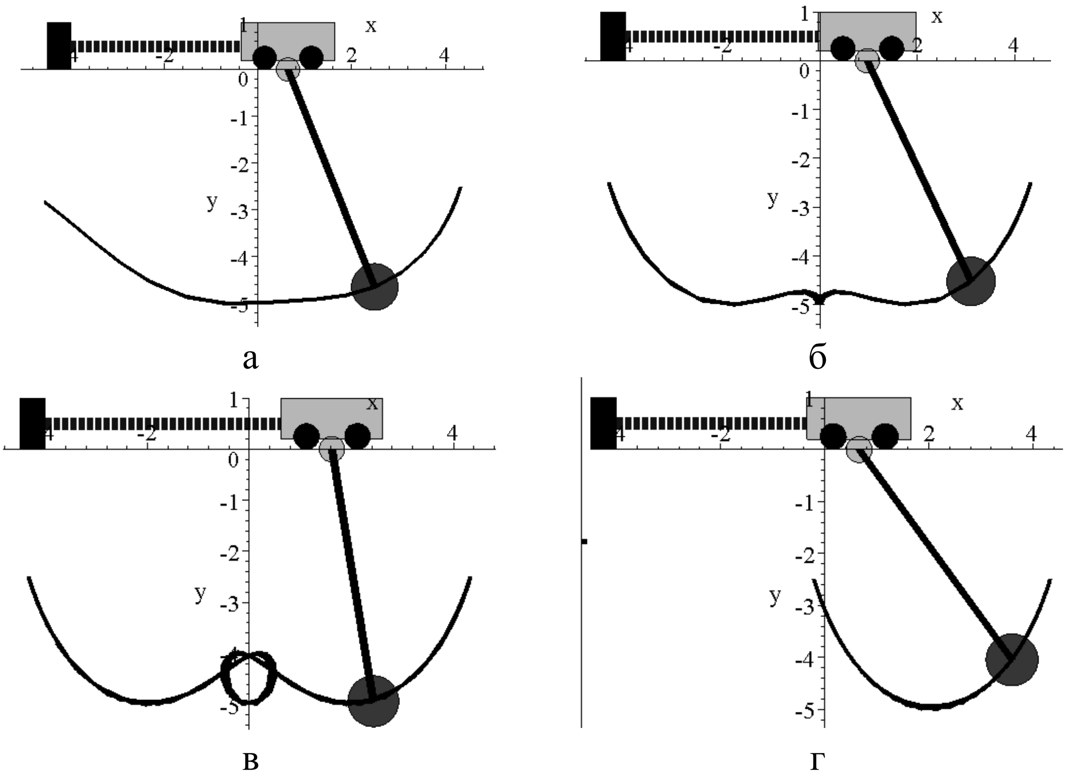


Fig. 4. Pendulum tracks in the value data:

a) k = 323,5; b) k = 61,6; c) k = 25,2; d) k = 0.

With consideration should be given preference option with parameters k = 323,5 (Fig. 4a). This is the minimum area formed by the phase trajectory measured by the number of pixels that constitute it and the necessary asymmetrical profile form liquid waves arising from longitudinal oscillations capacity.

Conclusions. The developed method allows recommending the parameters for non-chaotic trajectory of the pendulum in the trolley, which brings liquid form wave profile that results from the longitudinal oscillations capacity. Further research will be related to the choice of parameters pendulum oscillation cart forecast for experiments with form of trajectory.

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