USING OF DIFFERENT SCHEMES OF THE FIRST ORDER IN METHOD OF DISCRETE INTERPOLATION ON A BASIS VARIATIVE FORMATIONS DIFFERENCE SCHEMES OF ANGULAR PARAMETRES

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Summary. The proposed solution to the problem of interpolation DPC by method of variable angular parameters of difference schemes with additional conditions obtained by the superposition of additional conditions on ratio between the adjacent angles that leads to the formation of the first order difference schemes.

Keywords: **interpolation, difference schemes, angular parameters, variability discrete geometric modeling.**

Formulation of the problem. New features in the modeling and management of shape arbitrary DPC can be obtained by blending a variety of additional conditions on the contiguity ratio of the angles in the proposed variation scheme [1]. In addition, it is necessary to enable them to correct the shape of the modeled curve, possessed speed and simplicity of calculations and correspond to the internal geometry of the original DPC.

Analysis of recent research and publications. Most of the additional conditions [1-4] impose thickened DPC certain properties: equality of the angle at the corner adjacency to the point; equal angle at the corner adjacency at the point; equality adjacency angle at half the sum of the angles at the points and adjacency; equality of all adjacency angles at the points of condensation, and many other conditions. However, along with the available benefits, designed for today VDGM methods, taking into account the angular settings [1-4], still have prospects for further development and research that have been considered in Naydisha VM and his disciples, in the direction of expansion options to control the shape of the simulated curve. Therefore, the development of new methods, in the framework of the existing methods based on modeling VDGM angular parameters, in the absence of oscillations, is relevant.

Formulation of article purposes. It is proposed to expand the possibilities of modeling arbitrary DPC by imposing additional conditions on the new relationship between the angles of adjacency, which leads to the formation of the first-order difference scheme, with reference to the method of forming the difference schemes of variable angular parameters.

Main part. Consider the original plane curve (x_i, y_i), $i = 0,1,...,n$, which we need to thicken in the absence of oscillation. The main idea of the method of condensation forming on the basis of variable angular parameters of difference schemes is that the variability of the condensation circuit (1) has been proposed in [1].

$$(1-\eta_{i-1})\gamma^1_{i-0.5}+\gamma^1_i+\eta_i\gamma^1_{i+0.5}=\gamma^0_i, \quad i=\overline{1;n-1}. \qquad (1)$$

However, the expression (1) is not a differential circuit. Therefore, in order to find additional equations, you must apply one of the additional conditions, which will connect the corners of the adjacency source and thickened DPC. We write down an additional condition (2) to the contiguity ratio of the angles, which brings the ratio between the angles of the adjacency to the original on the thickened DPC while maintaining its geometric properties (Figure 1.)

$$\frac{\gamma^0_{i+1}}{\gamma^0_i}=\frac{\gamma^1_{i+0,5}}{\gamma^1_i}, \text{ thence } \gamma^1_i=\frac{\gamma^0_i}{\gamma^0_{i+1}}\cdot\gamma^1_{i+0,5} \quad i=\overline{0;n-1}. \qquad (2)$$

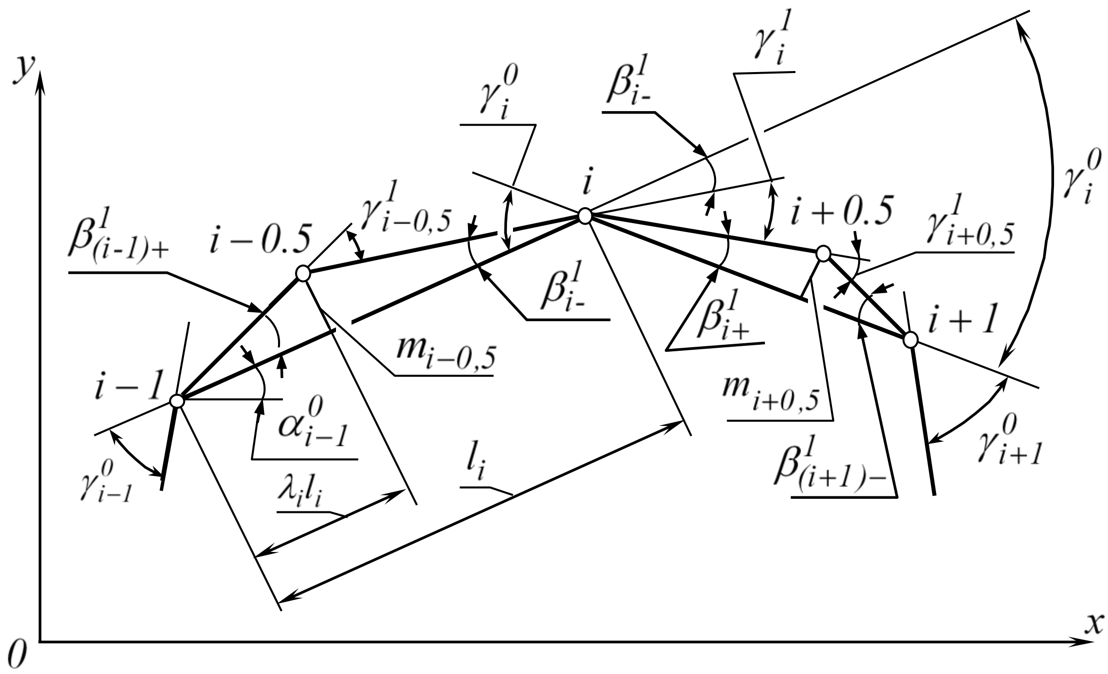

Fig. 1. General scheme of thickening

Expression (1) with (2) takes the following form:

$$(1-\eta_{i-1})\gamma^1_{i-0,5}+\left(\frac{\gamma^0_i}{\gamma^0_{i+1}}+\eta_i\right)\gamma^1_{i+0,5}=\gamma^0_i, \; i=\overline{1;n-1}. \qquad (3)$$

The resulting system of equations (3) is the difference scheme of the first order and relates the angles at the points of contiguity thickening thickened DPC with angles adjacency source DPC. We have equations with unknowns. One of the adjacency angles can be taken as a control parameter, and set it in such a way as to ensure the condition of absence of oscillations. Let us take as a control parameter is one of the angles at the point of thickening.

For the solution of the difference scheme (3), and the construction of condensation points apply the following method.

1. From the system of equations (3) are determined according to the adjacency between the corners at the points of condensation - the expression (4).

$$\begin{array}{ll} \gamma^1_{1,5} = A_{1,5} - B_{1,5} \cdot \gamma^1_{0,5} & \\ \gamma^1_{2,5} = A_{2,5} - B_{2,5} \cdot \gamma^1_{1,5} & i = \overline{1;n-1}, \\ \dots\dots\dots\dots & \\ \gamma^1_{i+0,5} = A_{i+0,5} - B_{i+0,5} \cdot \gamma^1_{i+0,5} & \end{array} \tag{4}$$

where $A_{i+0,5} = \dfrac{(\gamma^0_i + \gamma^0_{i+1}) \cdot \gamma^0_{i+1}}{\gamma^0_i + 2\gamma^0_{i+1}}$, $B_{i+0,5} = \dfrac{(\gamma^0_i + \gamma^0_{i+1}) \cdot \gamma^0_{i+1}}{(\gamma^0_{i-1} + \gamma^0_i) \cdot (\gamma^0_i + 2\gamma^0_{i+1})}$ – some coefficients.

2. One of the corners of the adjacency to the point of condensation is chosen as the control parameter, after which all the angles are expressed in adjacency points condensation through the control parameter.
3. Impose conditions of absence of oscillations [4] obtained in the preceding paragraph, depending.
4. Solve the resulting system of inequalities with respect to the control parameter. From the segment of solutions selected allowable value of the control parameter.
5. Calculate the value of the remaining corners of the adjacency to the point of thickening ($\gamma^1_{i+0,5}$, $i = \overline{0;n-1}$) by substituting the value of the control parameter obtained in (4) dependence. The values of the angles at the nodal points of contiguity ($\gamma^1_i, i = \overline{0,n-1}$) are from the expression (2).
6. Taking into account the basic geometric characteristics of the adjacency angles determined thickened curve [5]. Then determs coordinates $x_{i+0,5}, y_{i+0,5}$ of points of thickening under the basic algorithm of thickening.
7. The criterion for the end of the thickening is to achieve the conditions (5) at the k-th step of thickening

$$\max_{i=0..n-1} \left| \gamma^1_{i+0,5} \right| \le \varepsilon, \tag{5}$$

Where $\varepsilon \geq 0$ - arbitrarily small preassigned number.
If you want to continue the thickening point and then enumerated a number of the calculation is repeated.

8. Upon reaching the condition (5) in terms of thickened DPC connected segments, which is considered to be the ultimate form of the interpolating curve.

As an example, consider the solution of the problem of condensation discrete representation of a point series (Table 1) in the non-uniform grid with an accuracy of $\varepsilon = 0{,}4$, provided that the resulting discrete representation of the curve should be non-oscillating.

Table 1

The starting number line

i	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
x_i^0	10	25	50	70	100	120	110	90	70
y_i^0	20	50	80	90	90	75	50	35	50

Thickening test example was produced according to the methodology described above in accordance with the basic algorithm of the method of thickening [5].

As the angle of the control parameter is adopted adjacency $\gamma_{0,5}^1$ and expressed all the other angles $\gamma_{i+0,5}^1, i = \overline{1,7}$ through the angle $\gamma_{0,5}^1$ (Table 2, 2).

Superimposed on the resulting system of equations conditions for the absence of oscillations (Table 2, 3). The result is the following system of inequalities (Table 2, 4).

Decision of the resulting system of inequalities is a segment $[-26.481; 0]$ from which the selected value of the control parameter. It takes the value of the control parameter to $\gamma_{0,5}^1 = -10$.

Taking into account the obtained values of the control parameter determined by the values of the remaining corners of the adjacency (Table 2, 5).

Checks on the need for further thickening by the condition (5)

$$max\left|\gamma_{i+0,5}^1\right| = 0{,}635 > 0{,}4 .$$

This condition is not satisfied, therefore, a number of points are renumbered and made the next step of thickening.

Table 1

The results of the settlements methodology described above
for 1 step of thickening

ANGLE	Methods of solution of difference schemes			
	п.2	п.3	п.4	п.5
				$\gamma^{1}_{0,5}$ =**-10**
$\gamma^{1}_{1,5}=$	$-14{,}400-0{,}544\,\gamma^{1}_{0,5}$	$\gamma^{1}_{1,5}<0$	$\gamma^{1}_{0,5}>-26{,}48$	$\gamma^{1}_{1,5}=-8{,}962$
$\gamma^{1}_{2,5}=$	$-10{,}587+0{,}256\,\gamma^{1}_{0,5}$	$\gamma^{1}_{2,5}<0$	$\gamma^{1}_{0,5}<41{,}32$	$\gamma^{1}_{2,5}=-13{,}149$
$\gamma^{1}_{3,5}=$	$-18{,}399-0{,}119\,\gamma^{1}_{0,5}$	$\gamma^{1}_{3,5}<0$	$\gamma^{1}_{0,5}>-154{,}62$	$\gamma^{1}_{3,5}=-17{,}209$
$\gamma^{1}_{4,5}=$	$-31{,}851+0{,}084\,\gamma^{1}_{0,5}$	$\gamma^{1}_{4,5}<0$	$\gamma^{1}_{0,5}<378{,}41$	$\gamma^{1}_{4,5}=-32{,}693$
$\gamma^{1}_{5,5}=$	$-17{,}303-0{,}018\,\gamma^{1}_{0,5}$	$\gamma^{1}_{5,5}<0$	$\gamma^{1}_{0,5}>-949{,}64$	$\gamma^{1}_{5,5}=-17{,}120$
$\gamma^{1}_{6,5}=$	$-36{,}274+0{,}007\,\gamma^{1}_{0,5}$	$\gamma^{1}_{6,5}<0$	$\gamma^{1}_{0,5}<4882{,}17$	$\gamma^{1}_{6,5}=-36{,}349$
$\gamma^{1}_{7,5}=$	$-32{,}188-0{,}004\,\gamma^{1}_{0,5}$	$\gamma^{1}_{7,5}<0$	$\gamma^{1}_{0,5}>-9262{,}66$	$\gamma^{1}_{7,5}=-32{,}153$

Two steps of thickening was carried out to ensure the specified accuracy thickening. The second condition for condensation step (5) was observed.

$$max\left|\gamma^{1}_{i+0,5}\right|=0{,}318<0{,}4$$

MAPLE mathematical package was used to perform calculations of data.

Conclusions. The paper considers the new additional condition on the contiguity ratio of the angles, which resulted in the substitution scheme variative thickening (1) is obtained by the difference scheme of the first order. A feature of this scheme, firstly, is its simplicity, as a result of the solutions of the difference circuit have segment solutions instead polygon (simpler software implementation), and secondly, this additional condition imposes thickened DPC some specific properties, brings the ratio between the angles of the adjacency to the original on the thickened while maintaining its geometric properties.

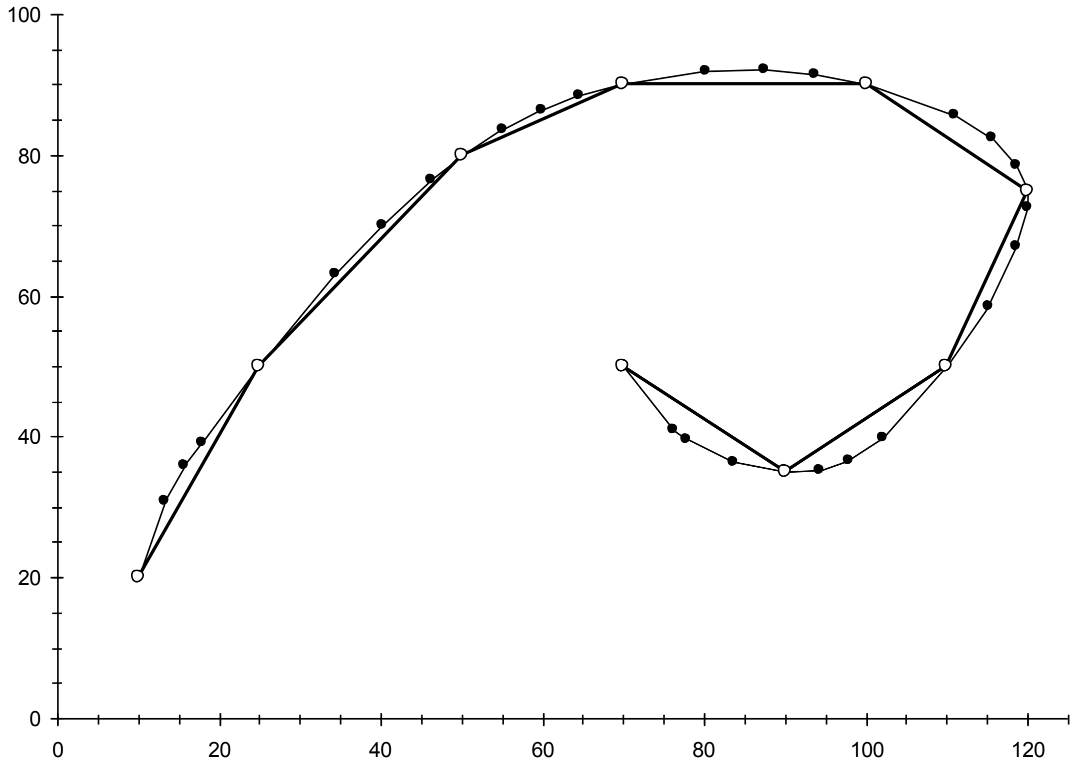

Fig.2. The result of thickening of test DPC at imposing an additional condition (2) in variative scheme (1).

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