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THE FUNDAMENTAL ISOTROPIC SPLINES

N. Ausheva

In the paper is highlighted the way to build the fundamental isotropic splines. The conditions of isotropy for spatial spline based on four points of point row are found. The example of calculated spline and graphics of curvature and twisting functions are given.

Keywords: fundamental spline, isotropic curve, isotropy conditions, Catmull-Rom spline.

Formulation of the problem. In constructing the absolute minimum surface on the basis of Weierstrass [1], the problem of the isotropic spatial curve, which significantly affects the resulting surface. When creating interactive user advisable to use Bezier curves that allow you to control curve shape, but then must perform additional calculations to calculate tangents. To avoid these calculations can apply fundamental splines, based only on information from a number of point [2].

Analysis of recent research and publications. Work [3] covers a method for constructing infinite family of minimal surfaces using Weierstrass equation with the guides in the form of isotropic Bezier curve of 3rd order. The conditions isotropy presented in the form of five nonlinear equations. In [4] proposed to use for modeling planar isotropic mesh curve for the travel time of Pythagoras (PH). Building a grid based on konfornoyi and kvizikonformnoyi replacement option. The paper [5] investigate the modification of periodic B-spline parameterization of normalized curves to create a zero length. Points characteristic polygon defined in complex form. The conditions for the formation of isotropic curves.

The wording of article purposes. The aim of this work is to develop a method of constructing isotropic curve based on the fundamental equation spline.

Main part. Construct a curve of zero length based on the fundamental spline equation [2]. The plot spline fundamental provisions given four points of the set point frame and the tangent at every point coordinates are calculated for two adjacent points. Let fundamental spline is given as:

$$\begin{aligned}\mathbf{r}(t) = {} & [(\mathbf{r}_{j+1} - \mathbf{r}_{j-1})(t^3 - 2t^2 + t) + (\mathbf{r}_{j+2} - \mathbf{r}_j)(t^3 - t^2)]s + \\ & + [\mathbf{r}_j(2t^3 - 3t^2 + 1) + \mathbf{r}_{j+1}(-2t^3 + 3t^2)],\end{aligned} \quad (1)$$

where $\mathbf{r}_{j-1}$, $\mathbf{r}_j$, $\mathbf{r}_{j+1}$, $\mathbf{r}_{j+2}$ - point given point frame (fig. 1), $s = \frac{1-u}{2}$, u - setting tension spline. If $u = 0$, we kind of fundamental spline and spline is Katmall Roma (Catmull-Rom slines).

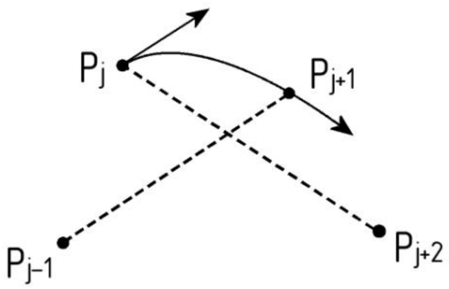


Fig. 1. Formation of the fundamental spline four points

Take derivative of expression (1):

$$\mathbf{r}'(t) = (\mathbf{r}_{j+1} - \mathbf{r}_{j-1})s + 2t[-2s(\mathbf{r}_{j+1} - \mathbf{r}_{j-1}) - s(\mathbf{r}_{j+2} - \mathbf{r}_j) + 3(\mathbf{r}_{j+1} - \\ - \mathbf{r}_j)] + 3t^2[s(\mathbf{r}_{j+1} - \mathbf{r}_{j-1}) + s(\mathbf{r}_{j+2} - \mathbf{r}_j) - 2(\mathbf{r}_{j+1} - \mathbf{r}_j)]. \quad (2)$$

Substituting in the original condition for zero length curves:

$$x(t)'^2 + y(t)'^2 + z(t)'^2 = 0. \quad (3)$$

Condition (3) is executed and does not depend on the value, if all the coefficients of equal powers of 0. To simplify the expressions, we introduce an additional condition:

$$\sum_{r=x,y,z} (r_{j+2} - r_j)^2 = 0. \quad (4)$$

Will receive the following factors:

- coefficients by t^0 :

$$\sum_{r=x,y,z} (r_{j+1} - r_{j-1})^2 = 0, \quad (5)$$

- coefficients by t^1 :

$$-s \sum_{r=x,y,z} (r_{j+1} - r_{j-1})(r_{j+2} - r_j) + 3 \sum_{r=x,y,z} (r_{j+1} - r_{j-1})(r_{j+1} - r_j) = 0, \quad (6)$$

- coefficients by t^2 :

$$18 \sum_{r=x,y,z} (r_{j+1} - r_j)^2 - 12s \sum_{r=x,y,z} (r_{j+2} - r_j)(r_{j+1} - r_j) + \\ s^2 \sum_{r=x,y,z} (r_{j+1} - r_{j-1})(r_{j+2} - r_j) = 0 , \quad (7)$$

- coefficients by t^3 :

$$-s \sum_{r=x,y,z} (r_{j+1} - r_{j-1})(r_{j+2} - r_j) + 3 \sum_{r=x,y,z} (r_{j+2} - r_j)(r_{j+1} - r_j) = 0, \quad (8)$$

- coefficients by t^4 :

$$4\sum_{r=x,y,z}(r_{j+1}-r_j)^2 - s\sum_{r=x,y,z}(r_{j+2}-r_j)(r_{j+1}-r_j) - s\sum_{r=x,y,z}(r_{j+1}-r_{j-1})(r_{j+1}-r_j)=0, \qquad (9)$$

Substituting expression (6) to (8) and we have:

$$\sum_{r=x,y,z}(r_{j+1}-r_{j-1})(r_{j+1}-r_j)=\sum_{r=x,y,z}(r_{j+2}-r_j)(r_{j+1}-r_j). \qquad (10)$$

If obtained dependence (9) and (10) substituted into the condition (7) then get expression that is equal to or (8), or (6). In result of the simplifications will isotropy condition:

$$\begin{cases} \sum_{r=x,y,z}(r_{j+1}-r_{j-1})^2=0, \\ \sum_{r=x,y,z}(r_{j+2}-r_j)^2=0, \\ -s\sum_{r=x,y,z}(r_{j+1}-r_{j-1})(r_{j+2}-r_j)+3\sum_{r=x,y,z}(r_{j+1}-r_{j-1})(r_{j+1}-r_j)=0, \\ \sum_{r=x,y,z}(r_{j+1}-r_{j-1})(r_{j+1}-r_j)=\sum_{r=x,y,z}(r_{j+2}-r_j)(r_{j+1}-r_j), \\ 2\sum_{r=x,y,z}(r_{j+1}-r_j)^2-s\sum_{r=x,y,z}(r_{j+1}-r_{j-1})(r_{j+1}-r_j)=0. \end{cases} \qquad (11)$$

Example. Construct a real and one imaginary curve from a family of curves isotropic if set to: $x_{j-1}=1+2i$, $x_j=3+3i$, $x_{j+1}=5+4i$, $y_{j-1}=1-2i$, $y_i=3+i$, $y_{j+1}=3-1i$, $z_{j-1}=4+1i$ setting and tension $u=-0.8$. Find the isotropic coordinates of the curve in terms (11): $z_{j+1}=1.764+5.472i$, $z_j=4.37+3.702i$, $x_{j+2}=11.861+9.389i$, $y_{j+2}=1.374-8.581i$, $z_{j+2}=-5.747+10.837i$. The length of the curve in the complex space is 0 and the length of the real and imaginary curves are equal to each other $R_{kr\mathrm{Re}}=R_{kr\mathrm{Im}}=3.54$. Figure 2. Showing real curve in figure 3. - graphics curvature and twisting for the real and imaginary parts. As we see difficulties schedules coincide.

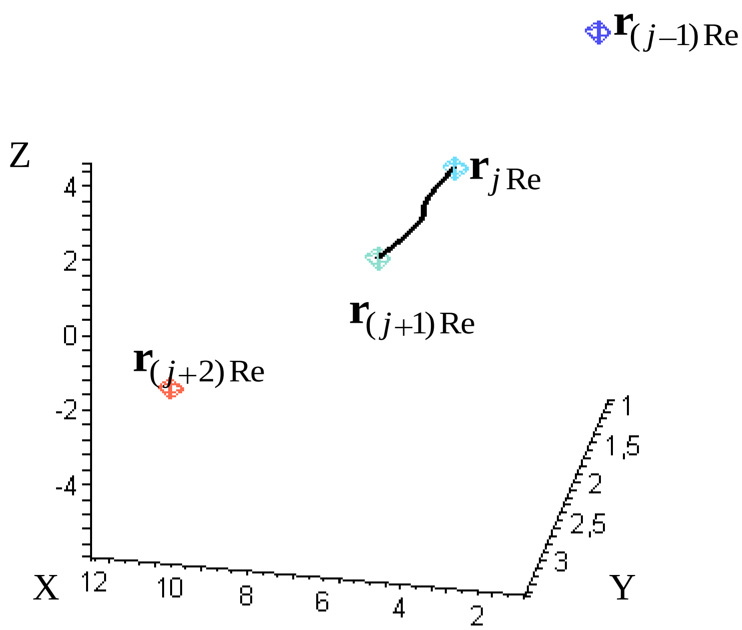
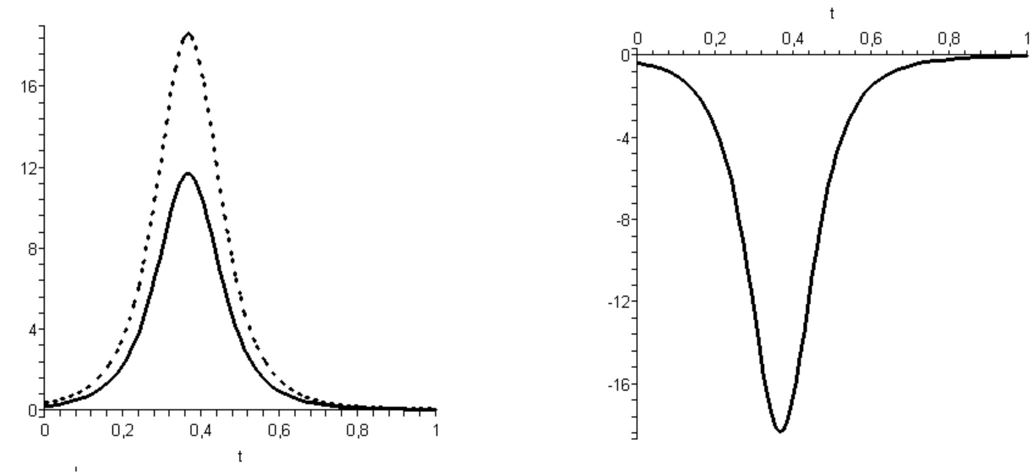


Fig. 2. The real part of the fundamental isotropic spatial spline



a) б)

Fig. 3. Graphs and functions) curvature, b) difficulties for real and fundamental parts isotropic spline

Conclusions. As a result, the research found isotropy conditions for fundamental spline. It was built of true isotropic spline and found the graphics curvature and twisting. Charts reflect difficulties for real and imaginary parts. Further research related to modeling rectangular portions based on isotropic curves.

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