УДК 514.18

REPLACEMENT OF THE SIMPLEX IN THE EQUATION OF PLANE CURVE AND ITS APPLICATION

I. Balyuba, E. Konopatskiy

In paper presents a way to replace the simplex in equation of plane curve, which allows you to convert equation of any arc plane curve and determine for each equation, the optimum in terms of arithmetic, simplex plane, as well as ways of its application in practice arcs plane curves modeling.

Keywords: BN-calculation, simplex plane, plane curve, dot equation, parabolic arc.

Formulation of the problem. In BN-calculus, the same curve in different simplexes has different point equations, from which one can choose more convenient for practical use. To simplify the calculations, it is important not only to choose the optimal parameter, but also the optimal simplex. The matter is that with a successful choice of a simplex at the stage of statement of the problem, the final point equation can be very much reduced, which in turn significantly affects the speed with the software implementation of such an equation. For example, a significant simplification of the point equation can be obtained if one of the angles of the simplex of the plane is assumed equal to $\frac{\pi}{2}$ (Cartesian coordinate system). In an equilateral flat simplex, there are also significant simplifications of the required point equation. A separate and very important task in BN-calculus is the problem of constructing curves arcs through preassigned points. To solve this problem, it is important to make maximum use of the given points as the vertices of a simplex. Proceeding from all the above, we can conclude that the problem of replacing a simplex in the equation of a plane curve is actual and can have wide practical application.

Analysis of recent research and publications. In analytic geometry [1], a similar problem is a coordinate transformation, which has found wide application and is effectively used in isolating the types of second-order curves from the general equation, determining their canonical equations, singular points, and the lines of these curves. On the other hand, for point equations, due to their specificity, the transformation of the coordinate system is not suitable. In affine geometry [2], which is closest to the BN-calculus, for the transformation of the affine coordinate system the matrix of the transition from one basis to another. But in the BN-calculus [3], there is no need to use the transition matrix, since the parallel projection invariant is used as the parameter in the equation. In this case, the problem of replacing a simplex is simplified and reduces to simple arithmetic operations on the points and functions of the parameter.

Formulation of the purpose of the article. Demonstrate by examples the possibilities of the method of replacing a flat simplex in BN-calculus.

Main part. In the general case, the problem can be formulated as follows.

A point equation of the arc of a curve in the simplex is given RPQ:

$$M=(P-R)p_P+(Q-R)q_Q+R, \tag{1}$$

where p_P, q_Q are functions of a certain parameter t.

It is required to determine the curve arc, which is determined by the current point M in the simplex CAB.

To solve the task, you need to perform the following steps: points P, Q, R express through dots C, A, B And substitute their values in equation (1). Thus, the equation takes the following form:

$$M=(A-C)p+(B-C)q+C. \tag{2}$$

where p, q are also functions of the parameter t.

Then the solution of the problem is reduced to the definition of functions p and q by specified functions p_P and q_Q.

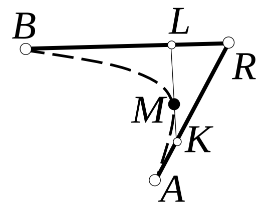

Рис. 1. Геометрическая схема конструирования дуги параболы

Let us consider several examples of the use of a simplex substitution for a plane arc of a curve.

Problem 1. A parabolic arc of a contour of the first order of smoothness is given:

$$M=(A-R)\bar{t}^{\,2}+(B-R)t^2+R,\ 0\le t\le 1. \tag{3}$$

It is required to determine the parabola equation, which is given by conjugate axes.

Decision. It is known from [4] that if $\frac{AK}{AR}=\frac{RL}{RB}=\frac{KM}{KL}=t$, Then equation (3) defines an arc of a parabola AMB (fig. 1). When $t=0\to M\equiv A,$ and when $t=1\to M\equiv B$, RA, RB – tangent to

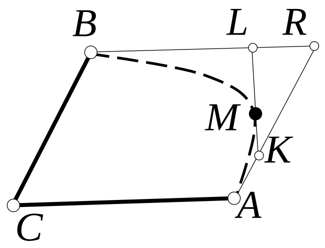

Рис. 2. Результат замены симплекса дуги параболы

the arc of a parabola.

Let us replace the simplex RAB for a new simplex CAB (fig. 2), where R is the fourth vertex of a parallelogram $BCAR$:

$$R = A + B - C.$$

We exclude R from the equation of the arc of the parabola (3), we obtain:

$$M = (A - A - B + C)\bar{t}^2 + (B - A - B + C)t^2 + A + B - C. \qquad (4)$$

Next, we transform equation (4) into the equation of the arc of the parabola in the new simplex CAB:

$$M = (A - C)\left[1 - t^2\right] + (B - C)t\left[2 - t\right] + C. \qquad (5)$$

The graphical algorithm (Figure 2) and the equation (5) determine the arc of the parabola previously defined by the graphic algorithm (fig. 1) and equation (3). For a parabola given by equation (5), the segments CA and CB are conjugate axes.

Problem 2. In accordance with [5], a geometric algorithm for constructing an arc of an ellipse along conjugate axes is given RA, RB (fig. 3): $SR = RA$, $M = AK \cap SQ$, $PK \parallel RB$. It is required to determine the equation of the arc of an ellipse M using this algorithm and replacing the simplex, transform this equation to an equation of the arc of an elliptical contour.

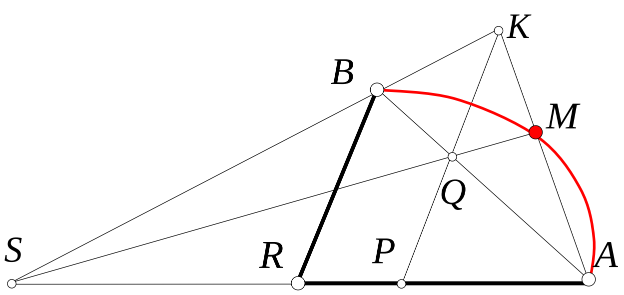

Рис. 3. Геометрический алгоритм построения дуги эллипса

Decision. We determine the equation of the ellipse according to the geometric algorithm (fig. 3). According to condition $SR = RA$ [5] determine the point S: $S = 2R - A = -(A - R) + R$.

As a parameter t we adopt the following relation:

$$t = \frac{PR}{AR} = \frac{QB}{AB} = \frac{KB}{BS}.$$

From these relations we determine the points Q and K:

$$\frac{QB}{AB} = \frac{Q - B}{A - B} = t \rightarrow Q = At + B\bar{t} = (A - R)t + (B - R)\bar{t} + R,$$

$$\frac{KB}{BS}=\frac{K-B}{B-S}=t\to K=(B-S)t+B=(A-R)t+(B-R)(t+1)+R.$$

Current point $M(p_R,q_R)$ with the help of the BN-calculus theorem:

$$\begin{vmatrix}M\\K\\A\end{vmatrix}=\begin{vmatrix}M\\Q\\S\end{vmatrix}=\begin{vmatrix}p_R & q_R & 1\\ t & t+1 & 1\\ 1 & 0 & 1\end{vmatrix}=\begin{vmatrix}p_R & q_R & 1\\ t & \overline{t} & 1\\ -1 & 0 & 1\end{vmatrix}=0\to$$

$$\begin{cases}p_R(t+1)+q_R\overline{t}=t+1\\ p_R\overline{t}-q_R(t+1)=-\overline{t}\end{cases}\to p_R=\frac{2t}{1+t^2},\ q_R=\frac{1-t^2}{1+t^2}.$$

Then the equation of the arc of the ellipse BMA in the simplex of conjugate axes RA and RB takes the following form:

$$M=(A-R)\frac{2t}{1+t^2}+(B-R)\frac{1-t^2}{1+t^2}+R,\ 0\le t\le 1. \qquad (6)$$

Let us pass to the second part of the problem. To determine the elliptic arc of the contour, we replace the simplex RAB for a new simplex CAB (fig. 4), where R is the fourth vertex of a parallelogram $BCAR$: $R=A+B-C$.

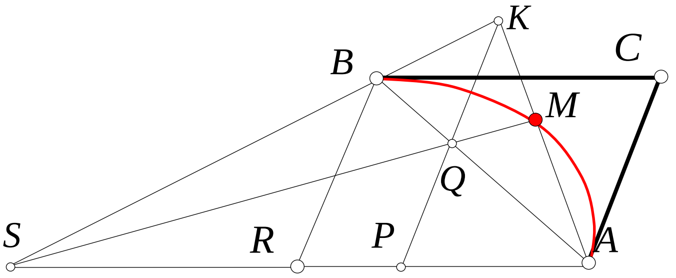

Рис. 4. Определение эллиптической дуги обвода

Eliminating the point R from equation (6), we get:

$$M=(A-A-B+C)\frac{2t}{1+t^2}+(B-A-B+C)\frac{1-t^2}{1+t^2}+A+B-C. \qquad (7)$$

After the transformations, we obtain the equation of the arc of the elliptic contour in the new simplex CAB:

$$M=(A-C)\frac{2t^2}{1+t^2}+(B-C)\frac{(1-t)^2}{1+t^2}+C,\ 1\ge t\ge 0. \qquad (8)$$

Statement. *The replacement in the simplex of the point equation of the arc bypassing the starting point by the parallelogram rule transforms it into an equation of a curve with conjugate axes. The inverse* *transformation is also valid.*

Problem 3. In accordance with [6], a geometric algorithm is proposed for constructing an arc of a contour of the second order curve by a ratio on the median, which has the following point equation:

$$M=(A-C)\frac{k\bar{t}^{2}}{k(1-2t)^{2}+2t\bar{t}}+(B-C)\frac{kt^{2}}{k(1-2t)^{2}+2t\bar{t}}+C, \qquad (9)$$

where $t=\frac{AT}{AB}$, $k=\frac{KC}{K_C C}$, $K_C=\frac{A+B}{2}$.

It is required to convert the equation of the arc of an ellipse M Using this algorithm and replacing the simplex, convert this equation to an equation of the arc of an elliptical contour.

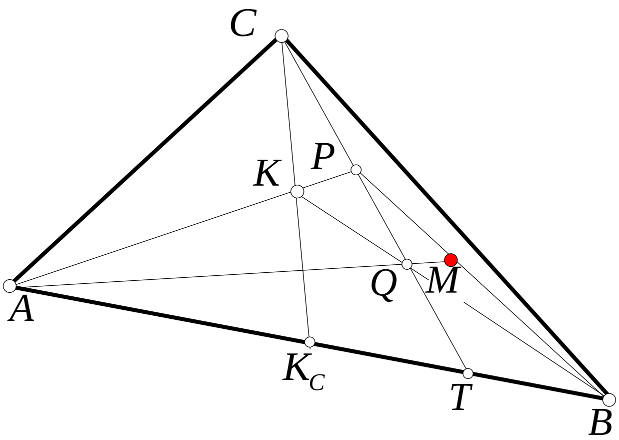

Рис. 5. Геометрическая схема конструирования дуги к2п

Decision. We define a point C from the following relationship:

$$k=\frac{KC}{K_C C}\to C\bar{k}=K-K_C k.$$

Further we substitute into the equation the point K_C:

$$C=K\frac{1}{\bar{k}}-\frac{A+B}{2}\frac{k}{\bar{k}}=-A\frac{k}{2\bar{k}}-B\frac{k}{2\bar{k}}+K\frac{1}{\bar{k}}. \qquad (10)$$

We replace the simplex in equation (9), substituting the point C equation (10). After some transformations, we get:

$$M=\left(A-K\right)\frac{k\bar{t}\left(1-2t\right)}{k(1-2t)^{2}+2t\bar{t}}+\left(B-K\right)\frac{kt\left(2t-1\right)}{k(1-2t)^{2}+2t\bar{t}}+K. \qquad (11)$$

It should be noted that the geometric properties of the arc of the curve, despite the replacement of points in the equation, are completely preserved and, in accordance with [6], with $k=\frac{1}{2}$ we obtain an arc of a parabola, with $k>\frac{1}{2}$ – arc of the ellipse, and where $k<\frac{1}{2}$ – arc of hyperbole.

Conclusions. In this paper, we propose a method for replacing a simplex in the equation of a plane curve, which allows us to transform the point equation of an arc of any plane curve in such a way that the arc of the curve possesses certain predetermined properties, and its equation has been optimized from the point of view of arithmetic computations. Also, the paper gives the equations of the arc of a parabola given by the conjugate axes, the elliptical arc of the bypass, and the arc of the curve of the second order passing through 3 predetermined points obtained by the method of replacing the simplex by the method proposed above, which is a separate result of the investigation and can have wide practical application in the construction of geometric objects On pre-determined conditions.

Literature

1. Александров П.С. Лекции по аналитической геометрии. / Александров П.С. – М.: Наука, 1968. – 912 с.
2. Яглом И.М. Идеи и методы аффинной и проективной геометрии. Часть 1. Аффинная геометрия. / Яглом И.М., Ашкинузе В.Г. – М.: Учпедгиз, 1962. – 248 с.
3. Балюба И.Г. Точечное исчисление: учебное пособие / И.Г. Балюба, В.М. Найдыш. – Мелитополь: МГПУ им. Б. Хмельницкого, 2015. – 236 с.
4. Балюба И.Г. Конструктивная геометрия многообразий в точечном исчислении: диссертация на соискание научной степени доктора технических наук: 05.01.01 / Балюба Иван Григорьевич – Макеевка: МИСИ, 1995. – 227 с.
5. Глаголев Н.А. Проективная геометрия / Глаголев Н.А. – М.: Высшая школа, 1963. – 344 с.
6. Конопацький Є.В. Геометричне моделювання алгебраїчних кривих та їх використання при конструюванні поверхонь у точковому численні Балюби-Найдиша. Дис. … канд. техн. наук: 05.01.01. Конопацький Євген Вікторович. – Мелітополь, 2012. – 164 с.