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USE BARYCENTRIC COORDINATES FOR THE CONSTRUCTION OF SPATIAL DISCRETELY REPRESENTED CURVE

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The work is devoted to construction of the spatial discretely presented curves. Formulas which allow to determine the position of the thickening points with the use of local systems of barycentric coordinates are obtained.

Keywords: discretely presented curve (DPC), local thickening, barycentric coordinates (BC).

Formulation of the problem. Local condensation is the most effective, and sometimes the only way to solve a number of problems of discrete geometric modeling. We list some of them.

If there are convex, concave, straight sections, areas of increasing and decreasing curvature along the curve, each section is formed according to a separate algorithm with subsequent joining of the sections.

In addition to these sites, local condensation is advisable when forming sites requiring a different number of condensations. For example, in the formation of DWD regions, where the number of initial points is small, and the curvature along the curve varies significantly, the number of condensations will be larger, in comparison with other areas.

The use of local condensation is one of the ways to solve problems arising in connection with the representation of the original point series on an uneven grid. In addition, local condensation can help simplify the algorithms for forming condensation points, reduce the number of necessary calculations, and improve the accuracy of calculations.

The use of local condensation involves the use of local coordinate systems. The choice and use of coordinate systems that provide an easy transition from the global coordinate system to local ones and vice versa, as well as the transition from one local coordinate system to another is a separate, complex task.

In the formation of a spatial DPC, the geometric scheme of the condensation of a point series is considerably more complicated, in comparison with the plane case. As a result, both the efficiency of applying local condensation and the complexity of using local coordinate systems in three-dimensional space increase.

To implement the point series thickening methods that provide the necessary accuracy of calculations, the possibility of local correction of the constructed curve, and also the imposition of a large number of additional conditions on the curve, it seems to us to use barycentric coordinates as local coordinate systems.

Analysis of recent research and publications. In work [3], the possibilities of using BC for the formation of planar PDCs are considered. The necessary formulas are obtained and a scheme of using the BC as local coordinate systems is formulated for the initial DPC in the global system of Cartesian coordinates. The properties of the BC are studied from the point of view of their use as an apparatus for realizing geometrical schemes of discrete interpolation. A number of algorithms for forming plane contours with a monotonous change in curvature are developed, using local systems of barycentric coordinates.

Formulation of the purpose of the article. The purpose of the article is to investigate the possibility of using the BC as an apparatus for calculating the geometrical schemes for modeling the spatial KDP.

Main part. BC of M point with respect to the vertices of the tetrahedron $i-1, i, i+1, i+2$ real numbers are called $M_{i-1}, M_i, M_{i+1}, M_{i+2}$, such that

$$M_{i-1} + M_i + M_{i+1} + M_{i+2} = 1,$$

And here the point is the center of mass of four material points - the vertices of the tetrahedron $i-1, i, i+1, i+2$, which are loaded by masses $M_{i-1}, M_i, M_{i+1}, M_{i+2}$, respectively [1].

To obtain the formulas for the transition from the Cartesian coordinate system to the BC, we use the definition of the BC as the ratio of the corresponding volumes:

$$M_{i-1} = \frac{V_{i-1}}{V};\ M_i = \frac{V_i}{V};\ M_{i+1} = \frac{V_{i+1}}{V};\ M_{i+2} = \frac{V_{i+2}}{V},$$

where $V, V_{i-1}, V_i, V_{i+1}, V_{i+2}$ – approximate volumes of tetrahedra $i-1, i, i+1, i+2$, $M, i, i+1, i+2$, $i-1, M, i+1, i+2$, $i-1, i, M, i+2$, $i-1, i, i+1, M$ respectively.

Oriented volume V базисного тетраэдра считается положительным. Ориентированный объём V_{i-1} tetrahedron $M, i, i+1, i+2$ is considered positive if the points M and i-1 Are located on one side with respect to the plane determined by the points $i, i+1, i+2$. Similarly, the sign of volumes V_i, V_{i+1}, V_{i+2}.

If the volumes $V, V_{i-1}, V_i, V_{i+1}, V_{i+2}$ Expressed in terms of the Cartesian coordinates of the vertices of tetrahedra [2], we obtain the formulas for the transition from the global system of Cartesian coordinatesт $Oxyz$ to the local BC system $i-1, i, i+1, i+2$:

$$M_{i-1}=\frac{\begin{vmatrix} x_i & x_{i+1} & x_{i+2} & x \\ y_i & y_{i+1} & y_{i+2} & y \\ z_i & z_{i+1} & z_{i+2} & z \\ 1 & 1 & 1 & 1 \end{vmatrix}}{\Delta};\quad M_i=\frac{\begin{vmatrix} x_{i-1} & x_{i+1} & x_{i+2} & x \\ y_{i-1} & y_{i+1} & y_{i+2} & y \\ z_{i-1} & z_{i+1} & z_{i+2} & z \\ 1 & 1 & 1 & 1 \end{vmatrix}}{\Delta}; \qquad (1)$$

$$M_{i+1}=\frac{\begin{vmatrix} x_{i-1} & x_i & x_{i+2} & x \\ y_{i-1} & y_i & y_{i+2} & y \\ z_{i-1} & z_i & z_{i+2} & z \\ 1 & 1 & 1 & 1 \end{vmatrix}}{\Delta};\quad M_{i+2}=\frac{\begin{vmatrix} x_{i-1} & x_i & x_{i+1} & x \\ y_{i-1} & y_i & y_{i+1} & y \\ z_{i-1} & z_i & z_{i+1} & z \\ 1 & 1 & 1 & 1 \end{vmatrix}}{\Delta},$$

where
$$\Delta=\begin{vmatrix} x_{i-1} & x_i & x_{i+1} & x_{i+1} \\ y_{i-1} & y_i & y_{i+1} & y_{i+2} \\ z_{i-1} & z_i & z_{i+1} & z_{i+2} \\ 1 & 1 & 1 & 1 \end{vmatrix}.$$

Solving the system of equations (1) with respect to the Cartesian coordinates of the point $M(x,y,z)$ We obtain the formulas for the transition from the local BC system to the global system of Cartesian coordinates.

$$\begin{aligned} x &= x_{i-1}M_{i-1}+x_iM_i+x_{i+1}M_{i+1}+x_{i+2}M_{i+2}; \\ y &= y_{i-1}M_{i-1}+y_iM_i+y_{i+1}M_{i+1}+y_{i+2}M_{i+2}; \\ z &= z_{i-1}M_{i-1}+z_iM_i+z_{i+1}M_{i+1}+z_{i+2}M_{i+2}. \end{aligned} \qquad (2)$$

Now let us consider the possibility of a transition from a BK system of a single tetrahedron (tetrahedron A,B,C,D) To the BK system of another tetrahedron (tetrahedron 1,2,3,4).

Let the BC points M in the tetrahedron system A,B,C,D – $M(M_A,M_B,M_C,M_D)$. Then we can write [1]:

$$M=AM_A+BM_B+CM_C+DM_D. \qquad (3)$$

In the system of another tetrahedron 1,2,3,4 the position of the points A,B,C,D determine the BC: $A(A_1,A_2,A_3,A_4,)$, $B(B_1,B_2,B_3,B_4,)$, $C(C_1,C_2,C_3,C_4,)$, $D(D_1,D_2,D_3,D_4,)$. Then we can write:

$$\begin{aligned} A &= 1A_1+2A_2+3A_3+4A_4; \\ B &= 1B_1+2B_2+3B_3+4B_4; \\ C &= 1C_1+2C_2+3C_3+4C_4; \\ D &= 1D_1+2D_2+3D_3+4D_4. \end{aligned} \qquad (4)$$

Substituting (4) into (3), after the transformations, we get:

$$
\begin{aligned}
M = 1(A_1 M_A + B_1 M_B + C_1 M_C + D_1 M_D) + \\
2(A_2 M_A + B_2 M_B + C_2 M_C + D_2 M_D) + \\
3(A_3 M_A + B_3 M_B + C_3 M_C + D_3 M_D) + \\
4(A_4 M_A + B_4 M_B + C_4 M_C + D_4 M_D).
\end{aligned}
$$

Given that $M = 1M_1 + 2M_2 + 3M_3 + 4M_4$, finally, we obtain the formulas for the transition of the tetrahedron BC system A, B, C, D to the BK tetrahedron system 1,2,3,4:

$$
\begin{aligned}
M_1 &= A_1 M_A + B_1 M_B + C_1 M_C + D_1 M_D; \\
M_2 &= A_2 M_A + B_2 M_B + C_2 M_C + D_2 M_D; \\
M_3 &= A_3 M_A + B_3 M_B + C_3 M_C + D_3 M_D; \\
M_4 &= A_4 M_A + B_4 M_B + C_4 M_C + D_4 M_D.
\end{aligned}
\tag{5}
$$

Conclusions. Formulas (1,2,5) allow us to conclude that it is expedient to use local BC systems in the formation of a spatial DPC. The coordinates of the condensation points can be calculated according to the following scheme. The initial DDC is specified in the global system of Cartesian coordinates. Each condensation point is defined in its own local BC system relative to the vertices of the tetrahedron, which are consecutive points of the original point series. The transfer of information from one local BC system to another is carried out using formulas (5). In this case, the TC of the vertices of tetrahedra in whose systems the condensation points are determined are calculated from formulas (1). After the BC determination of the condensation points, the transition to the initial system of Cartesian coordinates is carried out according to the formulas (2).

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