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CONTROL DISCRETE CURVATURE IN THE METHOD VARIABLE FORMATION DIFFERENCE SCHEME ANGULAR PARAMETERS

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We study how to manage the discrete curvature of the curve in the process of thickening based on the method of formation of the difference schemes of variable angular parameters.

Keywords: discrete representation of the curve, interpolation, variability discrete geometric modeling, discrete curvature, angle settings.

Formulation of the problem. An important element characterizing the shape of the curve is the "degree of curvature" or "curvature" of it at various points, which can be expressed by the number [7]. Naturally, the curvature of the curve in differential geometry is characterized by the angle of rotation of the tangent, calculated per unit arc length (Fig. 1) attitudes ω / σ. This ratio is called the mean curvature of the arc length.

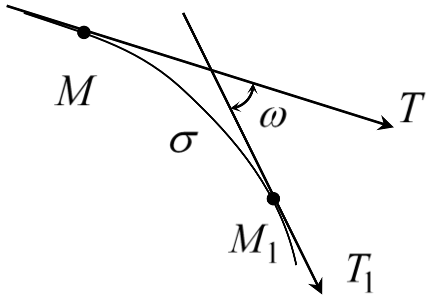

Fig. 1

In different parts of the curve, its average curvature will, generally speaking, be different (except for the circle). Therefore, from the concept of mean curvature of the arc MM_1, often go to the concept of curvature at a point defined as the limit to which the mean curvature of the arc tends MM_1, when the point M_1 along the curve tends to M [7], those.

$$k = \lim_{\sigma \to 0} \frac{\omega}{\sigma} \quad (1)$$

where k - curvature of the curve at a given point.

In differential geometry, in many studies it seems convenient to replace the curve near the point in question by a circle having the same curvature as the curve at this point (Fig. 2). This, the so-called circle of curvature at the point on it M, which the:

– tangent to the curve in t. M;
– Is directed by a convexity near this point in the same direction as the curve;

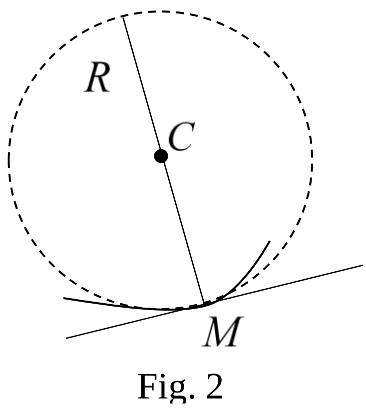

Fig. 2

– has the same curvature as the curve in p. M.

Centre C The circle of curvature is called the center of curvature and lies on the normal to the curve at the point under consideration from the convex side.

However, most of the initial data are presented discretely and, often, can not be represented by an analytical dependence, therefore, the issue of studying and managing the corresponding analogs proposed by [3] for the KDP becomes relevant.

Analysis of recent research and publications. Many scientists have considered the definition and application of the discrete curvature of the curve during the thickening process [1,2,4,6]. For example, it is proposed in [4] to form second-order contours of smoothness on the basis of a special function. In [1], a connection was found between the length of the links of the accompanying broken line and the angles of contiguity. The proposed method was used in [6], but it allows us to determine only the average curvature on a condensed section without the possibility of controlling it. Therefore, the study of the issue of change and the possibility of controlling the curvature of the KDP in the process of condensation is an actual problem.

Formulating of the article purpose. Control of the curvature of the DCC in the process of thickening using the method of variational formation of difference schemes of angular parameters.

Main part. Let us consider a curve in the context of variational, discrete geometric modeling, in which the curve is represented in the form of a KDP (x_i, y_i). Drawing an analogy between fig. 1 and fig. 3, it is natural to assume that the average curvature of a discrete curve can be characterized by an angle of contiguity at the point of condensation $\gamma'_{i+0,5}$ on a given site l_i, those attitudes $\frac{\gamma'_{i+0,5}}{l_i}$, $i=\overline{0,n-1}$.

An analogue of the circle of curvature, for a discretely represented curve, is a circle drawn through three successive points of the KDP (Figure 3).

Thus, in the process of condensation, we can control the value of the curvature radii of the simulated curve for the condensed sections. Since the angle in $\Delta(i-1)(i-0{,}5)i$

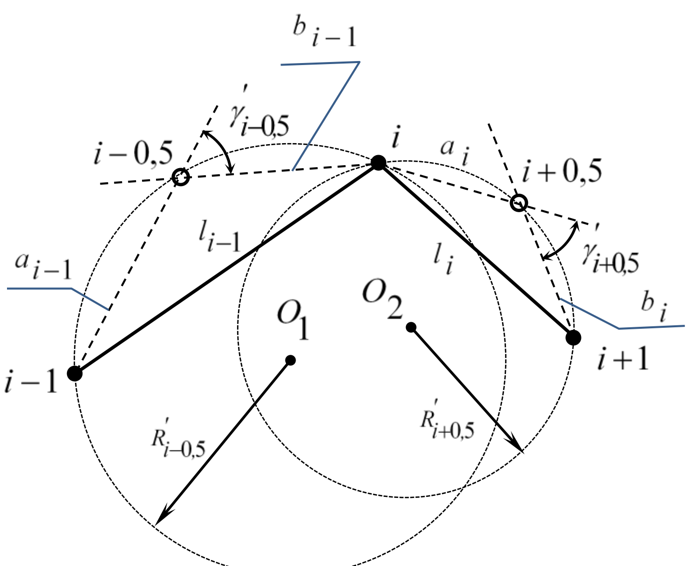

Fig. 3

at the top $(i-0{,}5)$, $i=\overline{1,n}$ Is blunt, then considered Δ Is obtuse. And, as is known from the course of analytic geometry, the center of the circumscribed circle around the obtuse triangle lies outside the triangle. The numerical value of the radius of the circumscribed circle (the radius of curvature) in this case can be determined from the sine theorem (for the link $i-(i+1)$):

$$2R'_{i+0{,}5}=\frac{a_i}{sin\beta_{(i+1)-}}=\frac{b_i}{sin\beta_{i+}}=\frac{l_i}{sin\gamma'_{i+0{,}5}},\quad i=\overline{0,n-1}, \tag{2}$$

where angles β_{i+} and $\beta_{(i+1)-}$, according to [5], make up some fixed part of the angle of contiguity $\gamma'_{i+0{,}5}$ at the point of condensation, i.e.

$$\begin{matrix}\beta_{i+}=\eta_i\cdot\gamma_{i+0{,}5}\\ \beta_{(i+1)-}=(1-\eta_i)\cdot\gamma_{i+0{,}5}\end{matrix},\ i=\overline{0,n-1}, \tag{3}$$

where η_i – the ratio of the angular parameters, determined from expression:

$$\eta_i=\frac{\gamma_i^0}{\gamma_i^0+\gamma_{i+1}^0},\ \ i=\overline{0;n-1}. \tag{4}$$

Thus, we are able to control the curvature at points of condensation of the curve during its thickening.

However, by controlling the process of thickening the KDP at sites $(i-1)-i$ и $i-(i+1)$, $i=\overline{1,n-1}$ With allowance for its discrete curvature (fig. 4), we do not control the value of the discrete curvature at the point i. And since for dynamical circuits one of the defining factors is the monotonicity of the change in curvature, then we will need to take this into account.

In fig. 4 shows a fragment of the DPC on which for both links $((i-1)-i)$ и $(i-(i+1))$ the centers of the circle of curvature are given $C_{i-0.5}$ and $C_{i+0.5}$, those $R_{i-0.5}<R_{i+0.5}$.

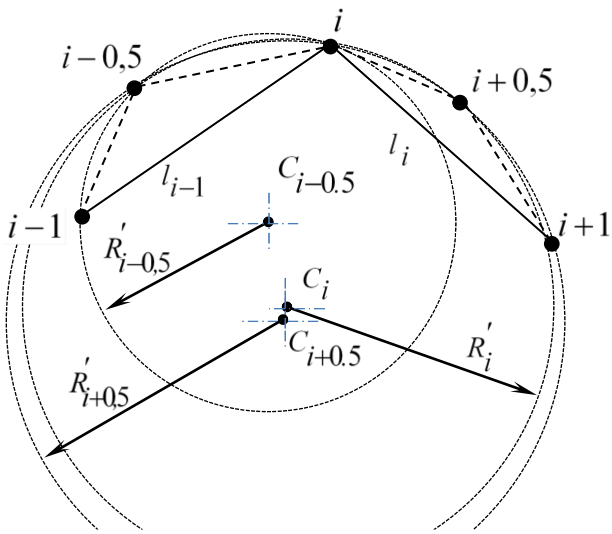

Fig. 4

In order to achieve a monotonous change in the curvature obtained during the thickening of the DPC, it is necessary to control it in three stages during the thickening process: $((i-1)-i)$, $((i-0{,}5)-(i+0{,}5))$, $(i-(i+1))$. In this case it is necessary to satisfy condition:

$$\begin{matrix} R_{i-0,5} < R_i < R_{i+0,5}, \\ R_{i-0,5} > R_i > R_{i+0,5}, \end{matrix} \text{ или } i = \overline{1,n-1}, \tag{5}$$

where the values of the radii of curvature are determined from expression (2)

$$2R_{i-0,5} = \frac{l_i}{sin\gamma'_{i-0,5}} => R_{i-0,5} = \frac{l_i}{2sin\gamma'_{i-0,5}};$$

$$2R_{i+0,5} = \frac{l_i}{sin\gamma'_{i+0,5}} => R_{i+0,5} = \frac{l_i}{2sin\gamma'_{i+0,5}};$$

$$2R_i = \frac{C_i}{sin\gamma'_i} => R_i = \frac{C_i}{2sin\gamma'_i}.$$

C_i can be expressed from $\Delta(i-0,5)i(i+0,5)$ using the cosine theorem:

$$C_i^2 = b_{i-1}^2 + a_i^2 - 2b_{i-1} \cdot a_i \cdot cos(180 - \gamma'_i), \tag{6}$$

where the lengths of the links a_i and b_{i-1} Are determined from expression

$$\frac{a_i}{sin\beta_{(i+1)-}} = \frac{l_i}{sin\gamma'_{i+0,5}} => a_i = l_i \frac{sin\beta_{(i+1)-}}{sin\gamma'_{i+0,5}} = l_i \frac{sin[(1-\eta_i)\gamma'_{i+0,5}]}{sin\gamma'_{i+0,5}};$$

$$\frac{b_{i-1}}{sin\beta_{(i-1)+}} = \frac{l_{i-1}}{sin\gamma'_{i-0,5}} => b_{i-1} = l_{i-1} \frac{sin\beta_{(i-1)+}}{sin\gamma'_{i-0,5}} = l_{i-1} \frac{sin[\eta_i \cdot \gamma'_{i-0,5}]}{sin\gamma'_{i-0,5}}.$$

Тогда

$$C_i = \sqrt{\left(l_{i-1} \frac{sin[\eta_i \cdot \gamma'_{i-0,5}]}{sin\gamma'_{i-0,5}}\right)^2 + \left(l_i \frac{sin[(1-\eta_i)\gamma'_{i+0,5}]}{sin\gamma'_{i+0,5}}\right)^2 + 2 \cdot l_{i-1} \cdot l_i \cdot \frac{sin[\eta_i \cdot \gamma'_{i-0,5}] \cdot sin[(1-\eta_i)\gamma'_{i+0,5}]}{sin\gamma'_{i-0,5} \cdot sin\gamma'_{i+0,5}} \cdot cos\gamma'_i}.$$

Taking into account the expression obtained, formula (5) takes the form:

$$R_i - R_{i-1} > 0, \text{ или } R_i - R_{i-1} < 0, \; i = \overline{1,n-1}$$

$$\frac{\sqrt{\left(l_{i-1} \frac{sin[\eta_i \cdot \gamma'_{i-0,5}]}{sin\gamma'_{i-0,5}}\right)^2 + \left(l_i \frac{sin[(1-\eta_i)\gamma'_{i+0,5}]}{sin\gamma'_{i+0,5}}\right)^2 + 2 \cdot l_{i-1} \cdot l_i \cdot \frac{sin[\eta_{i-1} \cdot \gamma'_{i-0,5}] \cdot sin[(1-\eta_i)\gamma'_{i+0,5}]}{sin\gamma'_{i-0,5} \cdot sin\gamma'_{i+0,5}} \cdot cos\gamma'_i}}{2sin\gamma'_i} - \frac{l_i}{2sin\gamma'_{i-0.5}} > 0 \tag{7}$$

The additional condition obtained in combination with the condition for the absence of oscillations:

$$\gamma_{i+0,5} > 0,\ \text{or},\ \gamma_{i+0,5} < 0,\ i = \overline{1, n-1} \qquad (8)$$

For difference schemes [5] form a certain area of the solution, from which we will select the values of the control parameters.

Conclusions. The proposed studies allow using the method based on the variational formation of the difference schemes of the angular parameters in the thickening process to take into account not only the absence of oscillations, but also to control the curvature of the DCC, which broadens the possibilities for the formation and design of the differential geometric characteristics of the WPC as a whole. Further research can be aimed at optimizing the choice of control parameters from a polygon of solutions, taking into account possible values of the boundaries of curvature.

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