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PLANE ISOMETRIC GRIDS FOR A SPECIFIED DISCRETE RANGEPOINTS

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In the paper, equations and images of plane isometric networks are given behind a preassigned discrete point series and the basic methods of interpolation and approximation.

Keywords: Lagrange curve, Bezier curve, fractional-rational curve, isotropic curve, isometric grid.

Formulation of the problem. The property of isometric (or even isothermal) grids (surfaces) is that their elementary cells are in the form of squares - the extreme coefficients of the 1 st quadratic form are the same. The use of isometric grids takes place in the simulation of heat propagation, drawing images on curvilinear forms with minimal distortions, and so on. In such problems, the formation of a plane isometric grid is sometimes useful for determining the coordinates of a discrete point series of a guide curve.

Analysis of recent research and publications. In work [2] construction of isometric grids for Bezier, Lagrange curves, fractional-rational curves is considered. In [3], the plane isometric grids for any directional curve given in a parametric form have been investigated.

Formulating the goals of the article. To develop applications to the system of computer mathematics Maple [1] the formation and study of flat isometric grids for a predetermined set of points.

Main part. Get an analytical description of a plane isometric grid with a parametric curve equation [3]. So, for any parametric plane curve on the real plane:

$$\boldsymbol{r}(t) = \boldsymbol{r}[x(t), y(t)], \quad (1)$$

the isotropic curve (the curve of zero length) will correspond if its parametric equation in the complex plane is written in the form:

$$\boldsymbol{r}_c(t) = \boldsymbol{r}_c[x(t) \pm y(t) \cdot I, y(t) \mp x(t) \cdot I], \quad (2)$$

where $I = \sqrt{-1}$ – imaginary unit.

From an isotropic plane curve $\mathbf{r}_c(t)$ two isometric grids can be obtained if in equation (2) the parameter is replaced by a complex variable $u+v\cdot I$:

$$\boldsymbol{R}_c(u,v) = \boldsymbol{R}_c[x(u+vI) \pm y(u+vI)I, y(u+vI) \mp x(u+vI)I], \quad (3)$$

and expressing from the last expression is valid $\boldsymbol{R}_{re}(u,v) = Re(\boldsymbol{R}_c(u,v))$ or imaginary parts $\boldsymbol{R}_{im}(u,v) = Im(\boldsymbol{R}_c(u,v))$.

If the directional curve of the plane isometric grid is a discrete point series $\boldsymbol{r}_j = [X_j, Y_j]$, then you must go to its parametric form $\boldsymbol{r}[x(t), y(t)]$, using known methods of interpolation and approximation (Fig. 1).

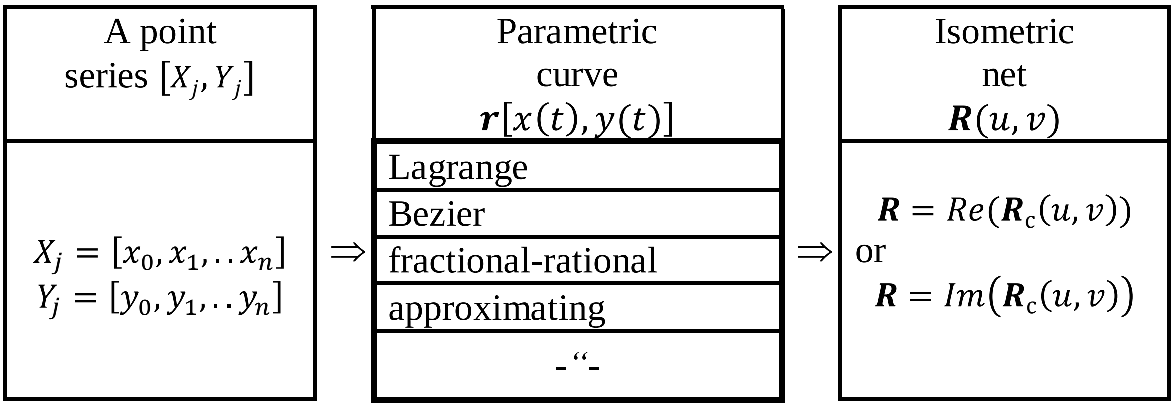


Fig.1 Scheme for the formation of isometric grids

A software for the Maple [1] symbolic algebra environment was developed for the automated formation and study of isometric grids for any number of points. $\boldsymbol{r}_j = [X_j, Y_j]$. To test the application, they scattered with five nodes of the discrete series with the following coordinates:

$$X_j = [1, 4, 7, 10, 15], \quad (4)$$

$$Y_j = [3, -7, 0, 13, 0]. \quad (5)$$

In 4 tables, the equation of the n-th order of the interpolation or approximation curve of the corresponding isometric grid is given in 4 tables. An equation of the nth order of the interpolation or approximation curve of the corresponding isometric grid is given. $\boldsymbol{R}(u, v) = Re(\boldsymbol{R}_c(u, v))$, its linear element ds^2 and the limits of the change of parameters u, v – grid coordinates. For the order *n*> *2* of the curve $\boldsymbol{r}(t)$ isometric mesh equation $\boldsymbol{R}(u, v)$ They are quite cumbersome and therefore they are not listed here. Since the coordinates of the nodes of the discrete series (4) - (5) are integers, then the coefficients of the equations $\boldsymbol{r}(t)$, $\boldsymbol{R}(u, v)$ and ds^2 are determined precisely and presented in the form of fractions.

The analysis of the results of computational experiments shows that the most predictable are the Lagrange and Bezier curves, and hence the flat isometric grids on their basis. Since the weight coefficients have a significant effect on the fractional-rational curve, the shape of the broken (4) - (5) visually does not coincide with the guide curve $\boldsymbol{r}(t)$.

Equations and images of isometric grids based on the Lagrange polynomial $\boldsymbol{r}(t) = \sum_{j=0}^{n} \boldsymbol{r}_j J_j(t)$, where $J_j(t) = \prod_{\substack{k=0 \\ k\neq j}}^{n} \frac{t-t_k}{t_j-t_k}$ is shown in Table 1.

Table 1

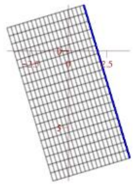
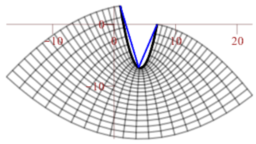
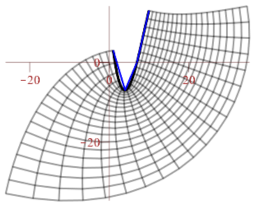
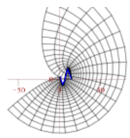
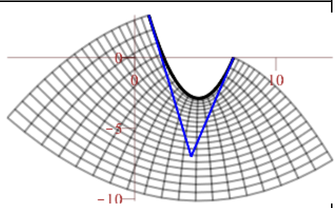
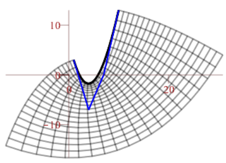
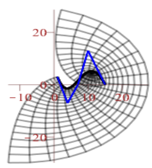
n	Equation	Image
n=1	$\boldsymbol{r}(t) = \begin{bmatrix} 3t+1, \\ -10t+3 \end{bmatrix};$ $\boldsymbol{R}(u,v) = \begin{bmatrix} 3u-10v+1, \\ -10u-3v+3 \end{bmatrix};$ $ds = 109(du^2+dv^2);$ $u = 0..1;\ v = 0..0.5$	
n=2	$\boldsymbol{r}(t) = \begin{bmatrix} 6t+1, \\ 34t^2-37t+3 \end{bmatrix};$ $\boldsymbol{R}(u,v) = \begin{bmatrix} 68uv+6u-37v+1, \\ 34u^2-34v^2-37u-6v+3 \end{bmatrix};$ $u = 0..1;\ v = 0..0.5$	
n=3	$\boldsymbol{r}(t) = \begin{bmatrix} 9t+1, \\ -\frac{99}{2}t^3+126t^2-\frac{133}{2}t+3 \end{bmatrix};$ $\boldsymbol{R} = \begin{bmatrix} -\frac{297}{2}u^2v+\frac{99}{2}v^3+252uv-\frac{133}{2}v+9u+1, \\ \frac{297}{2}uv^2-\frac{99}{2}u^3+126(u^2-v^2)-\frac{133}{2}u-9v+3 \end{bmatrix};$ $u = 0..1;\ v = 0..0.5$	
n=4	$\boldsymbol{r}(t) = \begin{bmatrix} \frac{64}{3}t^4-32t^3+\frac{44}{3}t^2-10t+1, \\ -224t^4+\frac{656}{3}t^2+70t^2-\frac{203}{3}t+3 \end{bmatrix};$ $u = 0..1;\ v = 0..0.5$	

Table 2 shows the equations and images of plane isometric meshes for the Bezier curve $\boldsymbol{r}(t) = \sum_{j=0}^{n} r_j J_{n,j}(t)$, where $J_{n,j}(t) = \frac{n!}{j!(n-j)!} t^j (1-t)^{n-j}$.

Table 2

n	Equation	Image
n=2	$\boldsymbol{r}(t) = \begin{bmatrix} 6t+1, \\ 17t^2-20t+3 \end{bmatrix};$ $\boldsymbol{R}(u,v) = \begin{bmatrix} 34uv+6u-20v+1, \\ 17u^2-17v^2-20u-6v+3 \end{bmatrix};$ $ds = 4(289(u^2+v^2)-340u+102v+109)(du^2+dv^2);$ $u = 0..1;\ v = 0..0.5$	
n=3	$\boldsymbol{r}(t) = \begin{bmatrix} 9t+1, \\ -11\,t^3+51\,t^2-30\,t+3 \end{bmatrix};$ $\boldsymbol{R}(u,v) = \begin{bmatrix} -33\,u^2v+11\,v^3+102\,u\,v+9\,u-30\,v+1, \\ -11\,u^3+33\,u\,v^2+51\,u^2-51\,v^2-30\,u-9\,v+3 \end{bmatrix};$ $u = 0..1;\ v = 0..0.5$	
n=4	$\boldsymbol{r}(t) = \begin{bmatrix} 2\,t^4+12\,t+1, \\ -21\,t^4-44\,t^3+102\,t^2-40\,t+3 \end{bmatrix};$ $u = 0..1;\ v = 0..0.5$	

Equations and images of flat isometric grids for a fractional-rational curve $\boldsymbol{r}(t) = \frac{\sum_{j=0}^{n} r_j w_j J_{n,j}(t)}{\sum_{j=0}^{n} w_j J_{n,j}(t)}$, where $J_{n,j}(t) = \frac{n!}{j!(n-j)!} t^j (1-t)^{n-j}$ with weighted coefficients $w_j = [1,2,2,1,1]$ is given in Table 3.

Table 3

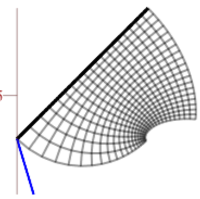
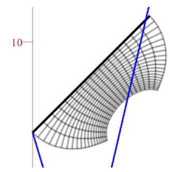
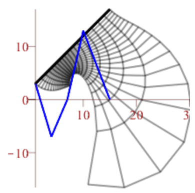
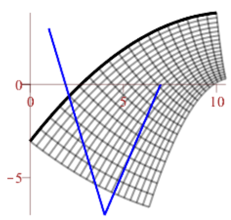
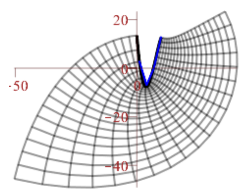
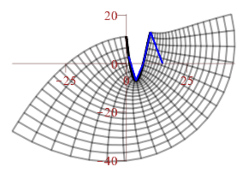
n	Equation	Image
n=2	$\boldsymbol{r}(t) = \begin{bmatrix} \frac{(1-t)^2+16t(1-t)+14t^2}{(1-t)^2+4t(1-t)+2t^2}, \\ \frac{12t(1-t)+12t^2}{(1-t)^2+4t(1-t)+2t^2} + 3 \end{bmatrix};$ $ds = \frac{288(du^2+dv^2)(u^2+(v-1)^2)(u^2+(v+1)^2}{(u^4-4u^3+(2v^2+2)u^2+(-4v^2+4)u+v^4+6v^2+1)^2};$ $u = 0..1;\ \ v = 0..0.8$	
n=3	$\boldsymbol{r}(t) = \begin{bmatrix} \frac{(1-t)^3+24\,t\,(1-t)^2+42\,t^2(1-t)+10t^3}{(1-t)^3+6\,t\,(1-t)^2+6\,t^2(1-t)+t^3}, \\ \frac{18\,t\,(1-t)^2+36\,t^2(1-t)+9t^3}{(1-t)^3+6\,t\,(1-t)^2+6\,t^2(1-t)+t^3} + 3 \end{bmatrix};$ $u = 0..1;\ \ v = 0..0.5$	
n=4	$\boldsymbol{r}(t) = \begin{bmatrix} \frac{(1-t)^4+32t(1-t)^3+84t^2(1-t)^2+40t^3(1-t)+15t^4}{(1-t)^4+8t(1-t)^3+12t^2(1-t)^2+4t^3(1-t)+t^4}, \\ \frac{24t(1-t)^3+72t^2(1-t)^2+36t^3(1-t)+14t^4}{(1-t)^4+8t(1-t)^\wedge+12t^2(1-t)^2+4t^3(1-t)+t^4} \end{bmatrix};$ $u = 0..1;\ \ v = 0..0.4$	

Table 4 shows the equation and image of the isometric networks of the approximation curve of 2-4 orders of the least squares method.

Table 4

n	Equation	Image
n=2	$\boldsymbol{r}(t) = \begin{bmatrix} t, \\ \frac{6442}{109437}t^2 - \frac{139720}{109437}t + \frac{111059}{36479} \end{bmatrix};$ $\boldsymbol{R}(u,v) = \begin{bmatrix} -\frac{12884}{109437}uv + u + \frac{139720}{109437}v, \\ -\frac{6442}{109437}(u^2-v^2) + \frac{139720}{109437}u - v - \frac{111059}{36479} \end{bmatrix};$ $u = 0..10;\ \ v = 0..5$	
n=3	$\boldsymbol{r}(t) = \begin{bmatrix} t, \\ \frac{49931987}{3711063} - \frac{137954065}{11133189}t + \frac{48090187}{22266378}t^2 - \frac{2068205}{22266378}t^3 \end{bmatrix};$ $\boldsymbol{R}(u,v) = \begin{bmatrix} u - \frac{137954065}{11133189}v + \frac{48090187}{11133189}uv - \frac{2068205}{7422126}u^2v + \frac{2068205}{22266378}v^3, \\ \frac{49931987}{3711063} - \frac{137954065}{11133189}u + \frac{48090187}{22266378}(u^2-v^2) - \frac{2068205}{22266378}u^3 + \frac{2068205}{7422126}uv^2 - v \end{bmatrix};$ $u = 0..10;\ \ v = 0..5$	
n=4	$\boldsymbol{r}(t) = \begin{bmatrix} t, \\ \frac{3323}{297} - \frac{53717}{5670}t + \frac{53699}{41580}t^2 - \frac{1}{315}t^3 - \frac{367}{124740}t^4 \end{bmatrix};$ $u = 0..10;\ \ v = 0..5$	

Conclusions. Conducted computational experiments on the formation and study of flat isometric grids for a predetermined discrete point number shows that without modern systems of symbol transformations, it is impossible to solve such problems because of the bulkiness of the analytical description. The use of fractional-rational curves with weighted coefficients for the formation of flat isometric grids is inappropriate because there is no visual relationship between the images of a broken discrete point series and the corresponding interpolation curve.

Literature

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ПЛОСКИЕ ИЗОМЕТРИЧЕСКИЕ СЕТИ ЗА ЗАДАННЫМ ДИСКРЕТНЫМ ТОЧЕЧНЫМ РЯДОМ

Аушева Н.Н., Несвидомина О.В.

В работе приведены уравнения и изображения плоских изометрических сетей за наперед заданным дискретным точечным рядом и основными методами его интерполяции и аппроксимации.

Ключевые слова: кривая Лагранжа, кривая Безье, дробно-рациональная кривая, изотропная кривая, изометрическая сетка.

ПЛОСКІ ІЗОМЕТРИЧНІ СІТКИ ЗА ДИСКРЕТНИМ РЯДОМ ТОЧОК

Аушева Н.М., Несвідоміна О.В.

В роботі наведено окремі рівняння та зображення плоских ізометричних сіток за вихідним дискретним рядом точок та основними методами його інтерполяції та апроксимації.

Ключові слова: крива Лагранжа, крива Без'є, дробово-раціональна крива, ізотропна крива, ізометрична сітка.