УДК 519.85

REPRESENTATION OF FUZZY MARKOV CIRCUITS AS A FUZZY TRANSITION MATRIX AND FUZZY COSTS OF STATES

Barishevskiy S.

The application to the representation of fuzzy Markov chains in the form of fuzzy transition matrix and fuzzy graph of fuzzy mathematics and fuzzy graph theory is considered.

Keywords: fuzzy mathematics, Gaussian fuzzy number, fuzzy probability, fuzzy matrix, fuzzy graph.

Formulation of the problem. The fuzzy Markov chain is one of the uncertainty models in which randomness and fuzziness are combined, which in turn leads to the appearance of the concept of fuzzy probability. In classical theory probability is a deterministic characteristic of the possibility of occurrence of an event under certain conditions. At the same time, in real life this possibility can depend uncontrollably on the totality of conditions that may themselves change. In these cases, the probability of naturally writing off a fuzzy number with the membership function, the parameters of which are estimated statistically by the set of tests [1]. Fuzzy Markov processes with discrete states are conveniently represented and illustrated with a fuzzy transition matrix [2] and a fuzzy graph of system states, since the system can arrive in one of *n* states and for each moment of time t you must specify n^2 transition probabilities P_{ij}.

Analysis of recent research and publications. In work [1] the fundamentals of the theory and practical applications of fuzzy mathematics are considered. In work [3] the bases of fuzzy discrete mathematics with the use of the apparatus of fuzzy logic are considered.

The paper [2] is devoted to an examination of the basic concepts that characterize a fuzzy Markov random process.

Formulation of the purpose of the article. In this paper, we propose an examination of the representation of fuzzy Markov goals in the form of a fuzzy transition matrix and fuzzy graph of states with the use of the apparatus of fuzzy mathematics and the theory of fuzzy graphs.

Main part. A fuzzy random process will be called a fuzzy Markov chain if for each k-th step the random sequence of events (states) S(0), S(1), ..., S(k), fuzzy transition probability from any state S_i at any S_j does not depend on when and how the system came to a state S_j. The initial state S(0) can be predefined or random.

Fuzzy probabilities of a Markov chain will be called probabilities $P_i(k)$ that after k-th step (and up to (k + 1) th), the system S will be in the state $S_i(i = 1,2, \dots, n)$. It is obvious that for any k

$$\sum_{i=1}^{n} P_i(k) \approx \tilde{1},$$

where $P_i(k)$ – fuzzy numbers, $\tilde{1}$ – fuzzy unit whose modal value (core) is equal to 1.

If the initial state of the system S is exactly known S(0)= S_i, then the initial probability $P_i(0) = 1$, and all the others are equal to zero.

The fuzzy transition probability (transition probability) at the k-th step from the state S_i into the state S_j we will call the fuzzy conditional probability that the system S after the k-th step will be in a state S_j provided that immediately before this (after a k-1 step) it was in the state S_i.

Since the system can reside in one of n states, for each time t it is necessary to set n^2 fuzzy transition probabilities P_{ij}, which can be conveniently represented as the following fuzzy matrix:

$$A = (P_{ij}) = \begin{pmatrix} P_{11} & \dots & P_{1n} \\ \dots & \dots & \dots \\ P_{n1} & \dots & P_{nn} \end{pmatrix}, \qquad (2)$$

where P_{ij} – fuzzy transition probability in one step from the state S_i into the state S_j; P_{ij} – fuzzy probability of delay in state S_i. Here P_{ij} are fuzzy Gaussian numbers with the corresponding membership functions:

$$\mu(P_{ij}) = exp\left\{-\frac{(P_{ij} - \mathring{P}_{ij})^2}{2\delta_{ij}^2}\right\}, \qquad \delta_{ij}^2 \to \sigma_{ij}^2,$$

where $\mathring{P}_{ij}$ - modal value (core) of fuzzy numbers $\mathring{P}_{ij}$, σ_{ij}^2 – concentration coefficients (carriers).

The matrix (2) is called a fuzzy transition matrix or a matrix of fuzzy transition probabilities.

If the fuzzy transition probabilities h depend on the step number (on time), but depend only on the state from which the transition takes place, then the corresponding fuzzy Markov chain is said to be homogeneous.

We note some features of the fuzzy matrix that form the transition probabilities of the fuzzy homogeneous Markov chain.

- Each line characterizes the selected state of the system, and its elements are fuzzy probabilities of all possible transitions in one step from the selected (from i-th) state, including the transitions into itself.

- Elements of columns show fuzzy probabilities of all possible transitions of the system in one step to a given (j-e) state (in other words, the lines characterize the fuzzy probability of the system transition from the state, the column to the state).

- The sum of the fuzzy probabilities of each line is unclearly equal to a fuzzy one, since the transitions form a complete group of incompatible events:

$$\sum_{j=1}^{n} P_{ij} \approx \tilde{1}, i = \overline{1,n}. \quad (3)$$

- On the main diagonal of the matrix of fuzzy transition probabilities there are fuzzy probabilities that the system will not leave the state S_i, and will remain in it.

Fuzzy Markov processes with discrete states are conveniently illustrated using a fuzzy graph of states. An example of a fuzzy graph of states of system S is shown in Fig. 1, where the circles denote states S_1,S_2,… system S, and fuzzy arrows (oriented edges) - possible transitions from state to state.

On a fuzzy graph only direct transitions differ, and not transitions through other states. Possible delays in the former state will be represented by a fuzzy arc, which is directed from the given state to it. The number of states of the system can be either finite or infinite (but countable).

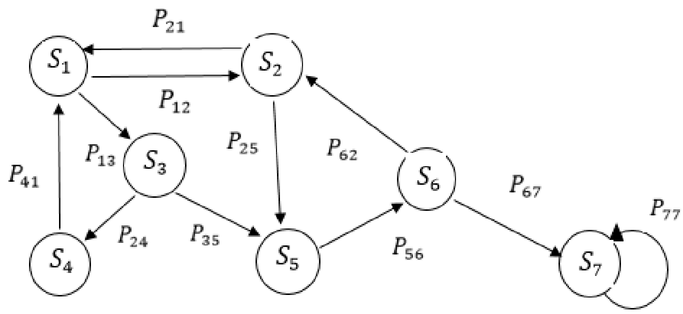

Fig.1. The fuzzy graph of the system S

If a fuzzy homogeneous Markov chain is given a fuzzy initial distribution of transitive probabilities (P_{ij}), then the fuzzy probabilities of the states of the system $P_i(k)$ $(i = \overline{1,n}; j = \overline{1,n})$ are defined by the recurrence formula:

$$P_i(k) \approx \sum_{j=1}^{n} P_j(k-1) \cdot P_{ij}, (i = \overline{1,n}; j = \overline{1,n}) \quad (4)$$

Conclusions. Since in this paper the probabilities are Gaussian fuzzy numbers, in order to find their numerical values, it is required to determine the basic operations on these numbers. In [1, 2] the rules for performing these operations are presented (rules of summation and subtraction, rules for multiplication and division).

Literature

1. Раскин Л.Г., Серая О.В. Нечеткая математика. Основы теории. Приложения. – Х.: Парус, 2008. – 352 с.
2. Гончар Т.А., Нечеткие однородные цепи Маркова / Т.А. Гончар, С.О. Барышевский // Международный студенческий научный

вестник. - №5. – Часть 4. – М.: Академия Естествознания 2017 – С. 470-471.
3. Берштейн Л.С. Нечеткие графы и гиперграфы/Л.С. Берштейн, А.В. Боженюк. – М.: Научный мир, 2005 – 256 с.
4. Барышевский С.О. Графоаналитический метод решения нечетким матричных игр. // Сучасні проблеми моделювання: зб. наук. праць. – Мелітополь: Видавництво МДПУ ім. Б. Хмельницького, 2016. – С.3-8.

ПРЕДСТАВЛЕННЯ НЕЧІТКИХ ЛАНЦЮГІВ МАРКОВА У ВИГЛЯДІ НЕЧІТКОЇ МАТРИЦІ ПЕРЕХОДУ І НЕЧІТКОГО ГРАФА СТАНУ

Баришевський С.О.

Розглянуто застосування до представлення нечітких ланцюгів Маркова у вигляді нечіткої матриці переходу і нечіткого графу стану нечіткої математики і теорії нечітких графів.

Ключеві слова: нечітка математика, гаусове нечітке число, нечітка ймовірність, нечітка матриця, нечеткий граф.

ПРЕДСТАВЛЕНИЕ НЕЧЕТКИХ ЦЕПЕЙ МАРКОВА В ВИДЕ НЕЧЕТКОЙ ПЕРЕХОДНОЙ МАТРИЦЫ И НЕЧЕТКОГО ГРАФА СОСТОЯНИЙ

Барышевский С.О.

Рассмотрено применение к представлению нечетких цепей Маркова в виде нечеткой переходной матрицы и нечеткого графа состояний нечеткой математики и теории нечетких графов.

Ключевые слова: нечеткая математика, гауссово нечеткое число, нечеткая вероятность, нечеткая матрица, нечеткий граф.