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MODELING OF OSCILLATORY PROCESSES IN THE SYSTEM OF LINEAR OSCILLATORS

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A mathematical model is offered for realization of analysis of shake processes in the system of the constrained harmonic oscillators with different descriptions at presence of the external loading. A program is developed that allows constructing vibration paths in the phase space, as well as finding the time dependencies of deviations and deviation rates for each element. The test tests of model are conducted and examples of phase trajectories of shake processes are made.

Keywords: programming language, mathematical model, program, oscillator system, oscillator, phase trajectories, phase portrait, numerical methods.

Formulation of the problem. The dynamic state of the system of connected oscillators is determined by the task of coordinates of each element in the form of functions from time $x_1(t)$, $x_2(t),\ldots x_n(t)$. The set of possible states of a vibrational system forms a phase space with n degrees of freedom. The method of studying oscillations using phase trajectories is proposed in the works of L.I. Mandelstam [1]. Each point S (t) of the phase space corresponds to the set of functions $x_1(t)$, $x_2(t),\ldots x_n(t)$. The state of the vibrational system is determined by the trajectory of moving the point S(t). The family of phase trajectories forms a phase portrait that determines the behavior of the system. Analysis of the phase portrait allows you to get useful information about the oscillatory system, even in those cases where the analytical decision of the problem is difficult or impossible. Therefore, the development of methods for constructing phase trajectories and phase portrait in this situation has practical and actual significance.

The theory of oscillations of mathematical pendulums with one and two degrees of freedom is set out in a number of publications [1, p.444-450], [2, p.157-171], [3, p.230-231]. An analysis of the dynamic processes in a system with 4 or more degrees of freedom for oscillating elements with different characteristics occurs with some difficulties. In this case, the analytical solution of the problem is not possible, so the numerical method is the only way to solve the problem [2, p. 186-207]. In the paper [4], based on a simple mathematical model, an algorithm is proposed and a computer program is developed that allows us to investigate the oscillations in the system of coupled oscillators. This paper is devoted to the illumination of the method for constructing phase trajectories in relation to the system of coupled harmonic oscillators with different characteristics with four and more degrees of freedom under conditions of external loading.

Analysis of recent research and publications. Phase portraits in a system with a number of degrees of freedom n = 1,2 are considered in detail in the writings [1; 2, pp.75-81]. With a sufficiently large value of n> 100, the problem is reduced to solving the wave equation for a solid solid, which is considered in many publications [5; 6]. An intermediate case with n = [4 ... 20] remains little investigated. Our studies allow us to qualitatively approach the study of oscillatory processes in this area, where obtaining an analytical solution meets great difficulties.

Formation of the purposes of the article. Consider a system of spring oscillators with different mass mi and coefficients of stiffness of the ki springs represented in Fig. 1. Let the left end of the spring of the first oscillator be fixed, as shown in Fig. 1, and force F (t) acts on the extreme right mass.

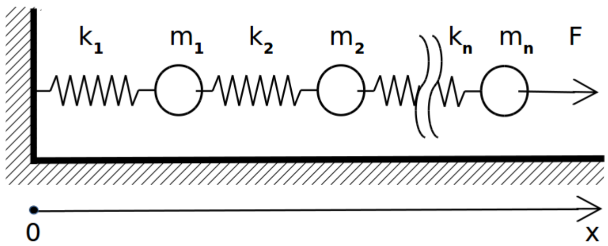

Fig.1 System of n oscillators with external stimulus

Oscillation oscillators with the same masses under the action of force of the form:

$$F(t) = \sum_{i=1}^{s} A_i \sin(\omega_i t + \varphi_i) \tag{1}$$

were investigated in work [4]. The case of related oscillators with different characteristics of elements under arbitrary power load is considered in the article. The task of the study was to develop a numerical algorithm for analyzing phase trajectories in a system of coupled harmonic oscillators.

Main part. According to the second law of Newton, the acceleration of the motion of the element with the mass mi, shown in Fig. 1, under the action of forces of elasticity without taking into account frictional forces is determined by the equation $m_i \ddot{x}_i(t) = \sum_i F_i$, where x_i, – offset element, $\sum_i F_i$ – amount of forces from the left and right springs, even $k_i(x_{i-1}-x_i)+ k_{i+1}(x_{i+1}-x_i)$, where k_i – spring hardness factor. In this case, the mathematical model of the oscillatory process in the system of n connected oscillators under the action of force F(t) has the form:

$$v_i(t) = \dot{x}_i(t);$$
$$\dot{v}_i(t) = \frac{1}{m_i}(k_i(x_{i-1} - x_i) + k_{i+1}(x_{i+1} - x_i) + \delta_n^i F(t); \quad i = 1,2,...n \qquad (2)$$
$$x_0 = 0, x_{n+1} = 0, k_{n+1} = 0;$$

where $v_i = \dot{x}_i(t)$ – deviation rate, $\dot{v}_i(t)$ – acceleration, δ_i^j – a symbol of Kronecker: δ_i^j=0 at $i \neq j$, δ_i^j=1 at $i=j$.

Algorithm of numerical solution of system of differential equations (2) with initial conditions $x_i(0)= x_{i0}$, $v_i(0)= v_{i0}$ was proposed earlier in the paper [4]. For computing, a computer program Oscil was developed using the C ++ programming algorithmic language in the integrated environment of C ++ Builder 2009. The program allows investigating the oscillatory process in a system of n oscillators with different characteristics under the action of external load. Initial data for conducting calculations are entered from the keyboard or from a text file. The results of the calculations are displayed on the display screen. In particular, it is possible to deduce the dependence of the velocity of the element with the mass mi on its displacement, the temporal dependence of its deviation and velocity, and other characteristics of the oscillatory system.

The testing of the program was carried out by comparing the calculated time dependences of the deviation x (t), the deviation rate v (t) and the trajectory of the phase portrait with the known analytical solution for one harmonic oscillator x(t) = Acos(ωt+φ)+Bcos(ωt+φ) [3,p.230-231].

In fig. 2 shows a graph of the time dependence and the phase portrait of the oscillations that are displayed on the display screen using the Oscil program for the following initial conditions in the normalized form x (0) =0.5, v (0) =0, k=1, m=1, ω=1, φ=0. The relative error of calculated deviations after 1000 oscillations did not exceed 0.001.

As an example, consider oscillations in a system of 4 oscillators with unit masses, with coefficients of rigidity 0.5, 1, 0.8, 1.5 under the influence of force F(t)=0.2cos(3t+0.1) – 0.1cos (5t-0.3). The initial deviations in the calculations were assumed equal to 0.2, 0.1, - 0.1, 0.1, initial speeds - equal to 0.5, - 0.5, 0.3, 0.1, observation time equal to 50. Phase portraits of oscillations are shown in Fig.2.

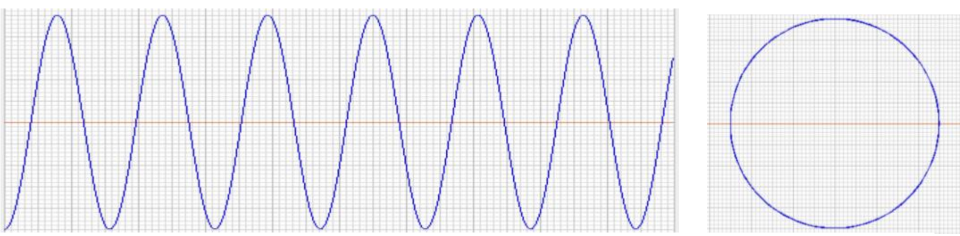

Fig.2 The graph of the time dependence of the deviation and the phase portrait, built using the program Oscil

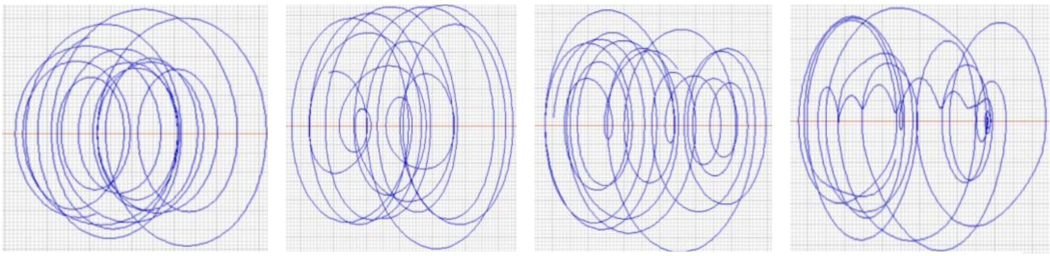

Fig. 3 Phase portraits of oscillations in a system with 4 oscillators with different characteristics under the action of force F(t)=0.2cos (3t+0.1) - 0.1cos (5t-0.3)

Conclusions. The oscillator oscillation processes in associated systems are accompanied by the mutual influence of the neighboring elements on each other due to the exchange of energies, after which the oscillations cease to be harmonic. Analytical solution of the corresponding problems in the case of a large number of degrees of freedom is impossible, so numerical methods can be considered the only way to analyze the trajectories of oscillations. The proposed mathematical model allows obtaining complete information on the time dependencies of deviations and the rate of deviations, as well as the phase portrait of each element. Test calculations, the results of which are shown in Fig. 2, showed high accuracy of calculations. Chimeric form of phase portraits, presented in Fig. 3, reflects a complex picture of oscillations in a system of several oscillators under conditions of external loading. In this example, there is no need to talk about periodic oscillations, although it can be argued that the oscillation trajectories of all oscillators are in the final region of the phase space. The proposed approach allows us to generalize the obtained data in the case of nonlinear oscillations, to take into account the friction in the system and to analyze the resonance phenomena.

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ФАЗОВЫЕ ТРАЕКТОРИИ В СИСТЕМЕ СВЯЗАННЫХ ЛИНЕЙНЫХ ОСЦИЛЛЯТОРОВ

Еремеев В.С., Кузьминов В.В, Шаров С.В.

Предложена математическая модель для проведения анализа колебательных процессов в системе связанных гармонических осцилляторов с различными характеристиками при наличии внешней нагрузки. Разработана программа, которая позволяет построить траектории колебаний в фазовом пространстве, а также найти временные зависимости отклонений и скоростей отклонения для каждого элемента. Проведены тестовые испытания модели и приведены примеры фазовых траекторий колебательных процессов.

Ключевые слова: язык программирования, математическая модель, программа, система осцилляторов, осциллятор, фазовые траектории, фазовый портрет, численные методы.

ФАЗОВІ ТРАЄКТОРІЇ В СИСТЕМІ ЗВ'ЯЗАНИХ ЛІНІЙНИХ ОСЦИЛЯТОРІВ

Єремєєв В.С., Кузьминов В.В., Шаров С.В.

Запропонована математична модель для проведення аналізу коливальних процесів в системі пов'язаних гармонійних осциляторів з різними характеристиками в умовах зовнішнього навантаження. Розроблена програма, що дозволяє побудувати траєкторії коливань у фазовому просторі, знайти тимчасові залежності відхилень та швидкості відхилення для кожного елементу. Проведені тестові випробування моделі і наведені приклади фазових траєкторій коливальних процесів.

Ключові слова: мова програмування, математична модель, програма, система осциляторів, осцилятор, фазові траєкторії, фазовий портрет, чисельні методи.