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COMPUTER IMPLEMENTATION OF THE MODEL OF POINSOT'S ROTATION OF THE OBJECT WITH A FIXED POINT

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A computer graphic model of Poinsot's interpretation of the rotation of a rigid body with a fixed point is presented to study the stability of its motion. For this, a rolling without sliding of the ellipsoid of inertia of this body along one of its tangent planes is simulated, as a result of which the corresponding rolling lines-herpolodia on the tangent plane and the poloidium on the surface of the ellipsoid are constructed.

Key words: Poinsot’s interpretation, moment of inertia, inertia ellipsoid, rolling of an ellipsoid, polhode, herpolhode.

Formulation of the problem. Investigation of the rotation after the inertia of the object with a fixed point (Euler's problem) is related to the definition of the stability of rotation of the solid body around the principal axes of the ellipsoid of inertia [1, 2]. Consideration of the geometric picture of Puanso allows us to conclude that the stability of said rotation. The essence of the geometric interpretation of Poonso is that, in order to study the stability of the body's rotation, the motion of its ellipsoid of inertia must be considered, which must roll without slipping on one of its tangent planes (the Poonso plane π). Then the stability of the motion of a solid is determined by the shape of the rolling line of the ellipsoid, which is formed on the tangent plane π and has the name of the haploid, as well as the shape and especially the location of the corresponding line on the surface of the ellipsoid (polodios). From here follows the important role of computer graphic technologies, which will make it possible to outline the geometric picture of Puanso, and thus contribute to solving the problem at a qualitative level of the specified range of tasks.

Analysis of recent research and publications. The classical description and definition of the geometric shape of the poles reduces to the use of elliptic integrals and is not simply formalized in the compilation of computer algorithms. In works [1,2] a detailed analysis of stability of rotations of a solid body is given, depending on the shape of the poles on the ellipsoid of inertia. But these results are based solely on the formulas and illustrated by the drawings. For engineering studies, it would be much more convenient to describe the properties of the literature in terms of the use of dynamic images created by means of graphic computer animations. The paper [3] presents a program for modeling the geometric Punso painting, compiled in the language of the MATHEMATICA package.

Implementation of it for the user is given in [4]. The paper [5] shows a similar program for the MatLab environment. But these programs are limited to the use of dependencies obtained in analytical form. For many similar software products (for example, [6]), the program texts are closed by interfaces. The same applies to the results presented on youtube (for example, [7]).

Formulating the goals of the article. Make a maple program for computer simulation of rotation of a solid, based on the geometric interpretation of Pounsot. In order to achieve this goal, it is necessary to describe and construct: a) a motionless, tactical plane of Puanso; b) the herpoloid on the Puansso plane; c) poloid on the surface of an ellipsoid of inertia; d) the process of rolling an ellipsoid of inertia along the Poonso plane; e) Examples of rolling an ellipsoid of inertia of a solid.

Main part. Let's denote through I_1, I_2 and I_3, – moments of inertia of the body relative to the fixed axes of the global coordinate system x, y and z, and through w_1, w_2 and w_3 – projections of the vector of instantaneous angular velocity of the body on these axes. It is known [1, 2] that the nature of the poludii and gerolodii essentially depends on the values of moments of inertia I_1, I_2 and I_3, as well as from integrals:

$$I_1^2 w_1^2 + I_2^2 w_2^2 + I_3^2 w_3^2 = K^2 = const\,; \qquad (1)$$

$$I_1 w_1^2 + I_2 w_2^2 + I_3 w_3^2 = 2T = const\,, \qquad (2)$$

where K – kinetic moment, and T – kinetic energy of the body of rotation.

Let the plane of Puanso π touches the ellipsoid of inertia at the point P, it is perpendicular to the invariant vector of kinetic moment K and will stand from the center of the ellipsoid at a constant distance $d = \sqrt{2T} / K$.

Fixed plane Puanso π, which depicts the herpoloid in the coordinate system space $Oxyz$, is described by the equation:

$$z = K\left(d - x(w_{10}I_1)/K - yx(w_{20}I_2)/K\right)/(w_{30}I_3)\,. \qquad (3)$$

Here through w_{10}, w_{20}, and w_{30} the initial values of angular velocities of the ellipsoid rotation around the corresponding coordinate axes are indicated.

Next, let's give a system of six differential equations that associate vector projections w_1(t), w_2(t) and w_3(t) instantaneous angular velocity of the body on the axis x, y and z, as well as Euler's corners $u(t)$, $v(t)$ and $w(t)$, which instantaneous axis of rotation forms with these axes of coordinates [1,2].

$$I_1 \frac{dw_1}{dt} = (I_2 - I_3) w_2 w_3\,;\; I_2 \frac{dw_2}{dt} = (I_3 - I_1) w_1 w_3\,;\; I_3 \frac{dw_3}{dt} = (I_1 - I_2) w_1 w_2\,; \qquad (4)$$

$$\frac{du}{dt}\sin w \sin v + \frac{dw}{dt}\cos v = w_1; \quad \frac{du}{dt}\sin w \cos v - \frac{dw}{dt}\sin v = w_2; \quad \frac{du}{dt}\cos w + \frac{dv}{dt} = w_3.$$

The solution of the system of equations (4) is carried out approximately by the Runge-Kutti method with the initial conditions $w_1(0)=w_{10}$, $w_2(0)=w_{20}$, $w_3(0)=w_{30}$. $u(0)=u_0$, $v(0)=v_0$ i $w(0)=w_0$.

Here are examples of execution of the completed program. This will take into account the values of moments of inertia of the body I_1, I_2 and I_3 relative to the fixed axes of the coordinate system x, y and z, for stable initial conditions of rotational velocities around the corresponding axes of an ellipsoid of inertia $w_{10}=1$; $w_{20}=2$; $w_{30}=1$, as well as constant initial values of the angles of rotation $u(0)=0.01$; $v(0)=0.01$; $w(0)=0.01$. To illustrate the results, frames of computer animations of the ellipsoid roller are selected.

Example 1. $I_1=2$; $I_2=12$; $I_3=15$. Рівняння дотичної площини $z=0,54 - 0,13x - 1,65y$. In fig. 1 shows three phases of rolling the ellipsoid in the tangent plane.

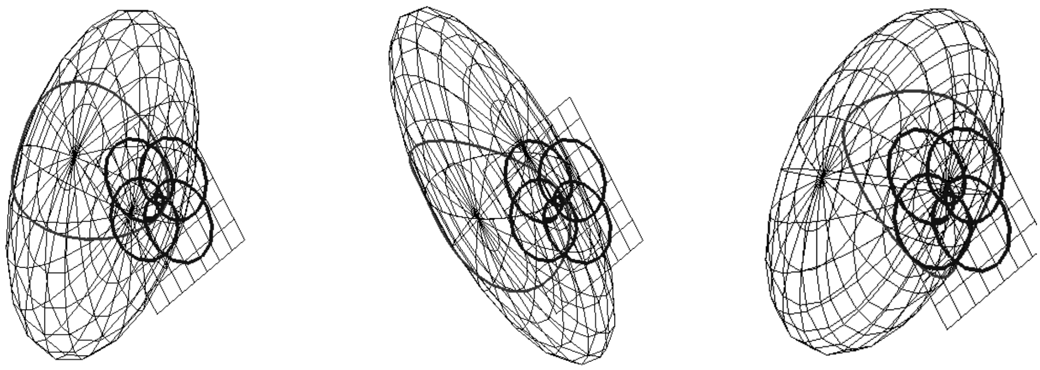

Fig. 1. Phases of rolling the ellipsoid in the tangent plane for example 1

Example 2. $I_1=8,2$; $I_2=5$; $I_3=13$. Equation of the tangent plane $z=0,49–0,63x–0,77y$. In fig. 2 shows three phases of the ellipsoid rolling.

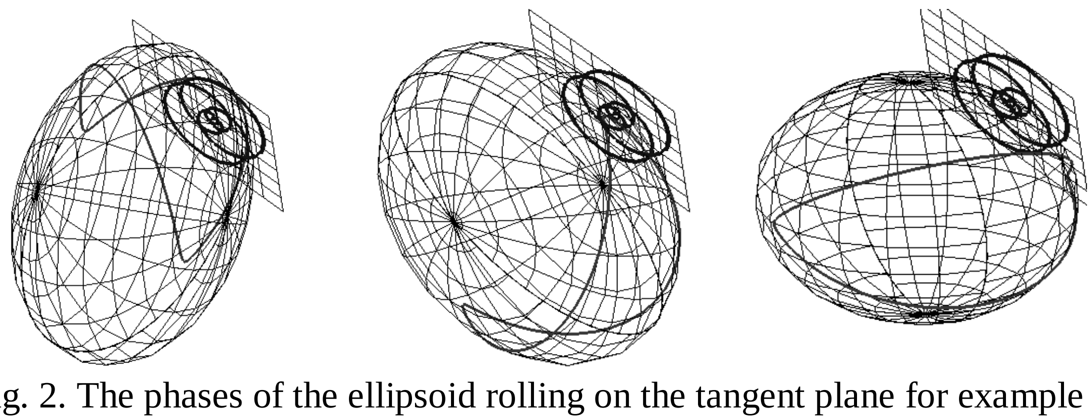

Fig. 2. The phases of the ellipsoid rolling on the tangent plane for example 2

Example 3. I_1=15; I_2=11; I_3=3. Equation of the tangent plane z=2,62 – 5x – 7,72y. In fig. 3 shows the phase of rolling the ellipsoid along the plane.

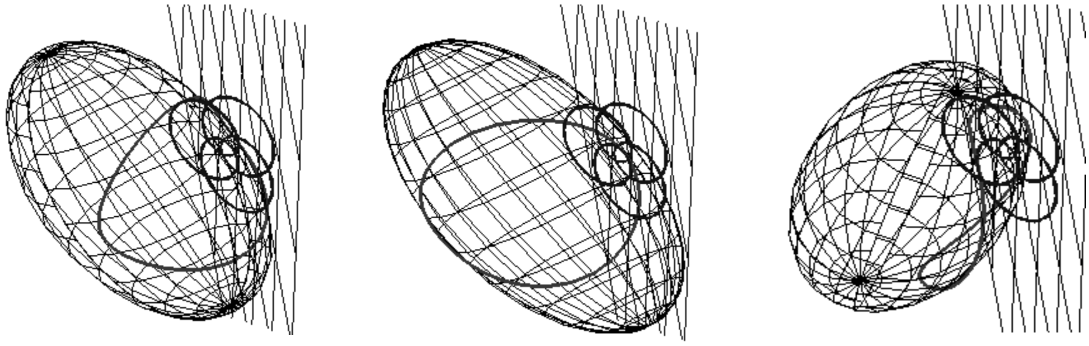

Fig. 3. The phases of the ellipsoid rolling on the tangent plane for example 3

In the animation mode, it can be shown that the rotation around the average magnitude of the axis of the ellipsoid of inertia is unstable. After all, with a slight perturbation of rotation around this axis, a new movement will be carried by the roll of an ellipsoid along the plane π, when the geometric point of the points of contact will serve as one of the poles, is quite close to the curve composed of any half of the two ellipses (separatrix). The ellipsoid movement will be equally probable for each of the relatively close positions in the four domains, on which the surface is separated by separatrices. This is typical of unstable rotation and substantially distinguishes this case from the rotation around the maximum and minimum values of the axes, when the perturbed movement is carried out by the roll of the ellipsoid of inertia along a fairly close posture that "envelops" one of these axes.

For calculations it is necessary to ask: the interval of integration time of the system of differential equations *Time* (not to be confused with the implementation time of the program), the number of intermediate positions of the rolling process (animation frames) *N*, initial values of vector projections $w_1(0)$, $w_2(0)$ i $w_3(0)$ instantaneous angular velocity of the body on the axis *x*, *y* and *z*, the initial values of Euler's corners *u(*0*)*, *v(*0*)* and *w(*0*)*, which instantaneous axis of rotation forms with these axes of coordinates.

Conclusions. In the work, the maple computer program for solid-state rotation of a fixed point based on the geometric interpretation of Puanso is compiled and tested.

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КОМПЬЮТЕРНАЯ РЕАЛИЗАЦИЯ МОДЕЛИ ПУАНСО ВРАЩЕНИЯ ОБЪЕКТА С НЕПОДВИЖНОЙ ТОЧКОЙ

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Приведено компьютерную графическую модель интерпретации Пуансо вращения твердого тела с неподвижной точкой для исследования устойчивости его движения. Для этого смоделировано качение без скольжения эллипсоида инерции этого тела по одной из своих касательных плоскостей, в результате чего построены соответствующие линии качения - герполодия на касательной плоскости и полодия на поверхности эллипсоида.

Ключевые слова: интерпретация Пуансо, момент инерции, эллипсоид инерции, качение эллипсоида, полодия, герполодия.

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Наведено комп’ютерну графічну модель інтерпретації Пуансо обертання твердого тіла з нерухомою точкою для дослідження стійкості його руху. Для цього змодельоване кочення без ковзання еліпсоїда інерції цього тіла по одній зі своїх дотичних площин, в результаті чого побудовані відповідні лінії кочення - герполодія на дотичній площині і полодія на поверхні еліпсоїда.

Ключові слова: інтерпретація Пуансо, момент інерції, еліпсоїд інерції, кочення еліпсоїда, полодія, герполодія