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INTERPOLATION CURVE ON THE BASIS OF SINGLE-SINGLE FUNCTIONS OF HYPERBOLIC SECANT

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The comparison of the interpolation curve on the basis of the sum of the functions of hyperbolic secant with other interpolation curves, in particular polynomials, is considered. It is shown that the proposed interpolation curve is not inclined to oscillations, regardless of the number of points through which it passes.

Keywords: interpolation curve, polynomial, hyperbolic secant, sum of one-type functions.

Formulation of the problem. When constructing contours or contours of flat shapes, there is the problem of conducting curves of continuous lines that must pass through a given point series. In systems of symbolic mathematics *Mathematica, Maple, MatLab, MathCAD* laid their interpolation methods, which have a number of features [1]. In addition to linear interpolation with straight lines, they offer different approaches to interpolating the point series as continuous curves, and piecewise interpolation with curve arcs. If, for piecewise interpolation, it is possible to reduce the oscillation, then there is another disadvantage - the connection of pieces of arcs with a given degree of smoothness. For example, the piecewise-cubic interpolation by the Hermite splines ensures the continuity of only the first derivative, and the second derivative is discontinuous [1]. In this connection, the problem arises to find an interpolation curve that is continuous and has minimal oscillations.

Analysis of recent research and publications. There is a research direction in which the discrete representation of the curve is thickened by intermediate points with a given density in the absence of oscillations, but there are no equations of the curve [2, 3]. Some authors drew attention to the curves, which is the sum of charts of bell-shaped curves [4-6]. Such interpolation functions allow the passage of a curve through a given number of points and due to its nature, under certain conditions, are not subject to oscillations. These functions include a value whose value can be influenced by the form of the interpolation curve.

Formulating the goals of the article. Consider the features of the interpolation of a point series by curves based on the sum of functions of hyperbolic secane and compare with polynomial interpolation.

Основна частина. Consider the curve, which is the sum of two functions of hyperbolic secane displaced along the axis *Ox* to a certain extent *h* (Fig. 1). The equation of the total function has the form:

$$y = y_1 + y_{2;} \quad y_1 = c_1 \operatorname{sech}(ax); \;\; y_2 = c_2 \operatorname{sech}\big(a(x-h)\big), \qquad (1)$$

where c_1, c_2 – steel values that specify the maximum coordinate value at the vertex of each graph;

a – became a value whose value affects the form of the graph.

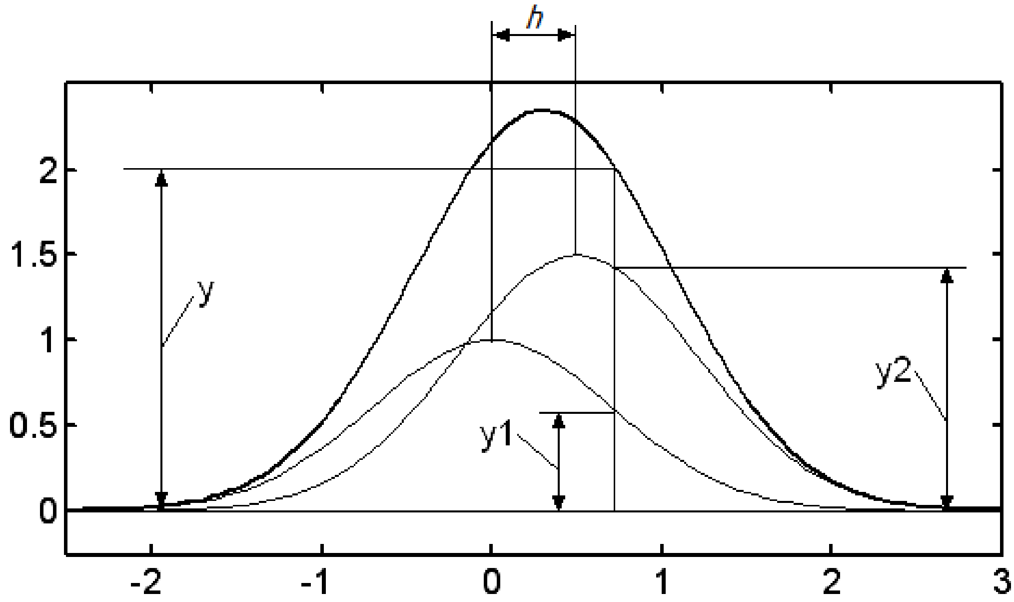

Fig. 1. Graphs of the functions of hyperbolic secana at $a = 1$; $c_1 = 1$; $c_2 = 1{,}5$; $h = 0{,}5$ and their total schedule

In fig. 1 shows two graphs of the hyperbolic section, which are shifted by the value of *h*, and their total graph. For $x = 0{,}8$ the general ordinates are obtained by adding ordinates of functions y_1 and y_2 . The figure also shows that with $x > 3$ *and* $x < -3$ the value of the ordinates of all functions is very small (practically equal to zero). Thus, the behavior of the total function is predicted: as the growth or decrease of the oscillation variable is eliminated, since the graph of the total curve practically coincides with the axis *Ox*.

The interpolation total function will be considered as parametric, ie in the form:

$$\begin{aligned} x &= c_1 \operatorname{sech}[a(t-t_1)] + c_2 \operatorname{sech}[a(t-t_2)] + \ldots + c_n \operatorname{sech}[a(t-t_n)]; \\ y &= d_1 \operatorname{sech}[a(t-t_1)] + d_2 \operatorname{sech}[a(t-t_2)] + \ldots + d_n \operatorname{sech}[a(t-t_n)], \end{aligned} \qquad (2)$$

where $c_1 \ldots c_n$, $d_1 \ldots d_n$ – coefficients that are provided that the curve (2) passes through *n* specified points;

a – the coefficient before an argument to which certain values can be given;

$t_{1 \ldots n}$ – ordinal point number: $t_1 = 1$; $t_2 = 2 \ldots$ $t_n = n$;

t – variable parameter, which at the given points takes the integer value (1, 2 ... *n*) by the number of the point, and between the points is a fractional.

To find the coefficients $c_1 ... c_n$, $d_1 ... d_n$ There are two linear systems of equations in *n* equations in each. For complex systems, the corresponding coordinates of each of *n* points are replaced by the current values of *x* and *y* in the left-hand side of each equation (2):

$$\begin{cases} x_1 = c_1 \operatorname{sech}[a(1-1)] + c_2 \operatorname{sech}[a(1-2)] + ... + c_n \operatorname{sech}[a(1-n)]; \\ x_2 = c_1 \operatorname{sech}[a(2-1)] + c_2 \operatorname{sech}[a(2-2)] + ... + c_n \operatorname{sech}[a(2-n)]; \\ . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ x_n = c_1 \operatorname{sech}[a(n-1)] + c_2 \operatorname{sech}[a(n-2)] + ... + c_n \operatorname{sech}[a(n-n)]; \end{cases} \tag{3}$$

$$\begin{cases} y_1 = d_1 \operatorname{sech}[a(1-1)] + d_2 \operatorname{sech}[a(1-2)] + ... + d_n \operatorname{sech}[a(1-n)]; \\ y_2 = d_1 \operatorname{sech}[a(2-1)] + d_2 \operatorname{sech}[a(2-2)] + ... + d_n \operatorname{sech}[a(2-n)]; \\ . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ y_n = d_1 \operatorname{sech}[a(n-1)] + d_2 \operatorname{sech}[a(n-2)] + ... + d_n \operatorname{sech}[a(n-n)]. \end{cases} \tag{4}$$

In systems (3), (4), instead of the variable *t* (point numbers), its value is set, and the coefficient *a* is considered to be predefined.
In [1], the interpolation of the point series on the test case of a discretely represented curve (DRC) according to the given coordinates of ten points (Table 1) is considered in the paper [1].

Table 1

Coordinates of points of the test DRC

	0	1	2	3	4	5	6	7	8	9
x	0	20	45	53	57	62	74	89	95	100
y	0	0	-47	335	26	387	104	0	100	0

The author of the paper [1] considered various methods of interpolation of the DRC using algorithms embedded in the common systems of computer mathematics listed at the beginning of the article. They are based on the use of algebraic curves: polynomials, splines, fractional-rational curves. The author concludes that the use of these types of interpolation does not produce the desired result. Let's compare the interpolation of the point series (Table 1) with a polynomial and an interpolation curve (2).
The degree of a polynomial will be one less than the number of points through which the curve must pass:

$$y = b_0 + b_1 x + b_2 x^2 + b_3 x^3 + ... + b_9 x^9. \tag{5}$$

We will alternate in (5) the coordinates of all ten points from the table. 1 and we get a system of ten linear equations.

The solution of the system will be the values of ten unknown coefficients $b_0, b_1, b_2, b_3, \ldots b_9$:

$$b=\begin{Bmatrix} 0; & 156456.8503966196; & -23827.25620877762; \\ 1492.1070451178582; & & -50.984919662155754; \\ 1.0493979889085476; & & -0.01340260414904959; \\ 0.00010416000035587583; & & -4.517635003471402*10\wedge\text{-}7; \\ 8.392710259937338*10\wedge\text{-}10 & & \end{Bmatrix} \qquad (6)$$

Analyzing the obtained coefficients of the polynomial (5) we conclude that their values in absolute value are in a large range of real numbers, requiring for operations with them corresponding electronic resources. For low-power computing machines, the construction of an interpolation curve for the considered algorithm becomes problematic.

In fig. 2 by an equation (5) with the substitution of the coefficients found in it (6), an interpolation curve is constructed which has at the beginning such an extremum, which made it show the curve itself on a reduced scale along the axis *Oy*.

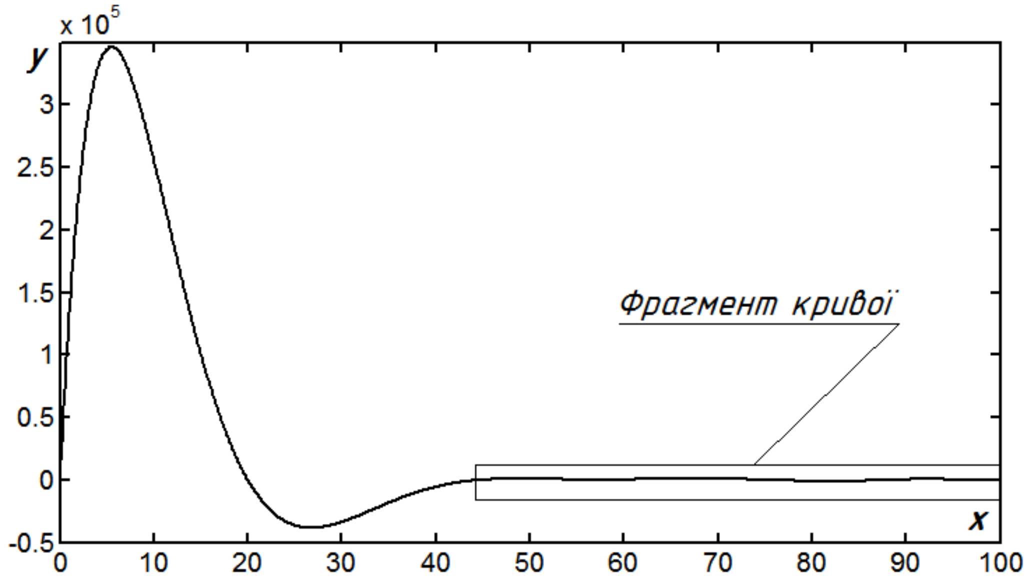

Fig. 2. Interpolation curve, constructed according to equations (5), which passes through a point series whose coordinates are given in Table 1.

We construct an interpolation curve in the same point series by equations (2). The coefficients c and d, obtained as a result of the solution of systems (3) and (4) for $a = 0.5$, have the following values:

$$c=\begin{Bmatrix} 35.8327; & -214.4113; & 351.2788; & -267.9846; & 172.29; \\ -27.6349; & -126.3872; & 278.046; & -245.7517; & 183.2027 \end{Bmatrix}$$

$$d=\begin{Bmatrix} -7451; & 24613; & -44984; & 59882; & -63371; \\ 54746; & -37053; & 18671; & -637; & 1113 \end{Bmatrix} \qquad (7)$$

The coefficients *c* and *d* in (7) are dimensional, which does not require high accuracy of calculations. In addition, the parametric equations (2), as well as the polynomial equation (5), include the same number of component functions, but the functions in (2) are the same as in (5), where the power index of each subsequent function increases.
In fig. 3, an interpolation curve was constructed by equations (2), and linear interpolation was performed.

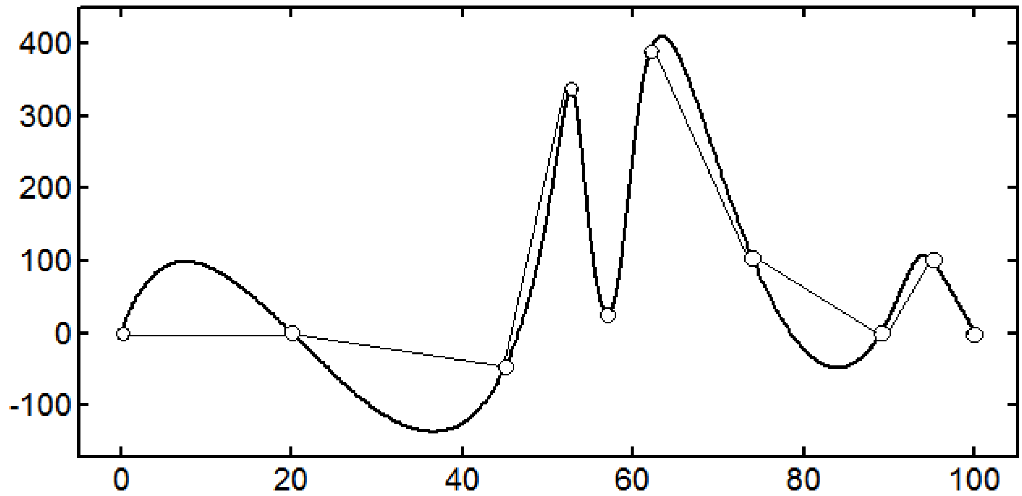

Fig. 3. Interpolation curve, constructed according to equations (2), which passes through a point series whose coordinates are given in Table 1.

Some segments of linear interpolation almost coincide with the arcs of the interpolation curve, so they are not shown. If you compare figures 2 and 3, then you can see that in Fig. 3 are some extremes, but they are much smaller than oscillations in rice. 2. Generally, if you combine graphs (2) and (3) into one single-picture with a common scale, then the interpolation curve (Figure 3) in the common drawing will look straight, parallel to the axis *Ox*. This indicates that the interpolation curve based on the sum of hyperbolic secane functions is not subject to oscillations, in contrast to the interpolation curve, which is a polynomial. In addition, the number of points of the duodenum does not affect the oscillation in contrast to the polynomial.

The fragment, isolated on rice, testifies to the dissipation of an interpolation curve based on the sum of hyperbolic secane functions to significant oscillations. 2. Compared to the split of the ordinates of the interpolation polynomial in the left part of the graph in Fig. The 2 right side is nearly straight. In fig. 4, where this fragment is depicted separately and on a different scale along the *Oy* axis, two interpolation curves are shown: 1 is an interpolation polynomial, and 2 is an interpolation curve based on the sum of hyperbolic secane functions. As can be seen from the figure, the interpolation curve on the basis of the sum of functions of the hyperbolic section in the sense of oscillations also wins on a limited section of the DRC.

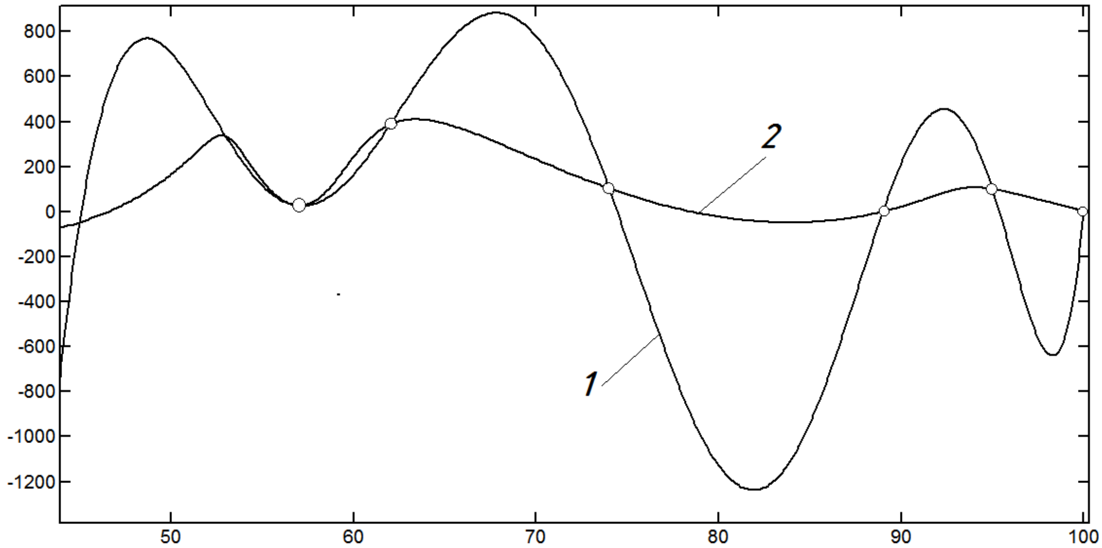

Fig. 4. Interpolation curves (1 - a polynomial of the 9th order (5); 2 is a curve based on the sum of functions of hyperbolic secana (2)) on a fragment of the DRC, isolated on rice. 2

Conclusions. The interpolation curve based on the sum of hyperbolic secane functions is less susceptible to oscillations than algebraic curves. It interpolates well enough the dot row, located on a straight line. To obtain the values of unknown coefficients in the parametric equations of the curve, it is necessary to solve two systems of linear equations, the number of which is equal to the number of points of the duodenum.

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ИНТЕРПОЛЯЦИОННАЯ КРИВАЯ НА ОСНОВЕ СУММЫ ОДНОТИПНЫХ ФУНКЦИЙ ГИПЕРБОЛИЧЕСКОГО СЕКАНСА

Пилипака С.Ф., Грищенко И.Ю., Кремец Т.С.

В работе рассмотрено сравнение интерполяционной кривой на основе суммы функций гиперболического секанса с другими интерполяционными кривыми, в частности полиномами. Показано, что предложенная интерполяционная кривая не склонна к осцилляциям независимо от количества точек, через которые она проходит.

Ключевые слова: интерполяционная кривая, полином, гиперболический секанс, сумма однотипных функций.

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Пилипака С.Ф., Грищенко І.Ю., Кремець Т.С.

В роботі розглянуто порівняння інтерполяційної кривої на основі суми функцій гіперболічного секанса із іншими інтерполяційними кривими, зокрема поліномами. Показано, що запропонована інтерполяційна крива не схильна до осциляцій незалежно від кількості точок, через які вона проходить.

Ключові слова: інтерполяційна крива, поліном, гіперболічний секанс, сума однотипних функцій.