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# FEATURES OF DETERMINING SELF-CROSSING POINTS IN DISCRETE MODELING

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***The paper proposes a method for determining the DPC self-intersection point in discrete modeling. An example of using the method in the method of thickening on the basis of variational formation of difference schemes of angular parameters is given.***

***Keywords: singular points, points of self-intersection, variational discrete geometric modeling (VDGM), absence of oscillations***

***Formulation of the problem.*** Points of self-intersection are special points that require additional resources in practical modeling [1,2]. The accuracy of their production, as well as the process of constructing an interpolating curve in their vicinity, significantly affects the accuracy of the results of geometric modeling, and therefore requires additional research. Methods of continuous interpolation are not always able to solve these problems in view of the diversity of the initial data set [3]. For example, in Fig. 1, several different tasks have been demonstrated, for which the spline curves integrated into the Solid Works package have a number of limitations. Conducted by Pugachev E.V. [1], V.M. Naidysh. and others [2], studies have shown the effectiveness of solving problems with singular points, in particular, with points of self-intersection by discrete interpolation methods. However, the possibilities for further development and improvement of this direction have not yet been exhausted.

***Analysis of recent research and publications.*** Pugachev E.V. was the first to pay attention to the problem of discrete interpolation of the DRC with singular points. [1]. Questions that were not considered in [1] concerning the close relationship of the tangent with tangents at other sites assigned at a singular point, as well as the order of approximation of the condensed accompanying broken line (ABL) to the tangent assigned at a singular point, were considered in studies by V. Naidish . and others [2,6]. However, this direction of research requires further development and improvement, in particular, about self-intersection points, and the possibilities of our method of discrete interpolation of the DRC developed on the basis of variational formation of difference angular parameter schemes [4] in solving the problem of condensation near self-intersection points.

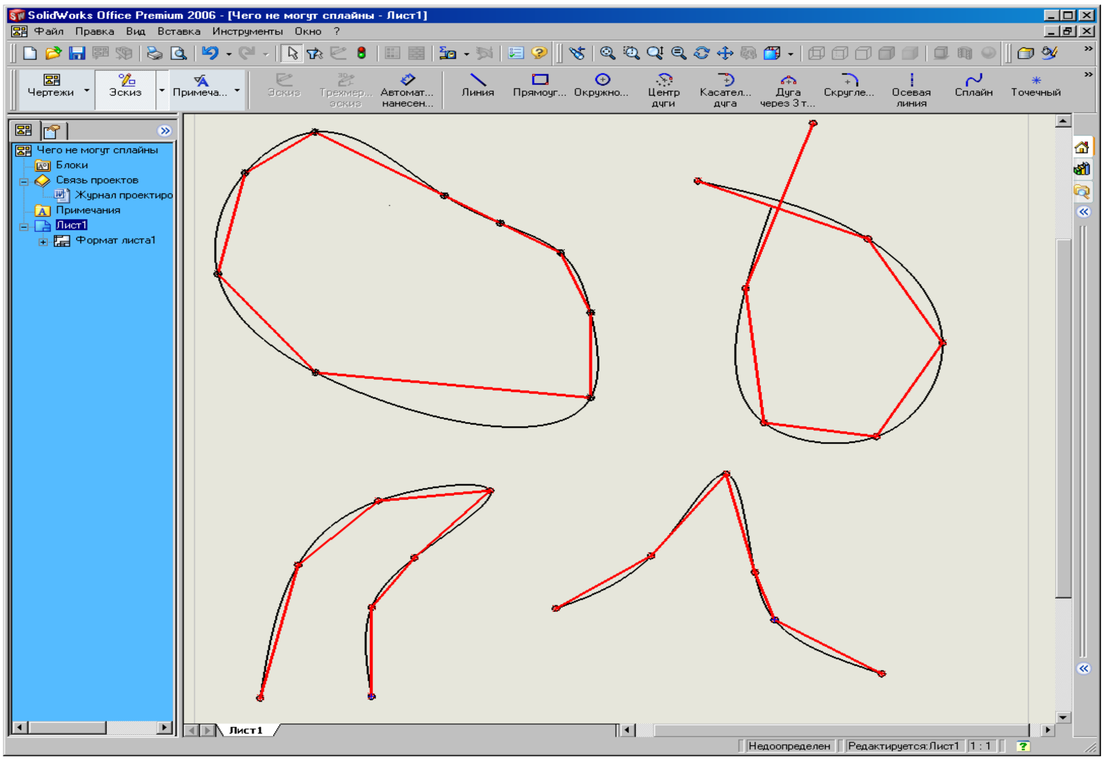


Fig. 1. Limitations when using splines:
a) DRC with rectilinear section; b) fragment of the DRC with a self-intersection point; c) a fragment of the DRC with a return point of the second kind; d) fragment of the DRC with a point of sharpening (fracture)

***Formulating the purpose of the article.*** The purpose of the paper is to determine the points of self-intersection in the preliminary estimation of the type of the discrete curve, as well as the use of the method considered in the work within the framework of the condensation method on the basis of the variational formation of the difference schemes of the angular parameters.

***Main part.*** Take the fragment of the DRC (Fig. 2). Consider the link $(i,i+1), i=\overline{0,n-3}$ and we will investigate it for the possibility of crossing with the other links of the DRC: $(N,N+1), N=\overline{i+2,n-1}$. The presence of an intersection point, we call it m, with coordinates $(x_m, y_m)$, can be determined from the condition:

$$\begin{cases} x_m \notin [x_i; x_{i+1}] \\ y_m \notin [y_i; y_{i+1}] \end{cases}. \qquad (1)$$

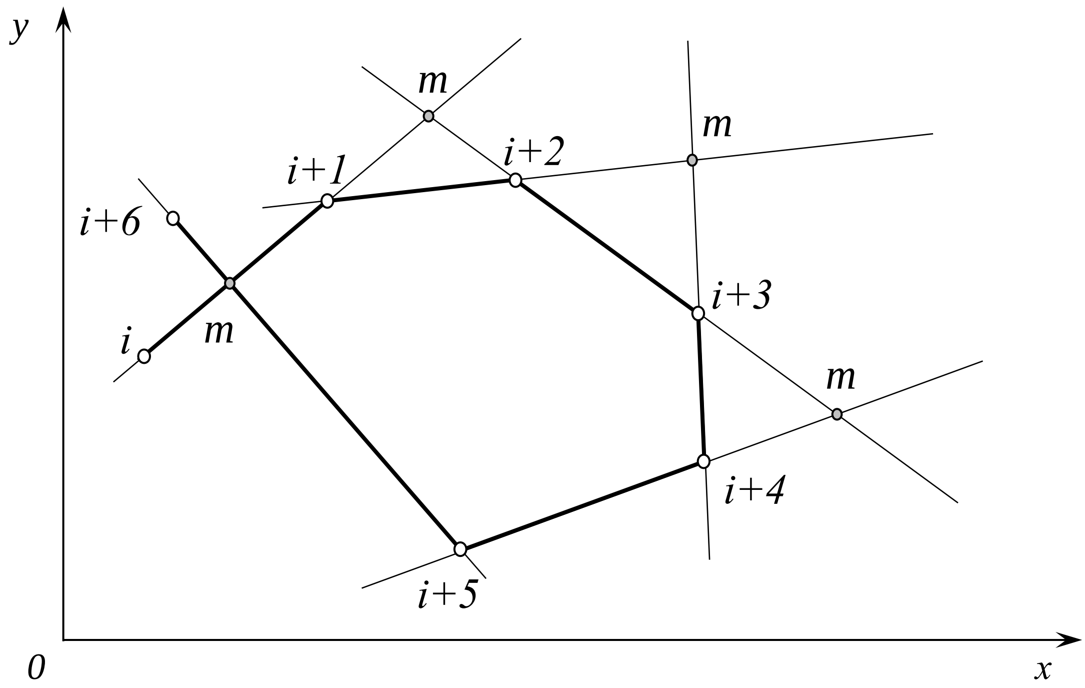


Fig. 2. Scheme for determining the point of self-intersection

If condition (1) is not satisfied, we check that both link nodes $(N,N+1)$ lay on one side of the link $(i,i+1)$, those, the following condition must be satisfied:

$$\begin{cases}\delta_N > 0 \\ \delta_{N+1} > 0\end{cases} \text{ or } \begin{cases}\delta_N < 0 \\ \delta_{N+1} < 0\end{cases}, \tag{2}$$

where $\delta_N, \delta_{N+1}$ – distance from points $N$ and $N+1$ respectively, to the line on which the link lies $(i,i+1)$.

$$\delta_N = \frac{Ax_N + By_N + C}{\sqrt{A^2 + B^2}}, \quad \delta_{N+1} = \frac{Ax_{N+1} + By_{N+1} + C}{\sqrt{A^2 + B^2}}, \tag{3}$$

where $A,B,C$ – coefficients of the equation of a straight line passing through a link $(i,i+1)$;

$x_N, y_N, x_{N+1}, y_{N+1}$ – coordinates of points $N$ and $N+1$ respectively.

If conditions (1) and (2) are not satisfied, then the intersection point $m$ belongs to a link $(i,i+1)$. Then the next link of the DRC is considered and the calculations are repeated. At the last step, the intersection of links $(n-3,n-2)$ and $(n-1,n)$.

The proposed method is easy to implement and allows for the preliminary analysis of the initial point series to determine the existence of points of self-intersection.

We consider the condensation of DRC by the method of variational formation of difference schemes of angular parameters [4] for the fragment of the DRC containing the point of self-intersection. Note that the presence of self-intersection points does not introduce any changes into the main algorithm of the method [4].

We show this by the example of a condensation of a discrete plane curve (Table 1) on a nonuniform grid with a predetermined accuracy $\varepsilon = 0{,}3$ subject to the receipt of a non-oscillating resultant DRC.

Table 1

Initial point number

| $i$ | $0$ | $1$ | $2$ | $3$ | $4$ | $5$ | $6$ | $7$ | $8$ |
|---|---|---|---|---|---|---|---|---|---|
| $x_i^0$ | 20 | 35 | 55 | 110 | 115 | 85 | 67 | 90 | 130 |
| $y_i^0$ | 60 | 35 | 25 | 50 | 70 | 85 | 60 | 20 | 10 |

The condensation of the given test example was carried out according to the main algorithm of the method [7]. To ensure the specified accuracy of the condensation, two steps of condensation were carried out. At the second step the condensation condition for the absence of oscillations [5] was observed

$$max\left|\gamma^{1}_{i+0,5}\right| = 0{,}283 < 0{,}3 .$$

After the second step of thickening, the points of the obtained DRC are connected by segments of ABL, which is considered the final form of the interpolating curve (Fig. 3).

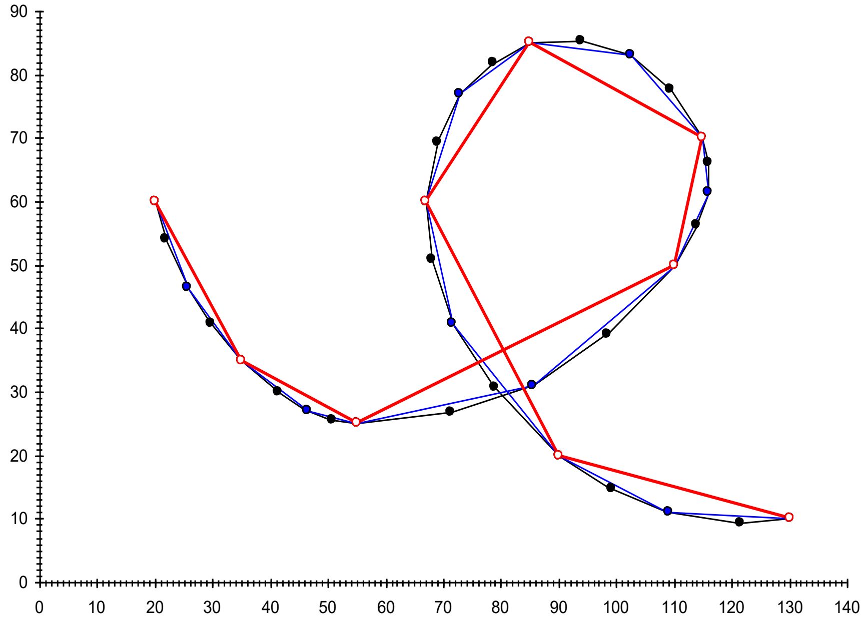


Fig. 3. Two steps of thickening a test DPC containing a self-intersection point that is not explicitly specified

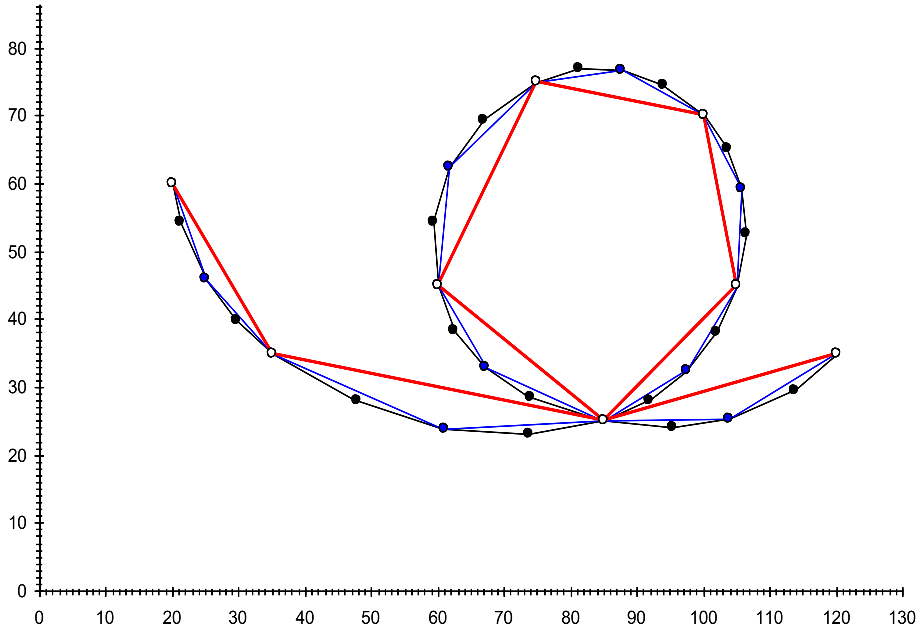


Fig. 4. Two steps of thickening a test DSC containing a self-intersection point, the coordinates of which are known

In Fig. 4 shows the result of two condensation steps for the DCS with a self-intersection point, the coordinates of which are known in advance. As in the previous example, the main algorithm of the method was used to thicken this test curve on the basis of variational formation of difference schemes of angular parameters [5].

***Conclusions.*** The method proposed in this paper is easy to implement and allows us to determine the existence of points of self-intersection when considering the initial point series. This can later be used both in discrete geometric modeling of plane curves with a number of features in geometry (points of self-intersection), as well as in continuous geometric modeling, since it allows one to determine the way the curve is specified.

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## ОСОБЛИВОСТІ ВИЗНАЧЕННЯ ТОЧОК САМОПЕРЕТИНУ В ДИСКРЕТНОМУ МОДЕЛЮВАННІ

Спірінцев Д.В., Найдиш А.В., Балюба І.Г., Лєбєдєв В.О.

***У роботі пропонується спосіб визначення точок самоперетину ДПК в дискретному моделюванні. Наведено приклад використання способу в рамках методу згущення на основі варіативного формування різницевих схем кутових параметрів.***

***Ключові слова: особливі точки, точки самоперетину, варіативне дискретне геометричне моделювання (ВДГМ), відсутність осциляції.***

## ОСОБЕННОСТИ ОПРЕДЕЛЕНИЯ ТОЧЕК САМОПЕРЕСЕЧЕНИЯ В ДИСКРЕТНОМ МОДЕЛИРОВАНИИ

Спиринцев Д.В., Найдыш А.В., Балюба И.Г., Лебедев В.А.

***В работе предлагается способ определения точки самопересечения ДПК. Приведен пример использования способа в рамках метода сгущения на основе вариативного формирования разностных схем угловых параметров.***

***Ключевые слова: особые точки, точки самопересечения, вариативное дискретное геометрическое моделирование (ВДГМ), отсутствие осцилляции.***