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CONSTRUCTION OF GEOMETRICAL MODELS DISTRIBUTION OF LOCAL CONTAMINATION COMPONENTS USING THE DISCRETE INTERPOLATION METHOD AND THE KOONS EQUATION

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The paper deals with the construction of geometric models for the distribution of certain components of various kinds of local contamination using the discrete interpolation method and using the Koons method, which makes it possible to model complex multiparameter systems, and environment.

Keywords: interpolation, one-parameter set, discrete interpolation method, interpolation node, Koons equation.

Formulation of the problem. Simulation of complex multi-parametric systems and environments such as ecological, hydrological, energy, geological, geomorphological, etc., from predicting their condition, determining the local pollution of these environments - is a rather complicated and at the same time an important engineering and practical, socio-social task that arises in the process of processing the monitoring results of such systems and environments and elaborates certain recommendations and methods for long-term forecasting of their condition, determination of anthropogenic impact.

In the practice of simulating complex multiparameter systems and environments it is impossible to obtain their analytical mathematical models. For example, such systems and environments as ecological, geological, hydrological, geomorphological, energy, etc., are characterized by a large number of parameters, and these parameters have, moreover, a diverse structure and different qualities, and the latter often quite often have some anisotropy in time and space Therefore, the above systems and environments relate to such a class of objects that it is practically impossible to describe analytically in the form of a continuum model.

It becomes clear that the task of developing mathematical models of such systems and environments, defining their parameters and properties, forecasting in time and space of their state is relevant.

Analysis of recent research and publications. In literary sources, in particular, the specialized, basically, are considered questions of monitoring of systems and environments, statistical processing of their results. Questions of the construction of their models are considered to a lesser extent, and they are usually chemical or physico-chemical. The question of constructing geometric models of such systems and environments is practically non-existent. In the previous papers of the author [1-4] the questions of simulation of complex technical objects, processes and environments using the discrete-interpolation method were considered. We emphasize that algorithms and methods of geometric modeling of complex multiparametric systems and environments with the construction of their discrete mathematical models in literary sources are practically absent. The above considerations allow us to formulate the following goals of this work.

Formulating the goals of the article. At present, the purpose of the study is to construct discrete geometric models of complex multiparameter systems and environments based on the discrete-interpolation method, as well as using the Kuns method.

Основна частина. Complex multi-parametric systems and environments such as ecological, energy, hydrological, meteorological, geomorphological, energy, etc. can be classified as stochastic systems by all features. This follows from the fact that such systems and environments are often interconnected and it is impossible to view a separate system in isolation. Accordingly, modeling, forecasting and controlling the state of such environments is a multi-parameter and stochastic process. For such systems and environments it is impossible to create their continual model, which leads to the use of discrete mathematical models, namely geometric, in the form of discrete numerical arrays, elements of which are certain components of systems and environments.

The discrete-interpolation method (hereinafter referred to as DIM) of modeling of multiparametric objects, systems and environments, developed by the author, is based on the use of certain discrete-interpolation schemes using the interpolation apparatus of the Lagrange.

The used DIM is original, and one of its components is the interpretation of the interpolation node not as a point, but as a more complex object, such as a line, surface, process or medium, represented in the form of some functionals, as a collection of their properties and parameters for a certain interpolation scheme. Such an approach to the simulation of multiparameter systems and environments allows one to include in a one-parameter set of functionals whose parameters have a different structure and properties, which is inherent in the above-mentioned systems and environments. Models in the form of a matrix set of certain parameters can be used for further simulation of complex multi-parameter objects and environments. Different interpolation schemes are used to construct these models on the basis of the DIM, with the help of which it is possible to obtain certain one-parameter sets, which will be exactly such models.

In the DIM, the Lagrange polynomials used are as follows:

$$\Phi(u)_n = \sum_{i=0}^{n-1} F_i(p_1, p_2, \dots, p_m) \prod_{\substack{j=0 \\ j \neq i}}^{n-1} \frac{u - u_j}{u_i - u_j}, \tag{1}$$

where u – interpolation parameter, $F_i(p_1, p_2, \dots p_k)$ - node function, $p_1, p_2, \dots p_k$ – parameters of the nodal function (indicators of pollution, the level of concentration of certain substances, taking into account the natural characteristics of the environment, etc.), n – number of interpolation nodes.

In the interpolation nodes there are certain discrete functions, and under the interpolation scheme, we continue to understand the layout of the arrangement of exactly its nodes.

If $F(p_1, p_2, p_3, \dots, p_k, \dots, p_m)$ – multiparameter implicitly given function, then its formation in the form of a certain functional $\Phi(p_{i,j})$, which is given by a matrix M[i, j] leads to such expression:

$$\Phi(p_{i,j}) = \sum_{i=0}^{n-1} M_i(i, j) \prod_{\substack{j=0 \\ j \neq i}}^{n-1} \frac{u - u_j}{u_i - u_j}, \tag{2}$$

where n - number of interpolation nodes, u - parameter $M_i[i, j]$, corresponding to the intermediate position or state,
$F(p_1, p2, p_3, \dots, p_k, \dots, p_m) = M[i, j]$, and

$$M[i,j] = \begin{pmatrix} p_{1,1} & p_{1,2} & \cdots & \cdots & p_{1,n} \\ p_{2,1} & p_{2,2} & \cdots & \cdots & p_{2,n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ p_{m,1} & p_{m,2} & \cdots & \cdots & p_{m,n} \end{pmatrix}.$$

That is M[i, j] is a node-based discrete-interpolation, for example, an ec matrix.

Expression (2), which is a generalized discrete-interpolation matrix, is a discrete geometric model of a particular system or environment (ecological, energy, hydrological, geological, meteorological, etc.).

Consequently, having the results of monitoring for measurement, for example, the level of concentration of a particular component of the medium, one can obtain a geometric model of its distribution or dispersion. Such models can be constructed for a certain number of components, which will allow a relatively complete integrated picture of the situation or state of a particular environment.

But it should be noted that the results of monitoring (measurements) are not always invested in a particular scheme. Their positional-spatial parameters can be arbitrary, for example, contour, that is, measurements were made for certain reasons within the boundaries of a certain area, and it was not possible to carry out internal measurements. In this case, we propose to use a DIM with the use of the Kuns method. It is known that the construction of the Kuns surface is based on a discrete contour basis in the form of four boundary curves whose coefficients are linear functions, and the equation has the following form:

$$r(u,v)=\sum_{i=0}^{1}a_i(u)\cdot r(i,v)+\sum_{j=0}^{1}a_j(v)\cdot r(u,j)+\prod_{i=0}^{1}a_i(u)\cdot a_j(v)\cdot r(i,j),$$

where $a_0(u) = 1 - u,\ a_1(u) = u,$

$$a_0(v) = 1 - v,\ a_1(v) = v,\quad 0 < u < 1,\ 0 < v < 1.$$

This way you can get not only certain missing values of the parameters p_1, p_2, p_3, …, p_k, … , p_m, but also applying the discrete-interpolation method, obtaining the Kuns equation for various intermediate positions, substituting these equations in (2) as interpolation nodes. This will allow us to obtain a more complete discrete geometric model for the distribution of certain components in a certain local area.

Висновки. The discrete geometric models of multi-parametric systems and environments built on the basis of DIM allow for more effective monitoring, simulation of their predicted state. The method differs in great variation and allows to include objects in a one-parameter set, which have even different structure and properties, which is practically impossible to combine in a continuum model. That is what makes it possible to simulate such complex multi-parametric systems and environments as ecological, hydrological, energy, geological, geomorphological.

Література

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ПОСТРОЕНИЕ ГЕОМЕТРИЧЕСКИХ МОДЕЛЕЙ РАСПРЕДЕЛЕНИЯ КОМПОНЕНТОВ ЛОКАЛЬНЫХ ЗАГРЯЗНЕНИЙ С ИСПОЛЬЗОВАНИЕМ ДИСКРЕТНО-ИНТЕРПОЛЯЦИОННОГО МЕТОДА И УРАВНЕНИЯ КУНСА

Холковский Ю.Р.

В работе рассматриваются вопросы построения геометрических моделей распределения определенных компонентов разного рода локальных загрязнений при использовании дискретно-интерполяционного метода и с использованием метода Кунса, что дает возможность моделирования сложных многопараметрических объектов, систем и сред.

Ключевые слова: интерполяция, однопараметрическое множество, дискретно-интерполяционный метод, узел интерполяции, уравнение Кунса.

ПОБУДОВА ГЕОМЕТРИЧНИХ МОДЕЛЕЙ РОЗПОДІЛЕННЯ КОМПОНЕНТІВ ЛОКАЛЬНИХ ЗАБРУДНЕНЬ ІЗ ВИКОРИСТАННЯМ ДИСКРЕТНО-ІНТЕРПОЛЯЦІЙНОГО МЕТОДУ ТА РІВНЯННЯ КУНСА

Холковський Ю.Р.

У роботі розглядаються питання побудови геометричних моделей розподілення певних компонентів різного роду локальних забруднень при використанні дискретно-інтерполяційного методу та з використанням методу Кунса, що дає можливість моделювання складних багатопараметричних об’єктів, систем та середовищ.

Ключові слова: інтерполяція, однопараметрична множина, дискретно-інтерполяційний метод, вузол інтерполяції, рівняння Кунса.