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COMPOSITION METHOD OF GEOMETRICAL MODELING: ESSENCE, FEATURES AND PROSPECTS OF APPLICATION

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This article explores the features of the composite geometric modeling method, developed on the basis of point BN-calculus shows principles of construction of parabolic response surfaces (B-surface) and algorithm of generalization of model factors. The following properties of B-surfaces allow effectively use them for multifactor processes modeling.
Keywords: point BN-calculus, multifactor, composite modeling method, the B-surface, parabolic surface.

***Formulation of the problem*.** In modeling processes, in particular in the field of energy efficiency, the problem of combining a large number of physically diverse factors often arises. Traditional methods for modeling processes and studying phenomena are combinational. Their components, as a rule, are mutually dependent, in this case, different approaches are used to find correlations between them..

The presence of a mutual dependence between the elements of the model at the initial stage of modeling, always brings to it certain restrictions on the number of factors, the size of the matrices, and so on. In addition, the need for the integration of physically diverse factors further complicates modeling, may increase the error, which entails making false decisions.

In order to increase the adequacy of the model, various ways of identifying the main factors, components of a geometric nature are developed, which provides some improvements to the model, however, while not limiting the number of source factors characterizing the process. Therefore, the problem of developing a simulation method that would have had the opportunity, when creating a model, to include the maximum amount of diverse factors of the investigated process.

In our opinion, solving the problem of integration of heterogeneous factors and increasing the possible source information in the simulation lies in the development of composite models, in which there is no mutually determined relationship between its elements.

By the composition, in our case, should be understood as the structure of uncorrelated heterogeneous elements, combined in one set, in order to obtain the desired. At the same time, the replacement (change) of any element of the composition does not entail a change in its other elements. This feature of the composite method of geometric modeling is important, because in practice, in order to construct an adequate model, it often becomes necessary to change qualitatively and quantitatively the source factors. Thus, the development of the compositional method of geometric modeling is relevant due to the possibility of unlimited increase in the number of source factors and their replacement, in the process of modeling, without changing the model.

Analysis of recent research and publications. The idea of developing composite surface modeling methods was based on the research of the point calculation of Baluba-Naidish.

The mathematical apparatus of a point BN-number, in particular, the construction of parabolic surfaces of the response, became the basis for the development of the composite modeling method proposed in the work.

Some questions which are components of the compositional method were considered in works [1,7,8]. In this work, the most complete studies on the development of the compositional method, which are relevant and original.

Formation of the purposes of the article. Identify the basics of the development of the composite method of geometric modeling in terms of the point calculation of Baliba-Naidish, to show its advantages and to determine the prospects of application.

Main part. In [1, 2], a geometric algorithm for obtaining a point equation of the arc of a parabola of the second order was given. But for completeness of teaching, let's give this algorithm.

It is known [4] that in the general case the curves of the second order are uniquely determined by five points, but because the parabola invariant *D*, which is a minor A_{33}, is equal to zero, then one of the roots of the characteristic equation is zero, which reduces the number of points from five to four for parabola, which are necessary for its unambiguous definition. In this case, three of them are valid, and one is inappropriate.

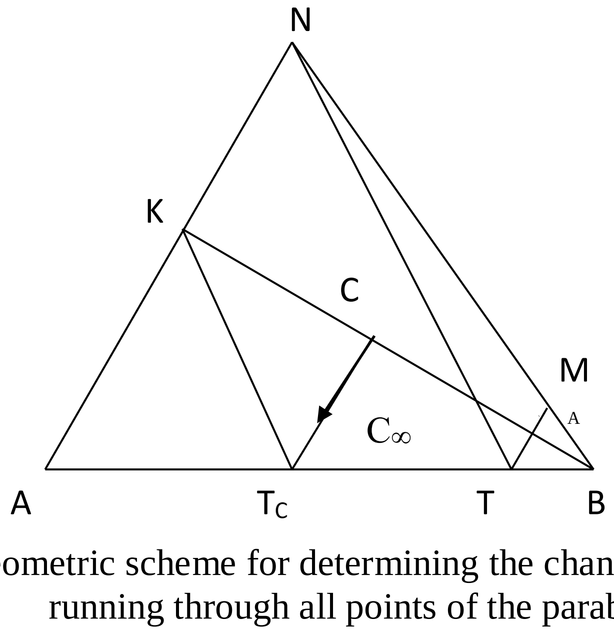

Fig.1 A geometric scheme for determining the changeable point M, running through all points of the parabola

In accordance with [2], the wrong point C_∞ (fig. 1) define as a direct line CT_C. Point C є вихідною, а точку T_C, according to [1, 3], we define the point equation:

$$T_C = (B\text{-}A)\, t_C + A. \tag{1}$$

Given that it is straightforward AK and T_CC parallel, we can write:

$t_C = \dfrac{T_C A}{BA} = \dfrac{CK}{BK}$, from where $t_C = \dfrac{C-K}{B-K}$, then we will define K as follows:

$$K = B\frac{-t_C}{\overline{t_C}} + C\frac{1}{\overline{t_C}}, \tag{2}$$

where $\overline{t_C} = 1 - t_{C.}$.

To construct a variable point M define the parameter t because of the relationship:

$t = \dfrac{TA}{BA} \rightarrow t = \dfrac{T-A}{B-A}$ - point representation form of the parameter t. From where we have:

$$T = (B-A)t + A. \tag{3}$$

In accordance with the geometric scheme (Fig. 1), define the point N using the relation:

$$\frac{KT_C}{NT} = \frac{T_C A}{TA}, \rightarrow \frac{K-T_C}{N-T} = \frac{T_C - A}{T-A} \rightarrow N = A\frac{(t_C - t)}{t_C} + B\frac{-t}{\overline{t_C}} + C\frac{t}{t_C \overline{t_C}}, \tag{4}$$

or in a different form:

$$N = (A-C)\frac{(t_C - t)}{t_C} + (B-C)\frac{-t}{\overline{t_C}} + C. \tag{5}$$

Based on similarities of triangles ΔBAN and ΔBTM, make a relationship:

$$\frac{TB}{AB} = \frac{MB}{NB}, \quad \text{or in point form:} \quad \frac{T-B}{A-B} = \frac{M-B}{N-B}, \tag{6}$$

From now on, taking into account (1), (2), (3), (5), we obtain the point equation of the parabola of the second order:

$$M = A\frac{\bar{t}(t_C - t)}{t_C} + B\frac{t(t_C - t)}{\overline{t_C}} + C\frac{t\bar{t}}{t_C \overline{t_C}}. \tag{7}$$

Applying the above algorithm three times in the direction of the parameter u and three times in the direction of the parameter v (Fig. 2), (in Figure 2, for the purpose of a smaller graphical load, shows the construction of two parabolas in the direction u and two parabolas in the direction v), we obtain a parabolic B-surface (Loose surface), which is given discretely in the form of separate six edges, which are parabolas whose equations are given in point form (7) in general form.

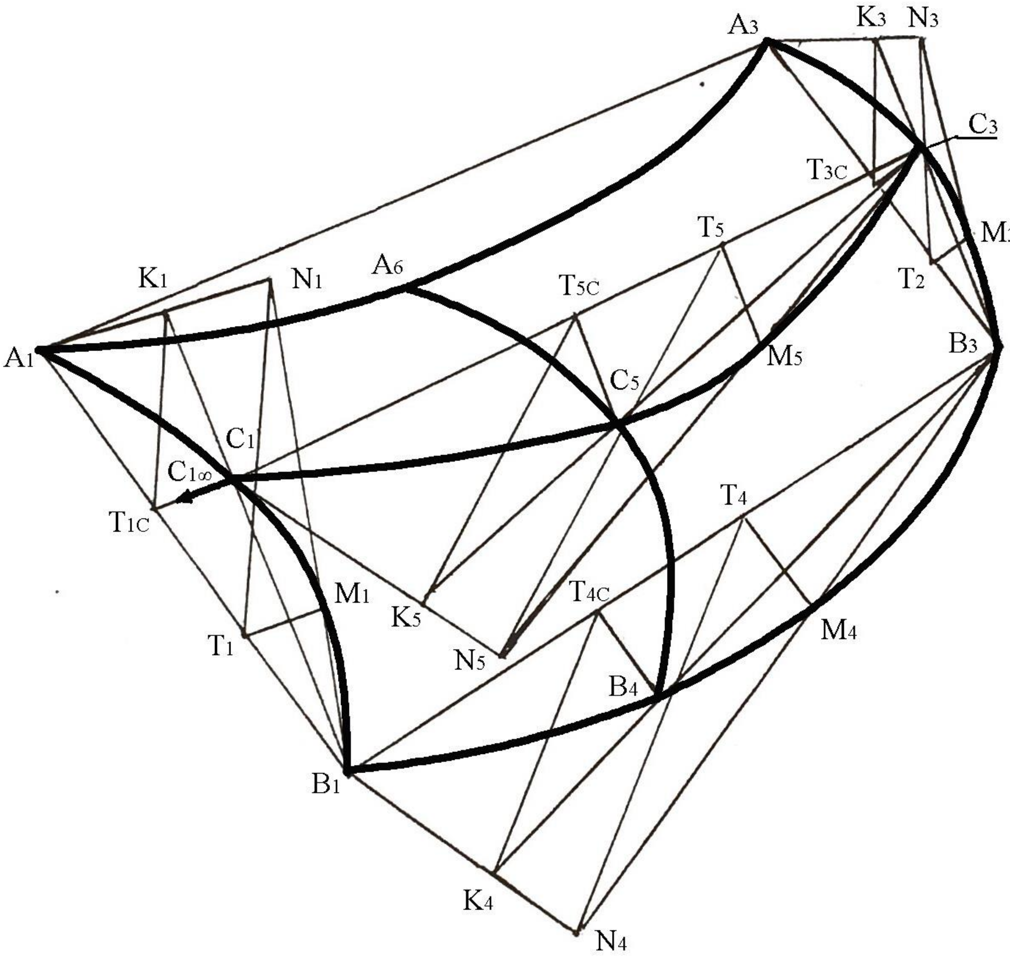

Fig. 2. Geometric scheme of formation of B-surface

We will write down these six point equations in accordance with the notation taken in Fig. 2 in the direction *u*:

$$M_1 = A_1 p_1 + B_1 q_1 + C_1 r_1, \text{ де } p_1 = \frac{\bar{u}(u_C - u)}{u_C}; \quad q_1 = \frac{u\bar{u}}{u_C \bar{u}_C}; \; r_1 = \frac{u(u - u_C)}{\bar{u}_C}; \tag{8}$$

where u - parameter along the B-curve, $u = \overline{0,1}$;

$\bar{u} = 1 - u$ - supplementary parameter u to unity;

u_C - parameter defining the direction of the parabola branches;

$\overline{u_C} = 1 - u_C$ - supplementary parameter u_C to unity.

$$M_2 = A_2 p_1 + B_2 q_1 + C_2 r_1, \quad M_3 = A_3 p_1 + B_3 q_1 + C_3 r_1, \tag{9}$$

and in the direction v:

$$N_1 = A_1 p_2 + A_2 q_2 + A_3 r_2, \text{ де } p_2 = \frac{\bar{v}(v_C - v)}{v_C}; \quad q_2 = \frac{v\bar{v}}{v_C \overline{v_C}}; \; r_2 = \frac{v(v - v_C)}{\overline{v_C}}; \tag{10}$$

Where v – parameter along the B-curve of the transverse direction, $v = \overline{0,1}$;

$\bar{v} = 1 - v$ - supplementary parameter v to unity;

v_C - parameter that defines the direction of the parabola branches;

$\overline{v_C} = 1 - v_C$ - supplementary parameter v_C to unity.

$$N_2 = B_1 p_2 + B_2 q_2 + B_3 r_2, \quad N_3 = C_1 p_2 + C_2 q_2 + C_3 r_2. \qquad (11)$$

If the point equation M_1 (8) instead of points A_1, B_1, C_1 substitute, respectively, the point equation N_1 з (10) and N_2 and N_3 з (11), which allows the method of moving simplex [7], we obtain a point equation of the segment of the empirical surface (Fig. 3). This surface may be discrete in the case of a discrete representation of the parameters *u* and *v*, or continuum in the case of continuity *u* and *v*.

If you enter generic notation: $A_1=x_{11}$, $B_1=x_{12}$, $C_1=x_{13}$, $A_2=x_{21}$, $B_2=x_{22}$, $C_2=x_{23}$, $A_3=x_{31}$, $B_3=x_{32}$, $C_3=x_{33}$, then the point equation of the segment B-surface will look:

$$M = x_{11}a_{11} + x_{21}a_{21} + x_{31}a_{31} + x_{12}a_{12} + x_{22}a_{22} + x_{32}a_{32} + x_{13}a_{13} + x_{23}a_{23} + x_{33}a_{33}, \qquad (12)$$

where $$a_{11} = \frac{\bar{v}(v_c - v)}{v_c} \cdot \frac{\bar{u}(u_c - u)}{u_c}; \quad a_{21} = \frac{v\bar{v}}{v_c \bar{v}_c} \cdot \frac{\bar{u}(u_c - u)}{u_c}; \quad a_{31} = \frac{v(v - v_c)}{\bar{v}_c} \cdot \frac{\bar{u}(u_c - u)}{u_c};$$

$$a_{12} = \frac{\bar{v}(v_c - v)}{v_c} \cdot \frac{u\bar{u}}{u_c \bar{u}_c}; \quad a_{22} = \frac{v\bar{v}}{v_c \bar{v}_c} \cdot \frac{u\bar{u}}{u_c \bar{u}_c}; \quad a_{32} = \frac{v(v - v_c)}{\bar{v}_c} \cdot \frac{u\bar{u}}{u_c \bar{u}_c}; \qquad (13)$$

$$a_{13} = \frac{\bar{v}(v_c - v)}{\bar{v}_c} \cdot \frac{u(u - u_c)}{\bar{u}_c}; \quad a_{23} = \frac{v\bar{v}}{v_c \bar{v}_c} \cdot \frac{u(u - u_c)}{\bar{u}_c}; \quad a_{33} = \frac{v(v - v_c)}{\bar{v}_c} \cdot \frac{u(u - u_c)}{\bar{u}_c}.$$

Or in another record

$$M = \sum_{i,j=1}^{3} x_{ij} a_{ij} . \qquad (14)$$

In this form of record, in further research, we will use the B-surface to represent the individual factors of the multifactorial processes or situations.

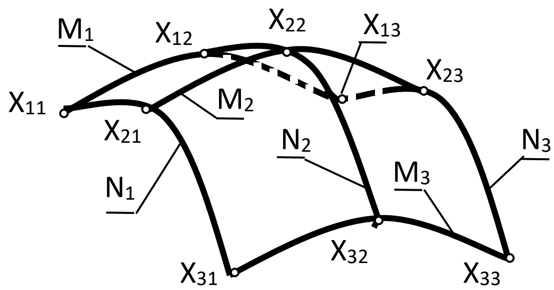

Рис. 3. Сегмент Б-поверхні

If the scheme (Fig. 3) representing the segment B-surface (12) is arbitrarily designated by a circle Q_{ij}, and at a certain technological level of such surfaces there will be three, on each of which, according to a technological criterion, to determine the optimal points Q_{11}, Q_{12}, Q_{13}, and then these points interpolate with (7), we will get the B-curve (Fig. 4).

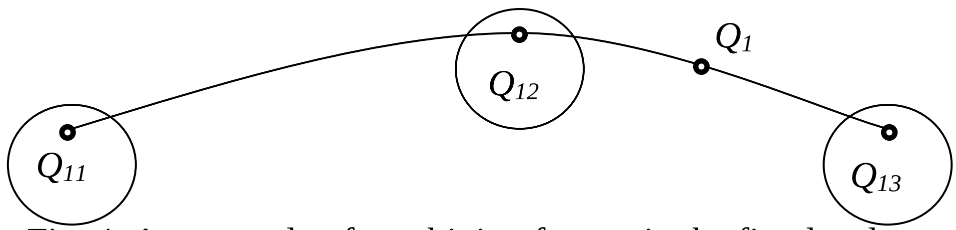

Fig. 4. An example of combining factors in the first level

However, depending on the complexity of the technological process, there may be a generalization of the factors of the following (graphic) form (Fig. 5).

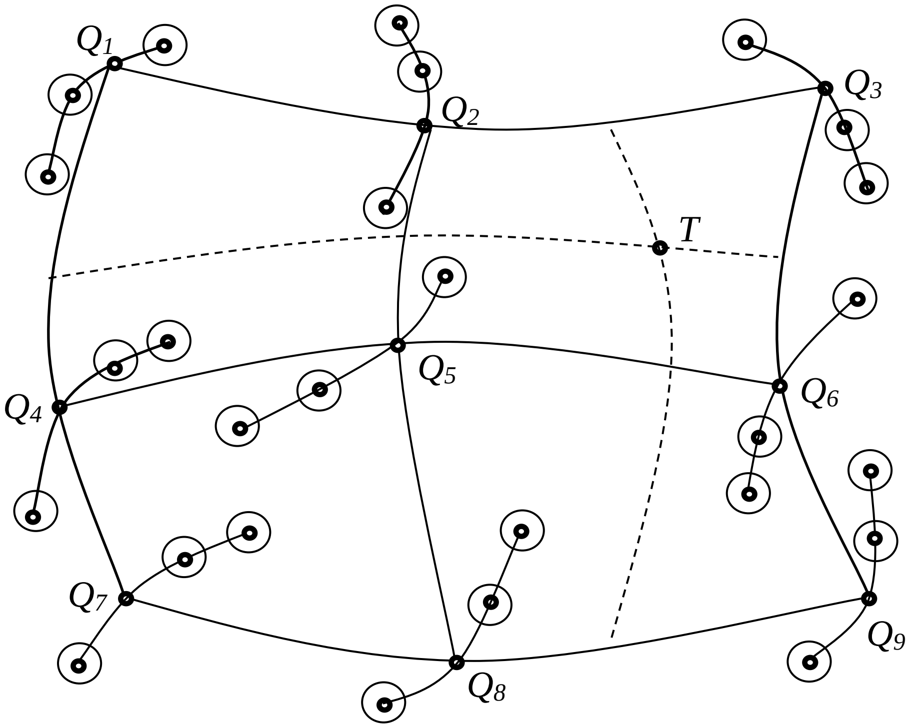

Fig. 5. The second step is the synthesis of factors

The scheme (Figure 5) can become an element for the next generalization step for the factors (Fig. 6).

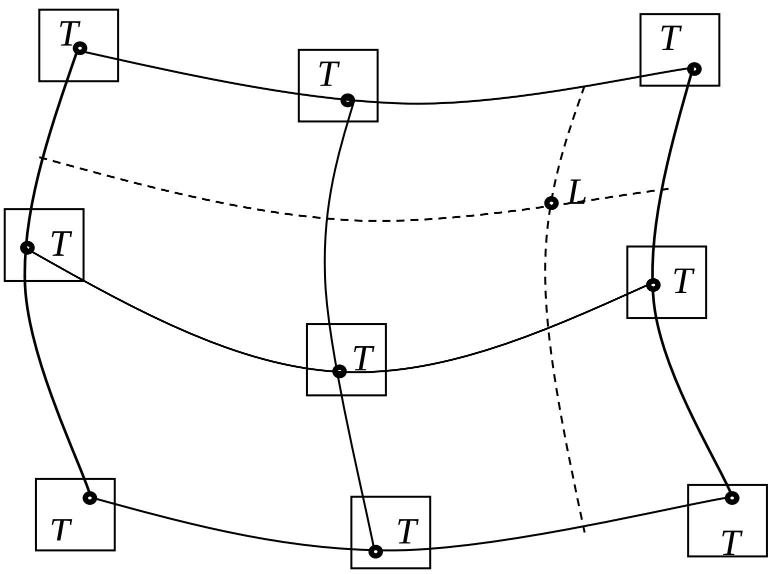

Fig. 6. The third step is a synthesis of factors

If the B-surface (Fig. 6) determines the point L, then it is possible to carry out the following - the fourth step of the generalization of factors (Fig. 7), for which the geometric scheme (Fig. 6) is an element.

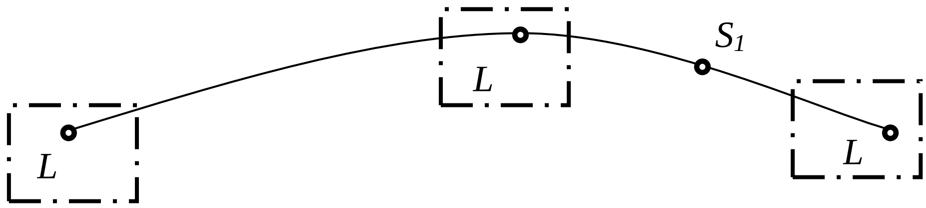

Fig. 7. Fourth step of generalization of factors

The presence of the optimal point S_i (Fig. 7), allows for the following generalization steps. And so on. The number of factors that may be involved in constructing a model is also not limited.

The above technique of generalization of factors, besides the fact that there are no restrictions on the number of homogeneous factors, it also allows for the generalization of heterogeneous factors.

It showed the technique of generalizing factors for the creation of models of processes and situations for the second order parabola, which required three valid points, and the B-surface was built on nine points, however, in [1] it is indicated that such technology operates and for generalization of factors by means of segments of B-surfaces constructed on twelve (3×4; 4×3), sixteen (4 × 4), twenty (4 × 5, 5 × 4), etc. Points

In order to study B-functions a_{ij} calculate their values for the parameters u:v = 0, 0.2, 0.4, 0.5, 0.6, 0.8, 1,0 and give the results of calculations in Table 1.

Table 1

Results of calculations of B-functions a_{ij}

	u, v						
a_{ij}	0	0,2	0,4	0,5	0,6	0,8	1
a_{11}	-2	-1,59744	-1,48608	-1,5	-1,52192	-1,33056	0
a_{12}	5	4,87424	5,40768	5,75	5,97632	5,14976	0
a_{13}	-4	-4,3008	-4,9536	-5,25	-5,3824	-4,4352	0
a_{21}	0	1,22304	2,11968	2,625	3,22752	4,70016	6
a_{22}	0	-3,73184	-7,71328	-10,0625	-12,67392	-18,19136	-22
a_{23}	0	3,2928	7,0656	9,1875	11,4144	15,6672	18
a_{31}	4	1,9344	0,8064	0,375	-0,0656	-1,2096	-3
a_{32}	-10	-5,9024	-2,9344	-1,4375	0,2576	4,6816	11
a_{33}	8	5,208	2,688	1,3125	-0,232	-4,032	9
$\sum a_{ij}$	1	1	1	1	1	1	1

Let's show graphically (Fig. 4) the results of calculations a_{ij}, shown in Table 1.

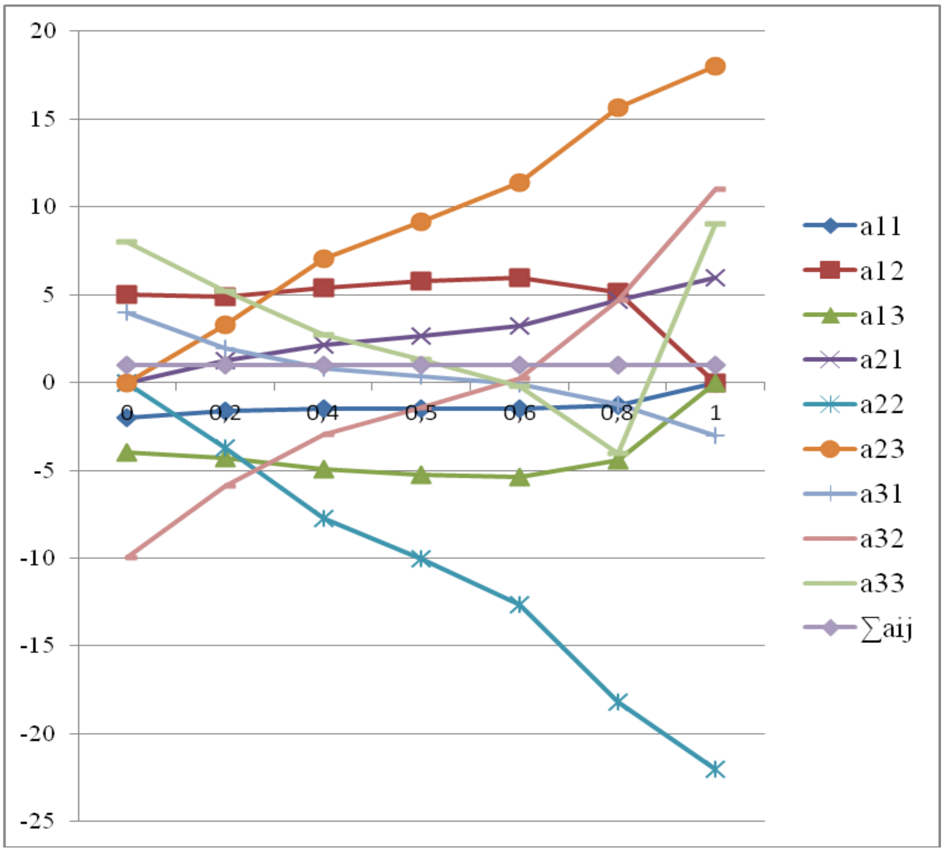

Fig. 4. Charts of B-functions a_{ij}

As you can see, a superposition $\sum_{i,j=1}^{3} a_{ij}$ for all parameter values u, v, which are taken in the table. 1, is equal to one (the last line of tab. 1).

Why at any value p_1, q_1, p_2, q_2 superposition of B-functions a_{ij} will be equal to units? Proceeding from the theory of a point BN-number [2], the point equations for all geometric figures have an area of values $0 \le t \le 1$. Therefore, if the options are p_i, q_i we can choose arbitrarily, then the third parameter r_i must be selected on condition $p_i+q_i+r_i=1$. If we take this into account we have:

$$\left\| \begin{matrix} p_1 p_2 & r_1 p_2 & q_1 p_2 \\ p_1 r_2 & r_1 r_2 & q_1 r_2 \\ p_1 q_2 & r_1 q_2 & q_1 q_2 \end{matrix} \right\| = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{33} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}. \qquad (15)$$

Let's consider separately each line of the matrix (15). Each of them is equal to the product unit for the corresponding parameter p_2, r_2, q_2:

$$(p_1+q_1+r_1)p_2=1\cdot p_2;$$
$$(p_1+q_1+r_1)r_2=1\cdot r_2;$$
$$(p_1+q_1+r_1)q_2=1\cdot q_2.$$

Taking into account that $p_2+r_2+q_2=1$, we have: $1\cdot p_2+1\cdot r_2+1\cdot q_2=p_2+r_2+q_2=1$, where is the superposition $\sum_{i,j=1}^{3} a_{ij}=1.$

Confirmation1. Superposition of function-parameters a_{ij} in any B-surface is always equal to one.

The presence of such a sign of the B-surfaces indicates that the parameters p_1, q_1 The first direction u on this surface can be chosen arbitrarily, but the choice r_1 you have to do, following a certain functional dependence.

For the other direction of the parabolic surface – v parameters p_2, q_2 Also selected arbitrarily, and the parameter r_2 are mutually determined with parameters p_2 and q_2 (that is their combination).

Consider the second property of the B-surfaces. If we determine the relation of the first row to the second matrix (15), then we get a matrix-string:

$$\left\{\frac{p_1p_2}{p_1r_2} \quad \frac{r_1p_2}{r_1r_2} \quad \frac{q_1p_2}{q_1r_2}\right\}=\{1 \quad 1 \quad 1\}\frac{p_2}{r_2}. \tag{16}$$

The result obtained from (16) indicates that the elements of the first and second lines of the matrix (15) are proportional to each other with a coefficient of proportionality $\frac{p_2}{r_2}$.

Similarly, we show the proportionality of the elements of the second and third lines of the matrix (15):

$$\left\{\frac{p_1r_2}{p_1q_2} \quad \frac{r_1r_2}{r_1q_2} \quad \frac{q_1r_2}{q_1q_2}\right\}=\{1 \quad 1 \quad 1\}\frac{r_2}{q_2}. \tag{17}$$

Also, the relation between the elements of the first and second, second and third columns of the matrix is proportional. (15):

$$\left\{\begin{matrix}\frac{p_1p_2}{r_1p_2}\\ \frac{p_1r_2}{r_1r_2}\\ \frac{p_1q_2}{r_1q_2}\end{matrix}\right\}=\left\{\begin{matrix}1\\1\\1\end{matrix}\right\}\frac{p_1}{r_1}; \qquad \left\{\begin{matrix}\frac{r_1p_2}{q_1p_2}\\ \frac{r_1r_2}{q_1r_2}\\ \frac{r_1q_2}{q_1q_2}\end{matrix}\right\}=\left\{\begin{matrix}1\\1\\1\end{matrix}\right\}\frac{r_1}{q_1}; \tag{18}$$

Proportional are also the first and third rows and columns. As is known [6], there is only a sufficient proportionality between two rows or columns of (15), then the determinant *detA* this matrix will be zero:

$$\det\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}=0.$$

Statement 2. Definition consisting of B-functions *a_{ij} any B-surface is zero.*

It should be noted that both the first and the second statement, each separately, are necessary but not sufficient for the determination of the SRV. Therefore, the assertion is formed **3**, which determines the necessary and sufficient conditions.

Statement 3. If superposition $\sum_{i,j=1}^{3} a_{ij}$ ***elements of the matrix*** $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ ***is equal to one (*** $\sum_{i,j=1}^{3} a_{ij} = 1$ ***),and a determinant detA=0, then the surface built on any source data*** x_{11}***, …, x33 is a B-surface.***

Thus, the compositional method of geometric modeling is as follows. Every point M_{ij} (12) B-surface is calculated as the sum of parts a_{ij} from the unit multiplied by the corresponding coordinate x_{ij}, that is, the composition of parts of the values of the coordinates of the output points. Since the output of the nine real points is independent, they can be chosen in such a way that the B-surface becomes a plane, a curve, a line, a straight line or a point. Existing mathematical methods do not provide such surface features that have a combination nature of formation, that is, those that are described by an equation or system of equations in which the change in the initial data can lead to a change in the equation itself.

Conversely, B-surfaces are of a composite nature, in which the independent coordinates (dots), as elements of the puzzle, make a picture based on an infinite number of tracks. B surfaces, depending on whether their parameters are discrete or continuously determined, are discrete or continuous.

Conclusions. A new type of surfaces of compositional nature - B-surface of the response is determined, their signs and rules for creating point equations describing them are formulated. The properties of their degeneration into a plane, a curve or a straight line are shown, which makes it possible to develop a method of deployment-collapse of cells [7], on the basis of which a composite method for the simulation of multifactorial processes will be developed, in which the possibility of combining heterogeneous source elements without restrictions in their number, as well as the ability to exclude unnecessary and include new factors without changing the model itself. Such opportunities will reduce the use of the model and increase its practicality.

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КОМПОЗИЦИОННЫЙ МЕТОД ГЕОМЕТРИЧЕСКОГО МОДЕЛИРОВАНИЯ: СУТЬ, ОСОБЕННОСТИ И ПЕРСПЕКТИВЫ ПРИМЕНЕНИЯ

Адоньев Е.А.

В статье исследованы особенности композиционного метода геометрического моделирования, разработанного на основе точечного БН-исчисления, показаны принципы построения параболических поверхностей отклика (Б-поверхностей), а также алгоритм обобщения исходных факторов модели. Показаны свойства Б-поверхностей, которые позволяют эффективно использовать последние для моделирования многофакторных процессов.

Ключевые слова: точечное БН-исчисление, многофакторный композиционный метод моделирования, Б-поверхность, параболическая поверхность.

КОМПОЗИЦІЙНИЙ МЕТОД ГЕОМЕТРИЧНОГО МОДЕЛЮВАННЯ: СУТЬ, ОСОБЛИВОСТІ ТА ПЕРСПЕКТИВИ ЗАСТОСУВАННЯ

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У статті досліджені особливості композиційного методу геометричного моделювання, розробленого на основі точкового БН-числення, показані принципи побудови параболічних поверхонь відгуку (Б-поверхонь), а також алгоритм узагальнення вихідних факторів моделі. Показані властивості Б-поверхонь, які дозволяють ефективно використовувати останні для моделювання багатофакторних процесів.

Ключові слова: точкове БН-числення, багатофакторний, композиційний метод моделювання, Б-поверхня, параболічна поверхня.