

GEOMETRICAL MODELLING OF THE LEFT HEART VENTRICLE ON THE BASIS OF R-FUNKTIONS

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Summary . The method of modeling of the simplified geometrical form of the left ventricle of heart, including manifestations of the post-infarction aneurysms, such as diffusion, sacculate, fungoid, is developed on the base of R-functions.

Keywords: geometric parameters of the heart, computer models of the heart, a method of R-functions postinfarction aneurysm, ventricular reconstruction forms.

Formulation of the problem. Visual observation of the structure of the heart is important in the practice of medical research and diagnosis of cardiovascular diseases. Depends on the accuracy of diagnosis and choice of treatment strategy. As a leading medical clinics of the world was a rule - to demonstrate the principle of transactions using computer technology 3D graphics and animation. These videos provide in order to find out stages of the operation, as current possibilities of 3D graphics and animations are able to show clearly and conditionally doctors and (most importantly) the patient treatment process [1,2]. Sketch. 1 (taken from the Internet) is an example of a computer model of the heart.

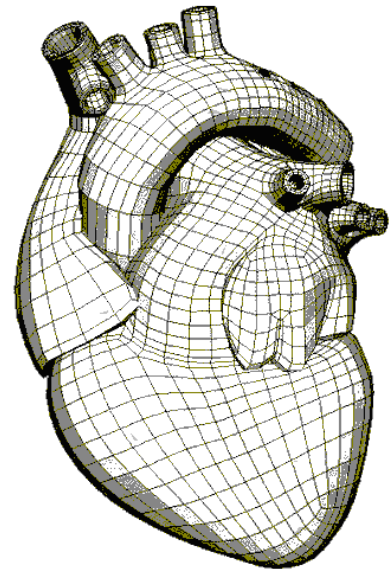


Fig. 1. A computer model of the heart.

Along with mathematical models aimed at precise (that is close to reality) description ventricular shape, often required in the simplified model, which takes into account only the features of the anatomical structure of the ventricles. In simplified models shown significant geometric characteristics, and they should be used primarily to identify and illustrate the parameters of physical fields and processes that accompany the heart.

Analysis of recent research. To simulate a three-dimensional shape of objects using different approaches. There are methods that perform simulation based on a combination of simple geometric objects whose relationship asked analytically or graphically as well as methods based on

the transformation and combinations analytically given surface [1,2]. By these methods include mathematical tools and R-functions [5].

In [3] presents a typical method for constructing three-dimensional models of the ventricles of the heart. Software package SolidWorks 2008 was created the basic model of the left ventricle (Fig. 2).

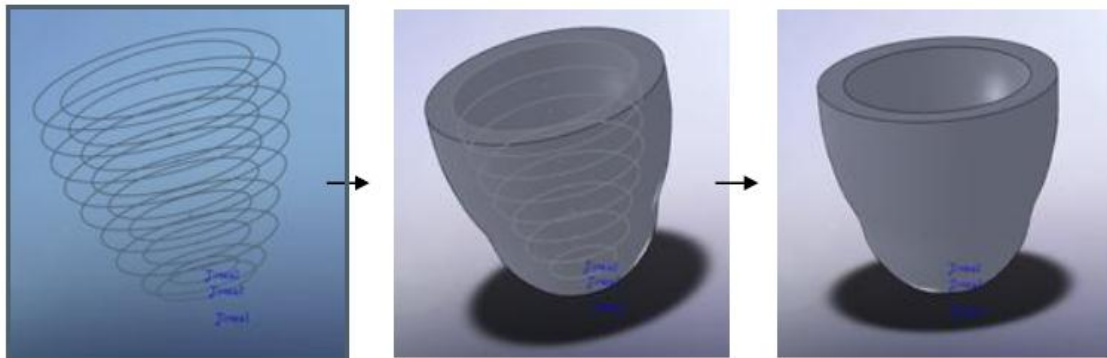


Fig. 2. Building a 3D model of the left ventricle.

Construction of models with postinfarction left ventricular aneurysm held in complex software Ansys. According to the classification [4] (Fig. 3) [3] were constructed model of the left ventricle postinfarction aneurysm perednobokovoyi wall, interventricular septum and ventricular apex (Fig. 4).

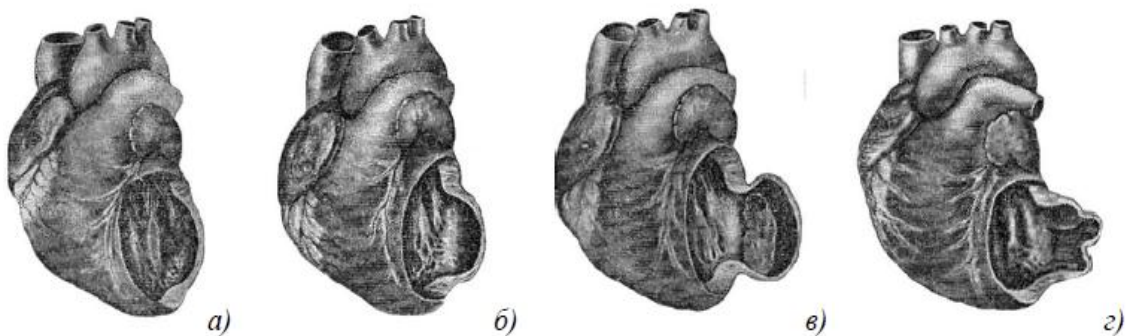


Fig. 3. Classification postinfarction left ventricular aneurysm:

- a) diffuse, b) Bursiform in) mushroom,
- d) "aneurysm in the aneurysm."

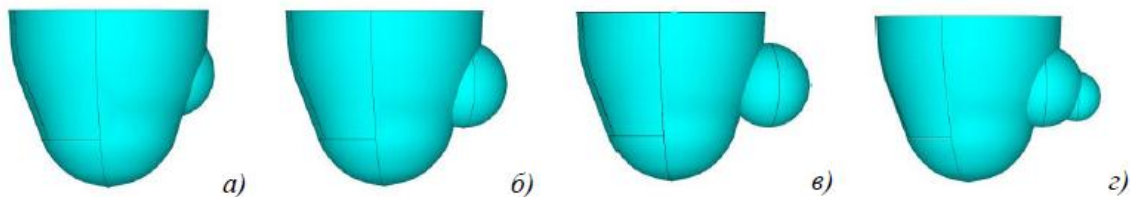


Fig. 4. Models with a postinfarction left ventricular aneurysms:

- a) diffuse, b) Bursiform in) mushroom,
- d) "in the aneurysm aneurysm".

Fig. 5 shows the 3D model of the wall of the left ventricle to calculate systolic phase of the cardiac cycle. Sketch. 6 shows a 3D model of the fluid to calculate systolic phase of the cardiac cycle.



Fig. 5. Model wall of the left ventricle to calculate diastolic phase of the cycle.

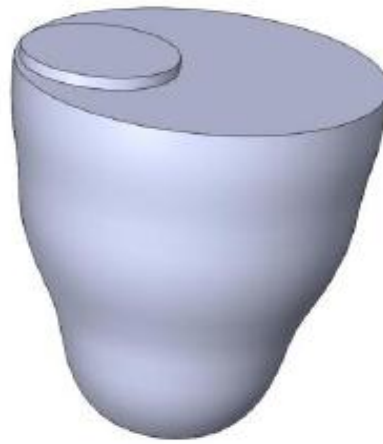


Fig. 6. Model for the calculation of the liquid surface diastolic phase of the cardiac cycle.

The wording of Article purposes. Develop using R-functions [5] simplified method of modeling the geometry of the left ventricle of the heart, including postinfarction diffuse manifestations, or the mushroom saccular aneurysms.

Main part. For algorithms based on R-R-functions necessary disjunction and conjunction R- set as a function procedures:

```
o := (a,b) -> (a+b+abs(a-b))/2;
p := (a,b) -> (a+b-abs(a-b))/2.
```

Supporting the region described by the proposed functions:

```
f1 := z - w^2/2 - sin(w)^10/2:
f2 := 0.7^2 - (x+A)^2 - y^2 - (z-1.5)^2:
```

where $w = \sqrt{x^2 + y^2}$, - shift the "center" of the aneurysm. Description simplified surface (similar to Fig. 4) of the left ventricle at $A = 2$ was carried out using the formula

```
F := p(o(f1, f2), 4-z),
```

or, in expanded form (here saved syntax maple):

$$\begin{aligned}
F := & -\frac{z}{4} - \frac{w^2}{8} - \frac{1}{8} \sin(w)^{10} + \frac{849}{400} - \frac{(x+2.)^2}{4} - \frac{y^2}{4} - \frac{\left(z - \frac{3}{2}\right)^2}{4} \\
& + \frac{1}{4} \left| z - \frac{w^2}{2} - \frac{1}{2} \sin(w)^{10} - \frac{49}{100} + (x+2.)^2 + y^2 + \left(z - \frac{3}{2}\right)^2 \right| - \frac{1}{2} \left| -\frac{3z}{2} + \frac{w^2}{4} \right. \\
& \left. + \frac{1}{4} \sin(w)^{10} + \frac{751}{200} + \frac{(x+2.)^2}{2} + \frac{y^2}{2} + \frac{\left(z - \frac{3}{2}\right)^2}{2} \right. \\
& \left. - \frac{1}{2} \left| z - \frac{w^2}{2} - \frac{1}{2} \sin(w)^{10} - \frac{49}{100} + (x+2.)^2 + y^2 + \left(z - \frac{3}{2}\right)^2 \right| \right|
\end{aligned}$$

Fig. 7 - 9 shows an image model of left ventricular (for clarity of the cut), prepared by maple-operator:

```

implicitplot3d( F, x=-3..3, y=-3..3, z=0..4,
view=0..4, style= patchcontour, color=yellow,
axes=NONE, grid=[70,70,70], lightmodel='light2');

```

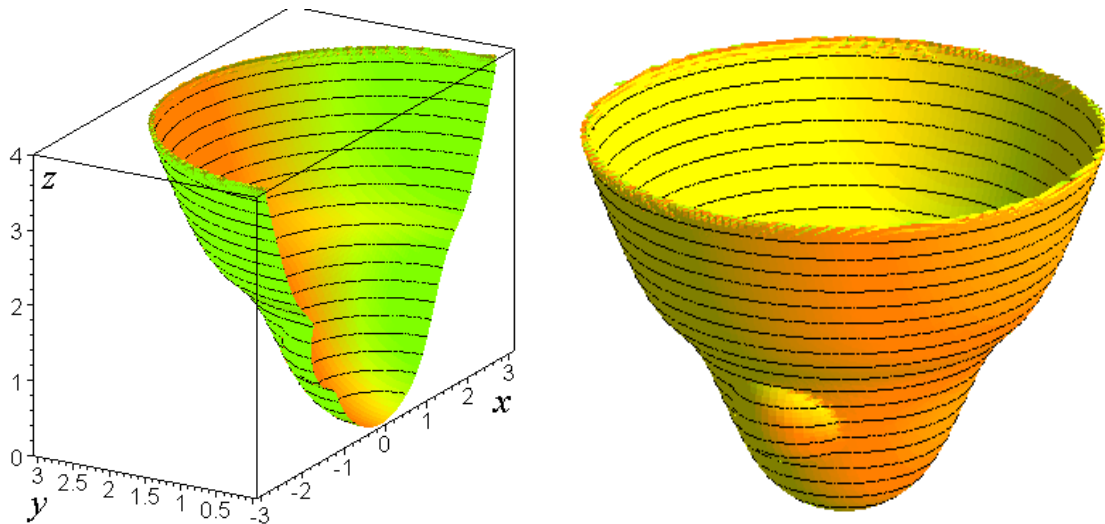


Fig. 7. The resulting 3D model of the left ventricle with diffuse postinfarction aneurysm.

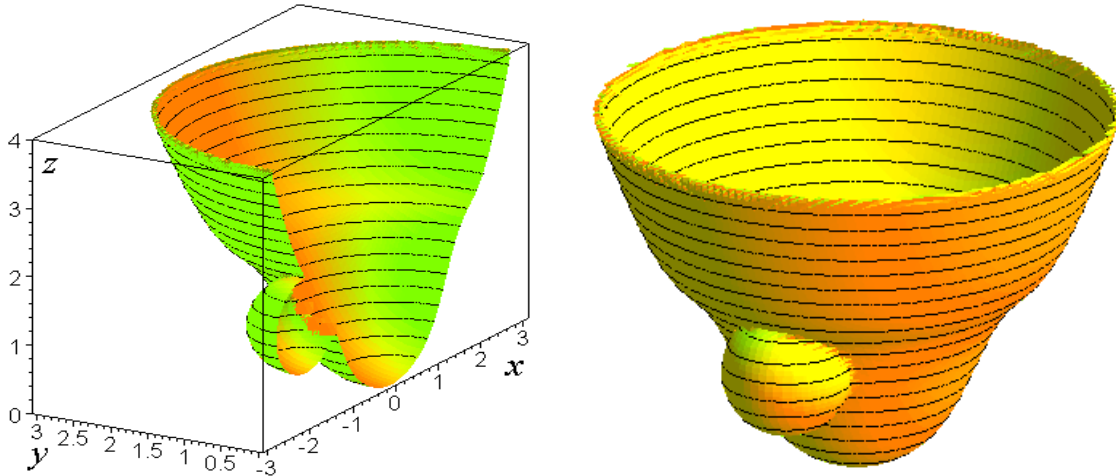


Fig. 8. The resulting 3D model of the left ventricle Bursiform with postinfarction aneurysm.

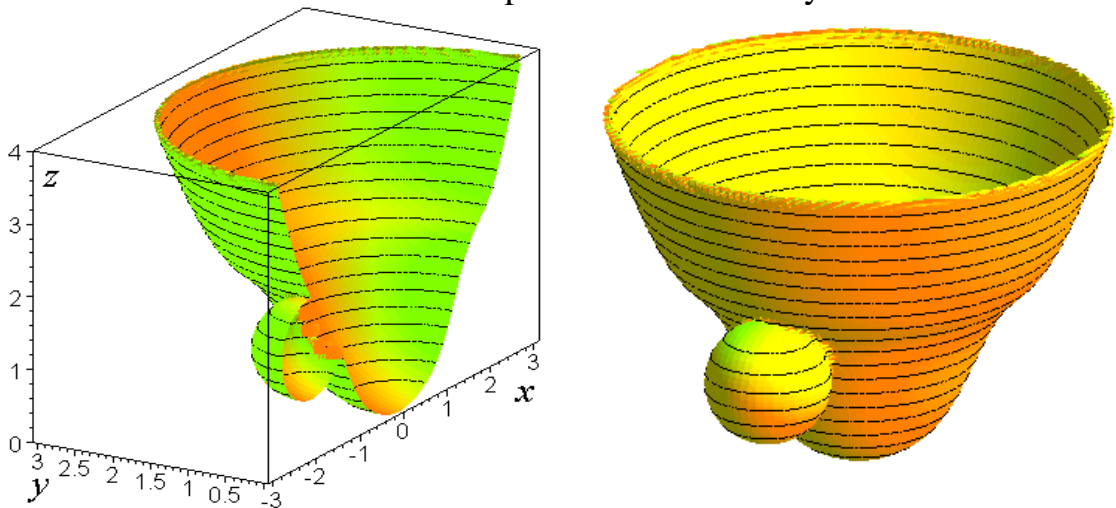


Fig. 9. The resulting 3D model of the left ventricle mushroom with postinfarction aneurysm.

Similarly, we can simulate the wall of the left ventricle to calculate diastolic phase of the cycle, as well as simulate the fluid surface to calculate the systolic phase of the cardiac cycle. As an example, consider the first of these problems when supporting area described by functions:

$$\begin{aligned} f1 &:= z - w^2/2 - \sin(w)^{10}/2; \\ f2 &:= R^2 - (x+2)^2 - y^2, \end{aligned}$$

where $w = \sqrt{x^2 + y^2}$, R - radius of the hole.

Description simplified surface (similar to Fig. 5) wall of the left ventricle when $R = 0,7$ was made by the formula

$$F := \circ(p(f1, 4-z), p(f2, z-3.5)),$$

or, in expanded form

$$\begin{aligned}
 F := & -\frac{w^2}{8} - \frac{1}{8} \sin(w)^{10} + \frac{99}{400} - \frac{1}{4} \left| -2z + \frac{w^2}{2} + \frac{1}{2} \sin(w)^{10} + 4 \right| - \frac{(x+2)^2}{4} - \frac{y^2}{4} + \frac{z}{4} \\
 & - \frac{1}{4} \left| -\frac{399}{100} + (x+2)^2 + y^2 + z \right| + \frac{1}{2} \left| \frac{w^2}{4} + \frac{1}{4} \sin(w)^{10} - \frac{701}{200} \right| \\
 & + \frac{1}{2} \left| -2z + \frac{w^2}{2} + \frac{1}{2} \sin(w)^{10} + 4 \right| - \frac{(x+2)^2}{2} - \frac{y^2}{2} + \frac{z}{2} - \frac{1}{2} \left| -\frac{399}{100} + (x+2)^2 + y^2 + z \right|
 \end{aligned}$$

Fig. 10 shows a model of the wall of the left ventricle, obtained with the above maple-operator `implicitplot3d`.

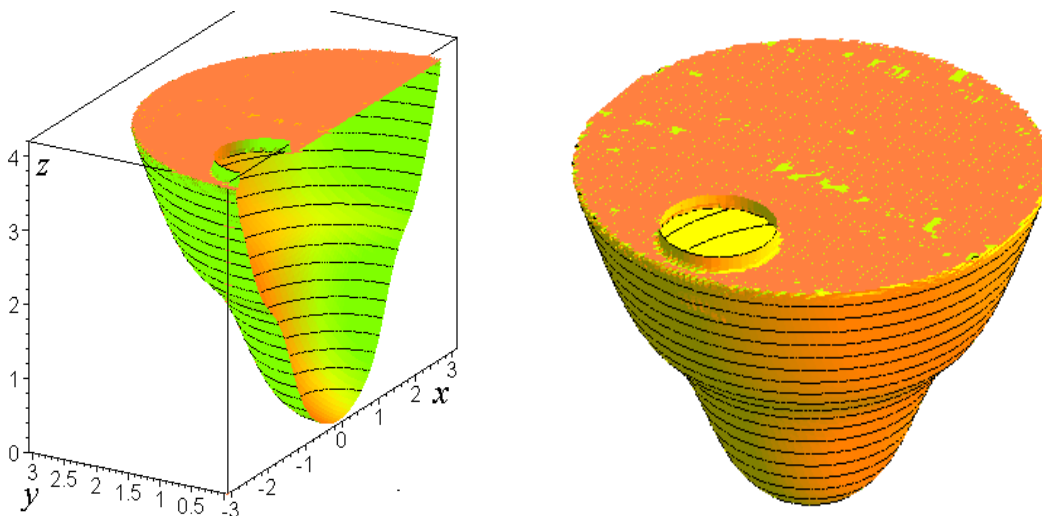


Fig. 10. The resultant left ventricular wall model to calculate the diastolic phase of the cycle.

Conclusions. Reproduced in the simplified method to simulate the shape of the left ventricle as a 3D object, described in Cartesian coordinates system appearance Oxyz equation $F(x, y, z) = 0$. This equation can be solved at the most formal level metric and positional 3D modeling problems. For example, a test to determine the position of the object relative terms, apply visual methods phase excitation wave motion on the surface of the heart, 3d object editing forms by converting expression $F(x, y, z) = 0$, and so on.

Literature

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