# THE DESCRIPTION AND CONSTRUCTION OF THE CONJUGATE CENTROIDS NONROUND GEARS

Ia. Legeta

***Summary.* Based on an analytical description of the means mathematical Maple package consider the geometric modeling of the centroids gears unconventional shapes.**

***Keywords:* noncircular gears, centroids, transfer function, parametric equations.**

*Formulation of the problem.* To transfer motion with variable velocity ratio used in engineering noncircular gears. In addition, the instrument noncircular gears are used for reproduction of non-linear functions, hydraulic engineering for designing pumps, etc. [1]. In the mechanism of noncircular gears relationship between the angles of rotation of driving and driven wheels is nonlinear. The interesting thing is that with a simple mechanism noncircular wheels can be played monotonically increasing function whose derivative in the interval playback is smooth function with limited positive values [2].

After a comparative analysis of cam mechanism and the mechanism of noncircular gears used for playback functions, you can reach the conclusion that the conditions of serial production of wheels and cams fidelity features noncircular wheels is higher than the cam mechanism, and played noncircular wheels monotonic function more complex. So urgent is the problem of analytical description centroid noncircular gears.

*Review of recent research.* In [1-3], the basic formula for calculating the centroid noncircular wheels – i.e. curves that touch each other and rolled without slipping when meshed gears. In addition, the centroid of driving wheels - a set of instantaneous centers of rotation in a coordinate system rigidly associated with this wheel.

Non-circular wheel usually asked:

- angle of rotation of the driving gear $\varphi_1 = \varphi_1(t)$;
- angle of rotation of the driven gear $\varphi_2 = \varphi_2(t)$;
- intercentral distance $a = const$ .

We assume that the rotation of the driving wheel is made constant (unit) speed. Let the angular velocity of the wheels are determined by the formulas:

$$\frac{d\varphi_1}{dt} = \omega_1 = 1; \quad \frac{d\varphi_2}{dt} = \omega_2 = \eta(t) . \tag{1}$$

The noncircular gear designed to convert rotary movement between machine parts for a given transfer function $\eta(t)$, where $t$ – time. The transfer function is defined as the ratio of derivatives:

$$\eta(t) = \frac{d\varphi_2}{dt} / \frac{d\varphi_1}{dt}. \tag{2}$$

With the transfer function of expression $\eta = \frac{\omega_2}{\omega_1} = \frac{R_1}{R_2}$ calculated instantaneous transmission radius:

$$R_1(t) = \frac{a\,\eta(t)}{1+\eta(t)} \text{ i } R_2(t) = \frac{a}{1+\eta(t)}.$$

Total variable radii $R_1$ and $R_2$ centroid at the point of contact is equal

$$R_1 + R_2 = a, \tag{3}$$

where $a$ – constant distance between the axles of the wheels.

Coordinates instantaneous centroid point of contact should be calculated by formulas:

$$r_1 = [R\cos\Phi_1, R\sin\Phi_1];\ r_2 = [R\cos\Phi_2, R\sin\Phi_2]. \tag{4}$$

where $\Phi_1$, $\Phi_2$ – transfer functions.

In [4,5] considered noncircular gears predominantly elliptical and "petal" shape. But deployments interest gears unconventional form.

In [6] the approximate description and method of construction of a closed curve, given the natural equation, subject to presentation of Fourier series integrand in its parametric equation.

*Formulation of Article purposes.* Using the formula elaborate geometric modeling centroid algorithm gears for alternative forms of literature based analytical description of tools mathematical package Maple.

*Main part.* Centroid algorithm for constructing noncircular gears give language syntax using Maple.

Depending on the transfer function $\Phi(t)$ variables determine the centroid radius at the point of contact:

$$R_1 := \frac{a\left(\frac{d}{dt}\Phi(t)\right)}{1+\left(\frac{d}{dt}\Phi(t)\right)}; \qquad R_2 := \frac{a}{1+\left(\frac{d}{dt}\Phi(t)\right)}. \tag{5}$$

Coordinates instantaneous centroid point of contact:

$$r_1 := \left[\frac{a\left(\frac{d}{dt}\Phi(t)\right)\cos(t)}{1+\left(\frac{d}{dt}\Phi(t)\right)}, \frac{a\left(\frac{d}{dt}\Phi(t)\right)\sin(t)}{1+\left(\frac{d}{dt}\Phi(t)\right)}\right]; \tag{6}$$

$$r_2 := \left[ \frac{a\cos(\Phi)}{1+\left(\frac{d}{dt}\Phi(t)\right)}, \frac{a\sin(\Phi)}{1+\left(\frac{d}{dt}\Phi(t)\right)} \right].$$

Parametric equations driving wheel determined by the formula:

$$x1 := \frac{a\left(\frac{d}{dt}\Phi(t)\right)\cos(t)}{1.+\left(\frac{d}{dt}\Phi(t)\right)}; \quad y1 := \frac{a\left(\frac{d}{dt}\Phi(t)\right)\sin(t)}{1.+\left(\frac{d}{dt}\Phi(t)\right)}. \tag{7}$$

Then parametric equations driven wheels will look like:

$$x2 := a - \frac{1.\,a\cos(\Phi)}{1.+\left(\frac{d}{dt}\Phi(t)\right)}; \quad y2 := \frac{a\sin(\Phi)}{1.+\left(\frac{d}{dt}\Phi(t)\right)}. \tag{8}$$

Build couple image centroid noncircular wheels for their parametric equations (7) and (8) using operators:

```
pic1 := plot([x1, y1, t=0..2*Pi], color = blue):
pic2 := plot([x2, y2, t=0..2*Pi], color = red):
```

Build phase rotation compatible pair of wheels, which corresponds to the parameter t, by using a sequence of operators:

```
theta := t:
PIC1 := rotate(pic1, -theta, [0, 0]):
PIC2 := rotate(pic2, eval(Phi), [a, 0]):
display([PIC1, PIC2], c1, c2);
```

Examples of images constructed through a complex program of animation frames centroid pair of gears alternative forms depending on the form of the transfer function *Φ(t)*. Fig. 1 shows a graph of the transfer function given by the formula

$$\Phi := \frac{t}{2} + \frac{1}{7}\sin(t) + \frac{1}{9}\sin(2\,t) + \frac{2}{31}\sin(3\,t). \tag{9}$$

Fig. 2 shows a graph of the first derivative $\Phi'$ transfer function $\Phi$, which is described by the equation

$$\eta := \frac{1}{2} + \frac{1}{7}\cos(t) + \frac{2}{9}\cos(2\,t) + \frac{6}{31}\cos(3\,t). \tag{10}$$

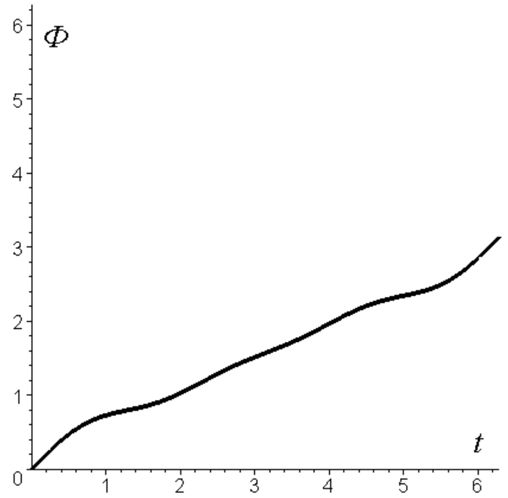



Fig. 1. Schedule transfer function $\Phi(t)$.

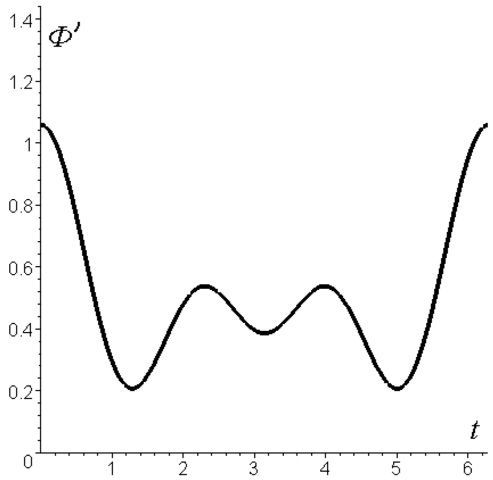



Fig. 2. Schedule the first derivative of the transfer function $\Phi(t)$.

The individual animation frames compatible pairs centroid rotation gears for alternative forms of a given transfer function $\Phi(t)$ shown in Fig.3.

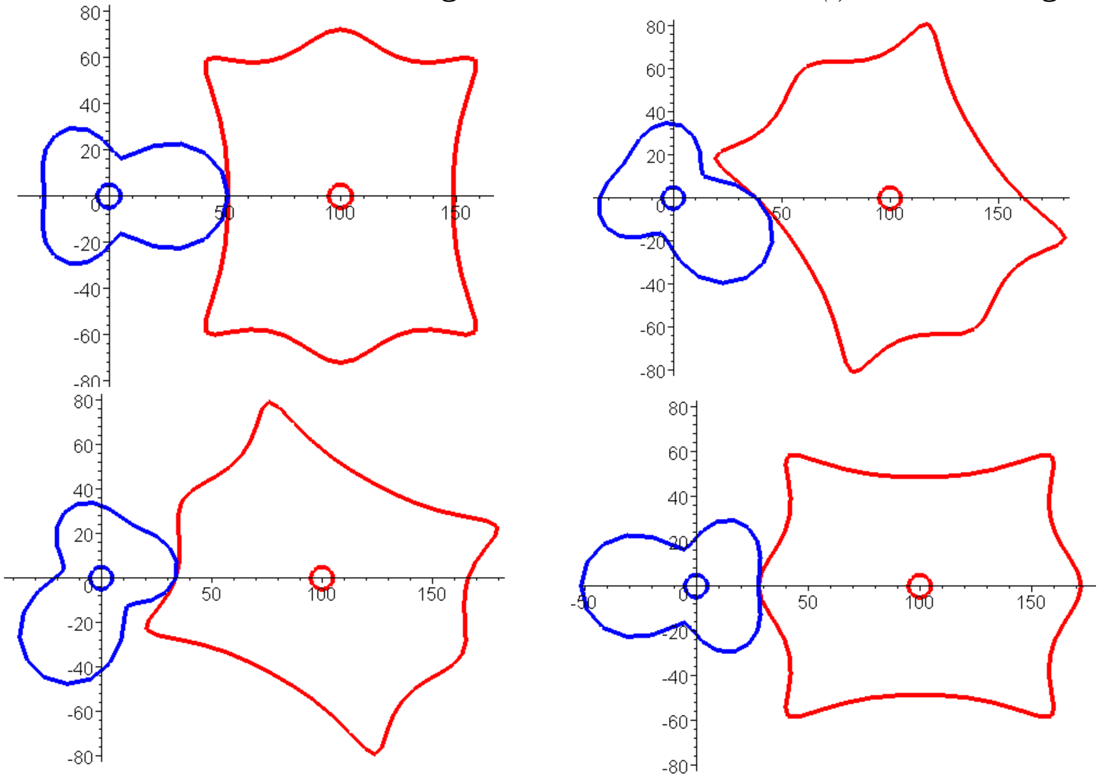



Fig. 3. Personnel joint rotation animation couples centroid for the transfer function (9).

Fig. 4, the animation frames for a couple centroid constructed for the transfer function

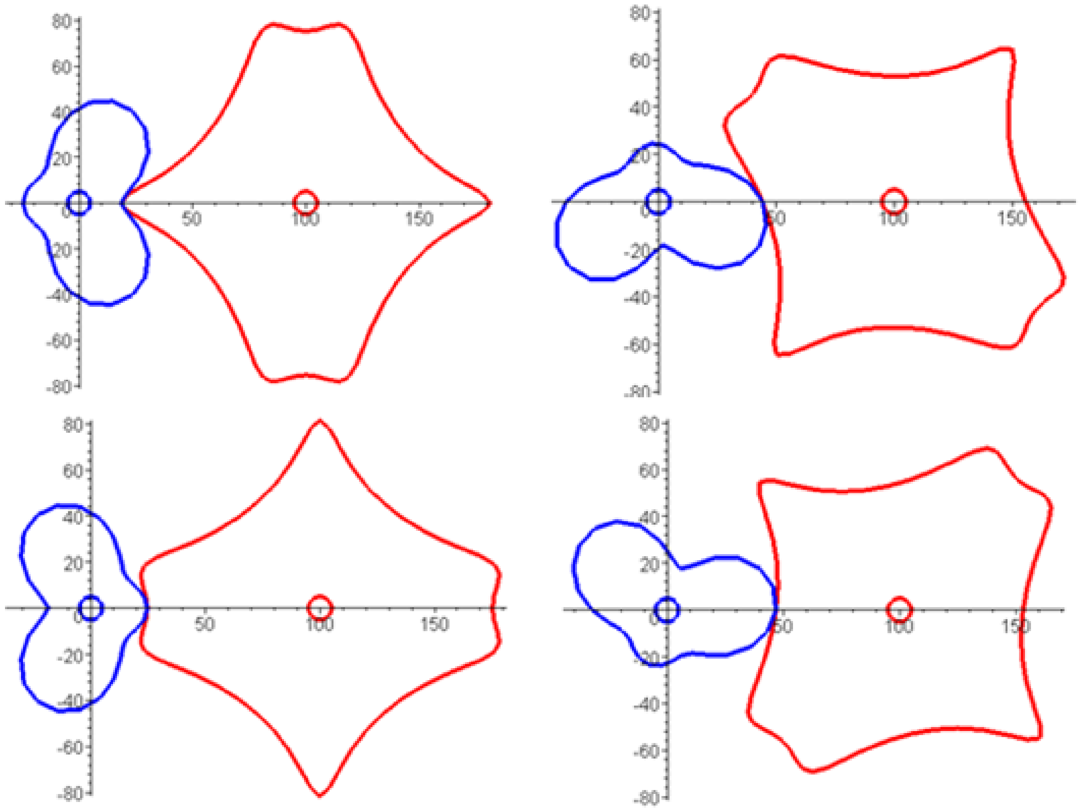
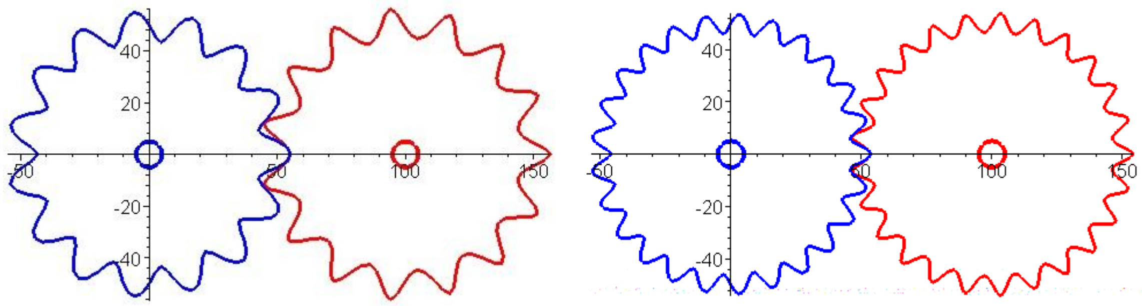



Fig. 4. Personnel joint rotation animation
couples centroid for the transfer function (11).

Figure 5 animation frames are compatible centroid rotation to transfer functions $\Phi := 1 + t + \frac{1}{65}\sin(15\,t)$ i $\Phi := 1 + t + \frac{1}{165}\sin(25\,t)$ .




*a* *б*

Fig. 5. centroid for transfer functions:
*a*) $\Phi := 1 + t + \frac{1}{65}\sin(15\,t)$ ; *б*) $\Phi := 1 + t + \frac{1}{165}\sin(25\,t)$ .

*Conclusions*. Analytical description centroid gears lets you analyze and adjust their unconventional geometric shapes.

Literature


1. *Артоболевский И.И.* Теория механизмов и машин. / И.И. Артоболевский. – М.: Наука, 1975. – 639 с.
2. *Литвин Ф.Л.* Некруглые зубчатые колеса. / Ф.Л. Литвин. – М.: Машгиз, 1956. – 312 с.
3. *Laczik B.* Measuring of Gears with General (Non–Circular) Pitch Curve / B. Laczik, Z. Szaniszló. – Helsinki, 2001. – P. 373–376.
4. *Утутов Н.Л.* Некруглые зубчатые колеса и основы синтеза уравнительных передач для цепных механизмов: дис. … д-ра техн. наук: 05.02.02 / Н.Л. Утутов, Восточноукраинский гос. ун–т. – Луганск, 1999. – 456 с.
5. *Киреев С.О.* Аналитическое определение профилей зубьев эллиптического колеса. / С.О. Киреев, Н.А. Падалко, А.П. Падалко // Техн. Науки: изв. вузов Сев.–Кавк. регион. – 2000. – №3.– С. 31–34.
6. *Легета Я.П.* Опис та побудова замкнутої кривої за її натуральним рівнянням / Я.П. Легета // Прикладна геометрія та інженерна графіка – К.: КНУБА, 2011. – Вип.88. – С. 219-228.