THE ANALYTICAL DESCRIPTION OF A TUBULAR SURFACE, REFERRED TO LINES OF CURVATURE, IN SYSTEM OF THE ACCOMPANYING THREE-EDGE OF A SLOPE CURVE

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***Summary.* The analytical description constructing of a tubular surface, referred to the lines of curvature, is carried out in the system of accompanying of a slope curve, which is set the curvature in the function of length of arc and size of corner of getting up.**

***Keywords:* cover Frenet-Serret formulas, the line slope coefficients of quadratic forms, lines of curvature.**

Formulation of the problem. Analytical description of surfaces, related to the lines of curvature, geometric applied is an important task. This is due to ease of use of the parameterization in the study of stress-strain state membranes and in determining the optimal trajectories processing tools products curvilinear forms. An obligatory condition to avoid superposition of numerical methods in further research is to find parametric equations surfaces without the use of approximate methods of mathematics.

Analysis of recent research. Among the methods of structural formation of coral surfaces assigned to the lines of curvature, the method of constructing the system cover surfaces tryhrannyka guide curve [1,3]. The task of the analytical conditions of formation and assignment of channels to the lines of curvature surfaces leads to differential equations that are generally not integrated in quadratures [1,3]. Therefore analytical description Surface channels assigned to the lines of curvature, considering the share of their education.

Formulation of Article purposes. Find analytical conditions for the formation of the tubular surface (such as a particular case surface channels), referred to the lines of curvature, forming the framework of cyclical sustainable community using radius, moving the specified law to cover normal plane trihedron line slope.

*Main part***.** Let the curve using braid f asked dependence on the length of the arc $k=k(s)$ and elevation angle $\beta = const$. Then her parametric equations have the form [2]:

$$x(s)=\cos\beta\cdot\int\cos\left(\frac{1}{\cos\beta}\int kds\right)ds;$$
$$y(s)=\cos\beta\cdot\int\sin\left(\frac{1}{\cos\beta}\int kds\right)ds; \qquad z(s)=s\cdot\sin\beta. \tag{1}$$

The vector equation of the slope of the curve as a function of the length of its arc has the form:

$$\bar{r}=\bar{r}(s)=x(s)\cdot\bar{i}+y(s)\cdot\bar{j}+z(s)\cdot\bar{k}, \tag{2}$$

where $\bar{i}$; $\bar{j}$; $\bar{k}$ – orts fixed coordinate system $Oxyz$ (рис.1).

When moving Frenet-Serret formulas from the top A on directing curve (2) of the circle centered at A, which lies in the plane γ, forms a cyclic skeleton surface. Let the plane γ passing through the unit vectors normal $\bar{b}$ trihedron accompanying guide curve f and forms a trihedron with the normal plane angle $\alpha=\alpha(s)$.

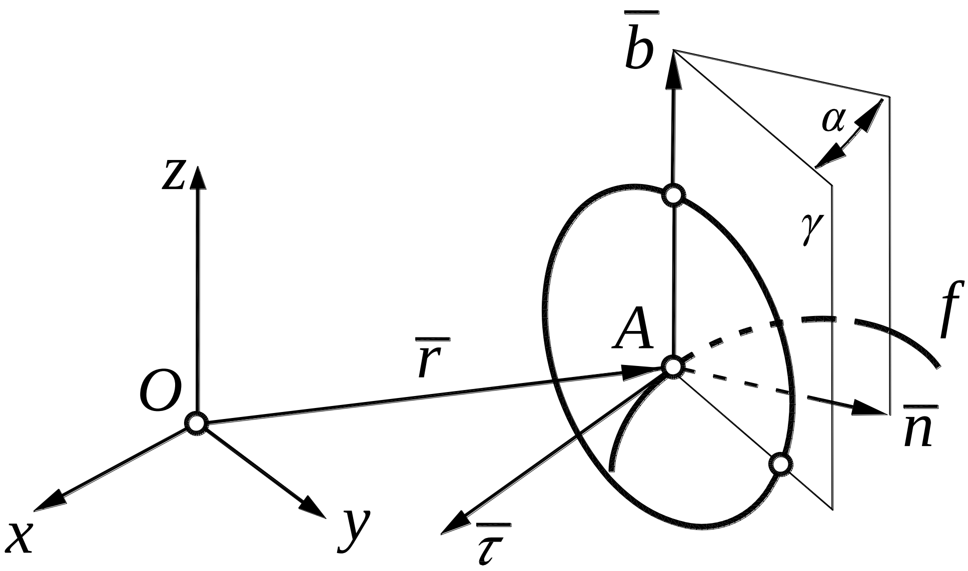


Fig.1. Generating circle that belongs plane rotating around ort $\bar{b}$ an angle $\alpha=\alpha(s)$.

In [1] found vector equation cyclic surface formed traffic circle located in a plane γ, which revolves around the ort $\bar{b}$ an angle $\alpha=\alpha(s)$:

$$\bar{R}(v,s)=\bar{r}(s)+\bar{\tau}\cdot\rho\cdot\sin\alpha\cdot\cos v+\bar{n}\cdot\rho\cdot\cos\alpha\cdot\cos v+\bar{b}\cdot\rho\cdot\sin v, \tag{3}$$

where $\rho=\rho(s)$ – the radius of the generating circle, which lies in the γ, s – arc length guide curve f, v – parameter, $0\le v<2\pi$.

Finding coefficients E, F first and L, M second quadratic form of the surface (3), and using sufficient condition referring to the cycle frame surface curvature lines [4] $L \cdot F - M \cdot E = 0$, in [1] defined analytic surface condition of the formation of channels:

$$\sigma \cdot \rho \cdot \sin^2 \alpha + \sin\alpha \cdot \cos\alpha \cdot \sin v - \rho \cdot \rho_S' (\alpha_S' - k) \cdot \sin v + \\ + \sigma \cdot \rho \cdot \rho_S' \sin\alpha \cdot \cos v = 0, \quad (4)$$

where $\sigma -$ roll guide curve f, $k -$ curvature of the guide curve f.

Roll slope line (2) to determine the expression [4]:

$$\sigma = k \cdot \mathrm{tg}\,\beta. \quad (5)$$

Analysis of condition (4) the formation of surface canals for spatial guide curve causes significant difficulties. In particular, spatial guide curve f $(\sigma \neq 0)$ Equation (4) is converted to the correct equality while the conditions: $\alpha = 0$ i This confirms the theory of surfaces known statement: cyclic circles frame tube surface forms a family of lines of curvature, and generating circle tube surface is normal to the plane of the center line.

When the conditions $\alpha = 0$ and $\rho = const$ only one family of coordinate lines (cyclic frame) are lines of curvature of the tube surface. To find a family lines, circles orthogonal to the set cycle frame, it is necessary to solve the differential equation of the orthogonal trajectories (the family formed at different values $s = const$) [4]:

$$F \cdot ds + E \cdot dv = 0\,. \quad (6)$$

Find the coefficients of the first quadratic form tubular surface, differentiating vector equation (3) at $\alpha = 0$ and $\rho = const$ and using the formula Freinet [4]. The coefficients of the first quadratic form tubular surfaces have a look: $E = \rho^2$, $F = \sigma \cdot \rho^2$. Putting them in the equation (6), we obtain the differential equation $\sigma(s) \cdot \rho^2 \cdot ds + \rho^2 \cdot dv = 0$, general solution which is an expression:

$$v = -\int \sigma(s) ds + u. \quad (7)$$

In expression (7): $\sigma(s) -$ roll guide curve slope f, $u -$ integration was to be a new parameter (instead v) vector equation tubular surface, referred to the lines of curvature, which in $\alpha = 0$ and $\rho = const$ looks like:

$$\overline{R}(u,\ s) = \overline{r}(s) + \overline{n} \cdot \rho \cdot \cos\left(-\int \sigma(s) ds + u\right) + \overline{b} \cdot \rho \cdot \sin\left(-\int \sigma(s) ds + u\right). \quad (8)$$

Substituting the parametric equation (1), expressions of directing cosines found in [2], and taking into account the slope of the line for equality (5) we obtain the parametric equations surface of the tubular line centers (1), which referred to the coordinate lines of curvature:

$$
\begin{aligned}
X(u,s) &= \cos\beta\int\cos\psi\, ds - \\
&\quad -\rho\left[\sin\psi\cos(u-\psi\sin\beta)+\sin\beta\cos\psi\sin(u-\psi\sin\beta)\right]; \\
Y(u,s) &= \cos\beta\int\sin\psi\, ds + \\
&\quad +\rho\left[\cos\psi\cos(u-\psi\sin\beta)-\sin\beta\sin\psi\sin(u-\psi\sin\beta)\right]; \\
Z(u,s) &= s\sin\beta+\rho\cos\beta\,\sin(u-\psi\sin\beta),
\end{aligned}
\tag{9}
$$

where $\psi(s)=\dfrac{1}{\cos\beta}\int k(s)ds,$ $\beta = const$ – angle of climb, $k = k(s)$ – dependence on the curvature of the arc length line in her hair (1).

Example. Let the center line of the tube surface is the slope of the curve, asked via addictions $k(s)=\dfrac{a}{s}$ (a – parameter curve) and elevation angle $\beta = const$. Then her parametric equation (1) is written as:

$$
\begin{aligned}
x(s) &= \frac{s\cos^2\beta}{a^2+\cos^2\beta}\cdot\left[\cos\beta\cos\left(\frac{a\ln s}{\cos\beta}\right)+a\sin\left(\frac{a\ln s}{\cos\beta}\right)\right]; \\
y(s) &= \frac{s\cos^2\beta}{a^2+\cos^2\beta}\cdot\left[\cos\beta\sin\left(\frac{a\ln s}{\cos\beta}\right)-a\cos\left(\frac{a\ln s}{\cos\beta}\right)\right]; \\
z(s) &= s\sin\beta.
\end{aligned}
\tag{10}
$$

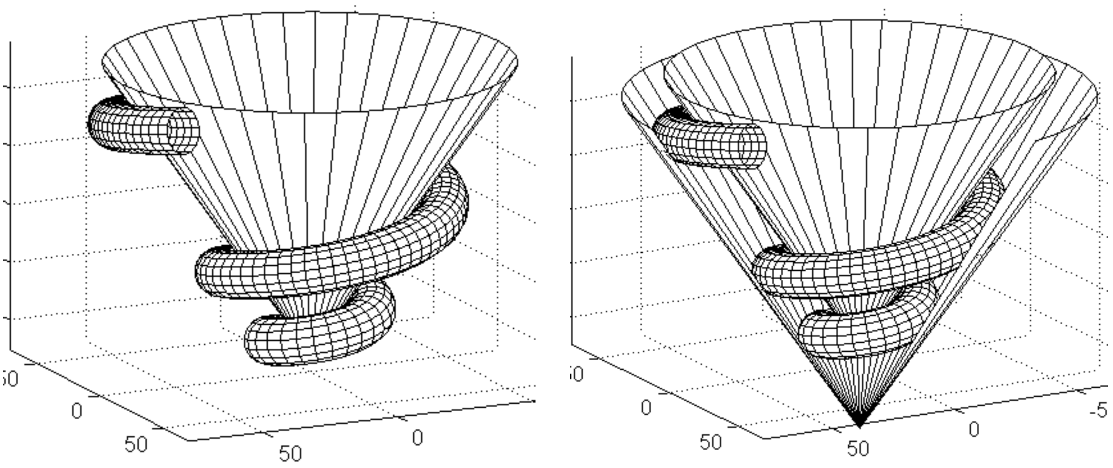
Substituting dependence $k(s)=\dfrac{a}{s}$ in a parametric equation (9), we obtain the parametric equations tubular surface of the center line (10), which used to coordinate lines of curvature:

$$
\begin{aligned}
X(u,s) &= x(s)-\rho\cdot\left[\begin{aligned}&\sin\left(\frac{a\ln s}{\cos\beta}\right)\cos(u-a\,\mathrm{tg}\,\beta\cdot\ln s)+ \\ &\quad +\sin\beta\cos\left(\frac{a\ln s}{\cos\beta}\right)\sin(u-a\,\mathrm{tg}\,\beta\cdot\ln s)\end{aligned}\right]; \\
Y(u,s) &= y(s)+\rho\cdot\left[\begin{aligned}&\cos\left(\frac{a\ln s}{\cos\beta}\right)\cos(u-a\,\mathrm{tg}\,\beta\cdot\ln s)- \\ &\quad -\sin\beta\sin\left(\frac{a\ln s}{\cos\beta}\right)\sin(u-a\,\mathrm{tg}\,\beta\cdot\ln s)\end{aligned}\right]; \\
Z(u,s) &= z(s)+\rho\cdot\cos\beta\cdot\sin(u-a\,\mathrm{tg}\,\beta\cdot\ln s).
\end{aligned}
\tag{11}
$$

Expressions $x(s); y(s); z(s)$ in parametric equations (11) are determined from equations (10).

Guide line (10) is the line slope, which lies on the cone. To find the equation of the surface of revolution (cone) find a pattern of change of distance $\mu(s)$ the axis of the cone to a point on the line in the conical braid:

$$\mu(s) = \sqrt{x^2 + y^2} = \frac{s\cos^2\beta}{\sqrt{a^2 + \cos^2\beta}}. \quad (12)$$



a) b)

Fig. 2. tubular surface (11), assigned to coordinate lines of curvature, whose center line is the curve slope (10)

angle given rise $\beta = \frac{\pi}{18}$ and natural equation $k(s) = \frac{8}{s}$:

a) tangent to the inner surface of the cone;
b) surface tangent to two cones: internal and external.

Independence (12) and $z(s) = s \cdot \sin\beta$ forming a parametric equation meridian surface rotation. Eliminating data of equations *s*, we obtain a linear dependence, which sets meridian cone (direct):

$$z(\mu) = \frac{\sin\beta}{\cos^2\beta} \cdot \mu. \quad (13)$$

Thus, the meridian is a direct surface rotation (13), the cone generatrix which is inclined to the horizontal plane Oxy at an angle equal to: $\mathrm{arctg}\left(\frac{\sin\beta}{\cos^2\beta}\right)$.

Fig. 2 shows a tubular surface, referred to the curvature of lines, tangent to cones. This surface is based on equations (11) with the center line (10) at $k(s) = \frac{8}{s};\ \beta = \frac{\pi}{18}$.

Conclusions. Construction of the surface of the tubular system cover trihedron line slope made it possible to find out analytical terms referring to the lines of curvature of the surface. Found parameterization allows you to design technical form of simplified analytical description of the tube surface.

Literature

1. *Пилипака С.Ф.* Конструювання циклічних поверхонь сім'ями ортогональних ліній в системі супровідного тригранника напрямної кривої/ С.Ф. Пилипака // Прикладна геометрія та інженерна графіка.– К.: КДТУБА, 1998.– № 63. – С. 159–161.
2. *Пилипака С.Ф.* Конструювання лінійчатих поверхонь загального виду в системі супровідного тригранника напрямної просторової кривої / С.Ф. Пилипака, М.М. Муквич // Праці Таврійської державної агротехнологічної академії. – Мелітополь: ТДАТА, 2007. – Вип 4. Прикладна геометрія та інженерна графіка. – Т. 35. – С. 10–18.
3. *Фролов О. В.* Каналові поверхні та їх віднесення до ліній кривини / О.В. Фролов // Праці Таврійської державної агротехнологічної академії. – Мелітополь: ТДАТА, 2003. – Вип. 4. Прикладна геометрія та інженерна графіка. – Т. 22. – С. 112–120.
4. *Выгодский М.Я.* Дифференциальная геометрия / М.Я. Выгодский. – М. – Л.: ГТТИ, 1949. – 511 с.