

DEFINITION OF THE TRAJECTORY OF ROUND OF THE OBSTACLE BY THE ROBOT BY MEANS OF THE GEODETIC

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Summary. The way of the description and creation of a trajectory of movement on the plane of the robot which has to bypass an obstacle is developed based on use of the geodetic line of an auxiliary (enveloping) surface.

Keywords: the trajectory of work, geodesic line, enveloping surface, the system of equations for the geodesic and surveying solving the equations.

Formulation of the problem. One of the main problems arising in driving mobile robot is to build trajectories, given the possible obstacles to the movement of the robot. This is considered a priori that it is desirable to determine the shortest path. Also found trajectory must meet certain conditions [1]. For example, a robot that automatically examines some zone at the end of the road is expected to reach a known place (including for recharging batteries).

Analysis of the known results. In [2] an algorithm determining the trajectory of work as interpretation "movement on non-flat surfaces", an idea which is as follows. Flat maps to the presence of obstacles are invited to submit an abstract as a non-flat surface, replacing barriers smooth "hills". If toll free from interference zones (for example, on asphalt, the sand, the grass, in swampy areas, etc.), costs associated with different resources, at the above non-flat map these zones can also specify additional conventional hills and hollows.

For algorithmic implementation of such an approach is desirable to determine the trajectory of analytical methods. In [3] were considered ways to solve this problem by using geodetic lines. It noted the practical difficulty of achieving the goal of management. In [2] proposed simplified approach: find a path using variational methods [4].

Let not the flat area of interest expressed by function $z = f(x, y)$, where z – imaginary "height" a certain point trajectory; x and y - Cartesian coordinates projection surface point on the plane. It is necessary to

$$T = \int_{x_s}^{x_e} \frac{\sqrt{1 + (y')^2}}{f(x, y)} dx$$

minimize the functional

with boundary conditions:

$y(x_s) = y_s; y(x_e) = y_e$, where x_s and y_s - cartesian coordinates starting point; x_e and y_e - cartesian coordinates of the final (target) point.

When searching for extremum said functional artificial demand is assumed to have a clear desired curve equation $y = y(x)$. This greatly reduces the solution of the problem, as are straight, parallel to the axis Oh, can cross the curve at several points of the solutions [4].

For expansion factor "explicit equation" functionality to convert using parametric curve tasks:

$$T = \int_0^{t_e} \frac{\sqrt{1 + \left(\frac{y'}{x'}\right)^2}}{f(x, y)} x' dt \quad (1)$$

with boundary conditions: $y(0) = y_s; x(0) = x_s; y(t_e) = y_e; x(t_e) = x_e$.

Then the problem is reduced [4] to solve a system of two differential equations of 2nd order:

$$\begin{cases} \frac{\partial^2}{\partial x'^2} F \cdot x'' + \frac{\partial^2}{\partial x \partial x'} F \cdot x' + \frac{\partial^2}{\partial t \partial x'} F - \frac{\partial}{\partial x} F = 0 ; \\ \frac{\partial^2}{\partial y'^2} F \cdot y'' + \frac{\partial^2}{\partial y \partial y'} F \cdot y' + \frac{\partial^2}{\partial t \partial y'} F - \frac{\partial}{\partial y} F = 0, \end{cases} \quad (2)$$

where F – integrand in formula (1).

Algorithm "driving on a flat surface" provides passage barriers on the boundaries fast only simple interference, in which a small number of "corners". If the goal is unattainable and the number of obstacles to the number of faces is large, then the implementation of the algorithm may not be enough resources evaluator. However, this algorithm has a great advantage over other algorithms because it can take into consideration various factors terrain. Cons obvious - a numerical solution of equations essentially depends on the configuration of the surface. In [5] a method to track the average (geodesic) curve, which is located between two boundary curves subsidiary. The method is designed for regular surfaces and is based on the numerical solution of system of equations for the geodesic surface in the parameter space.

In [6] the way search ended close geodetic difference as a "method of shots", which in practice is sensitive to the choice of angles in the initial starting point.

The wording of Article purposes. Develop based on using the geodesic line auxiliary (enveloping) surface description and method of constructing the trajectory of the plane on the work that has to get around the obstacle.

Main part. First, consider the function:

$$f(u) = h \exp\left(1 - \frac{(u - a)^2}{r^2}\right), \quad (3)$$

which charts at $r = 1$ i $a = 0$ depending on h shown in Fig.1.

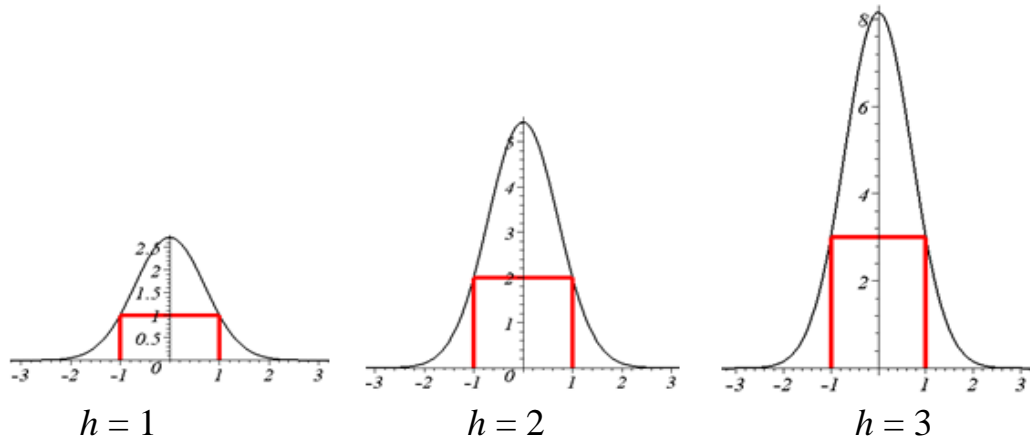


Fig. 1. The graph of the function (3) depending on the parameter h .

With (3) form a function of two variables

$$f(u, v) = h \exp\left(1 - \frac{(u - a)^2 + (v - b)^2}{r^2}\right), \quad (4)$$

in which schedule $r = 1$, $a = 1$, $b = 1$ i $h = 1$ shown in Fig. 2.

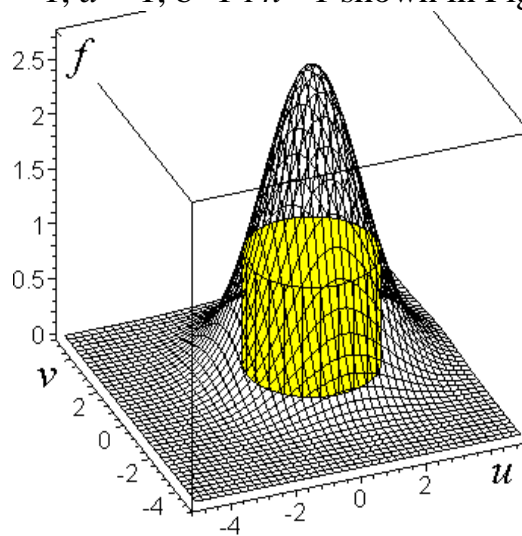


Fig. 2. Schedule function (4) at $r = 1$, $a = 1$, $b = 1$ and $h = 1$.

From Fig. 2 shows, as a graph of this function (4) "enveloping" the surface of the cylinder formed by the rotation around the vertical axis "stepped" function from Figure 1.

We emphasize that the above is conditional cylinder object; further use only its image in the horizontal plane and the enveloping surface - auxiliary facility, which will be built geodesic line.

Problem. Determine the trajectory of the robot from point A to point B subject to G circumvent obstacles to the movement in a circle (Fig. 3).

As the paradigm algorithm for determining the trajectory of the robot's choose a desired trajectory T is the projection on a horizontal plane geodesic curve, based on the respective enveloping surfaces.

Moreover, geodesic curve has a point A' and B' connect to enveloping surface whose projections on the plane will be point A and B.

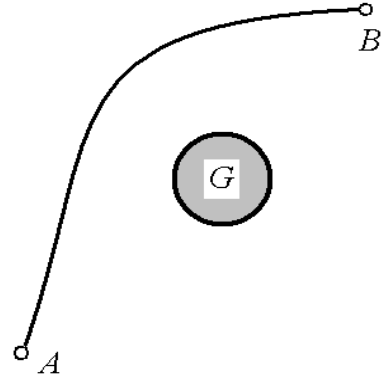


Fig. 3. Trajectory Working conditions

Describe the geodesic line by using the system of differential equations [7]:

$$\begin{aligned} u'' + \Gamma_{11}^1 u^2 + 2\Gamma_{12}^1 uv + \Gamma_{22}^1 v^2 &= 0, \\ v'' + \Gamma_{11}^2 u^2 + 2\Gamma_{12}^2 uv + \Gamma_{22}^2 v^2 &= 0. \end{aligned} \quad (5)$$

Given the initial conditions:

$$u(0)=u_0, v(0)=v_0, u'(0)=du_0, v'(0)=dv_0.$$

system (5) has a unique solution. This means that through each point of the surface in a given direction is only one geodesic.

Expressions Γ_{ij}^k Krystoffelya called symbols; they should record the coefficients of the second quadratic form surface:

$$\begin{aligned} \Gamma_{11}^1 &= (E_u G - 2F_u F + E_v F) / (2(EG - F^2)); \\ \Gamma_{11}^2 &= (2F_u E - 2E_v E - E_u F) / (2(EG - F^2)); \\ \Gamma_{12}^1 &= \Gamma_{21}^1 = (E_v G - G_u F) / (2(EG - F^2)); \\ \Gamma_{12}^2 &= \Gamma_{21}^2 = (G_u E - E_v F) / (2(EG - F^2)); \\ \Gamma_{22}^1 &= (2F_v G - G_u G - G_v F) / (2(EG - F^2)); \\ \Gamma_{22}^2 &= (G_v E - 2F_v F + G_u F) / (2(EG - F^2)); \end{aligned} \quad (6)$$

Thus, the study of geodesic lines surfaces is necessary to solve systems of non-linear ordinary differential equations. Most of nonlinear differential equations have analytical solutions, so widely used computer methods of geodesic equations.

Maple capabilities allow to get an orderly system of equations based on surveying and numerical solutions of this system and graphical representation of the solutions of the system for arbitrarily defined surfaces and initial conditions.

By means of the package Maple: Parametric equations for surface formed geodetic system of equations; created by numerical integration procedure geodetic system of equations for given initial conditions; created procedure Graphic representation of solutions of equations of geodesic. It was compiled maple- program determining geodesic curve.

Test example. Consider the surface described by the equation

$$z = f(0,0,3)+f(2,2,2)+f(-2,-2,2)+f(-1,2,3)+f(1,-2,2)+f(2,0,2)], \quad (7)$$

where

$$f(a,b,h) = h \cdot \exp(-(u-a)^2 - (v-b)^2).$$

Fig. 4 shows the result of the program - the geodetic image on the surface, and its projection on the horizontal plane.

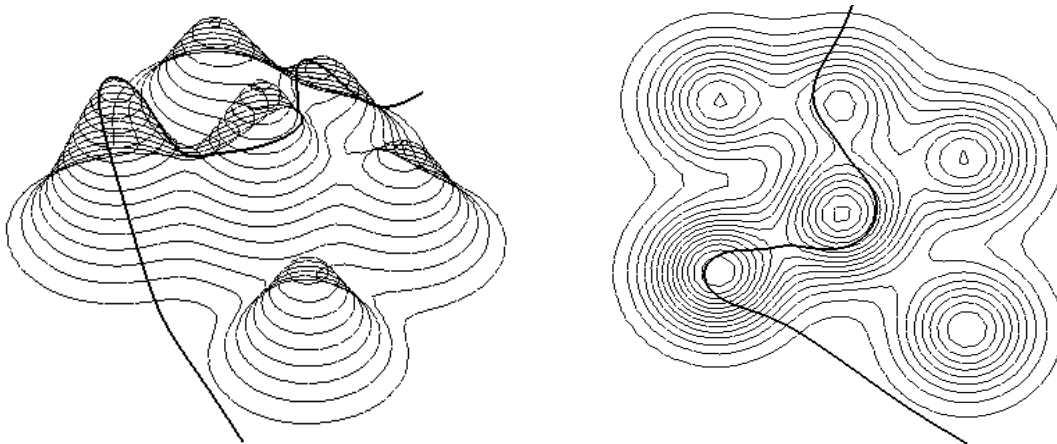


Fig. 4. The result of the program.

In developed algorithm for tracing work is to influence the shape of the trajectory change parameter h in the formula (4).

Example. Fig. 5 - 7 are some options for $r = 2$. This starting point has coordinates $\{u_0 = -5; v_0 = -5\}$. The direction of movement wondered vector $\{Du_0 = 3; Dv_0 = 1.5\}$.

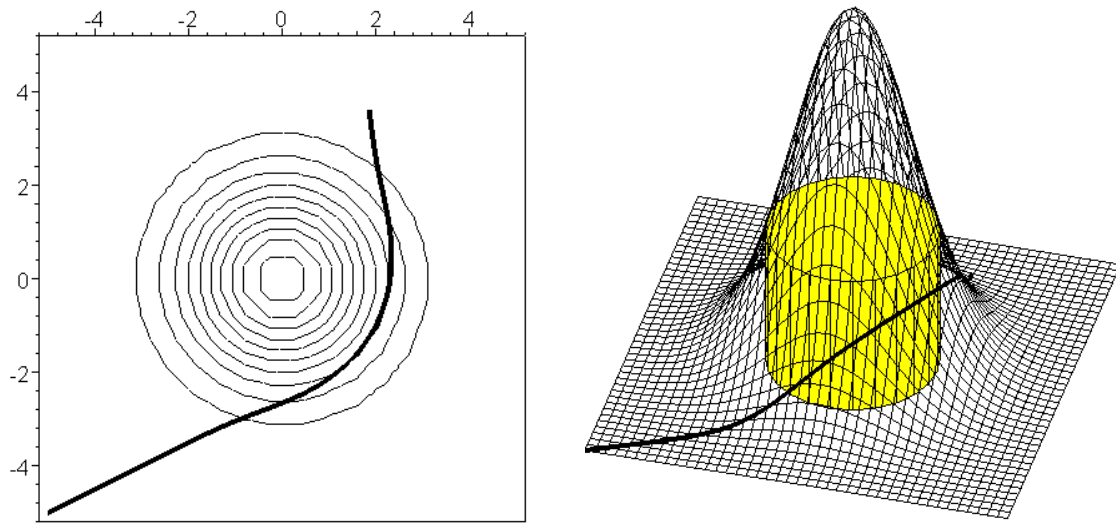


Fig. 5. Trajectory for setting $h = 2$.

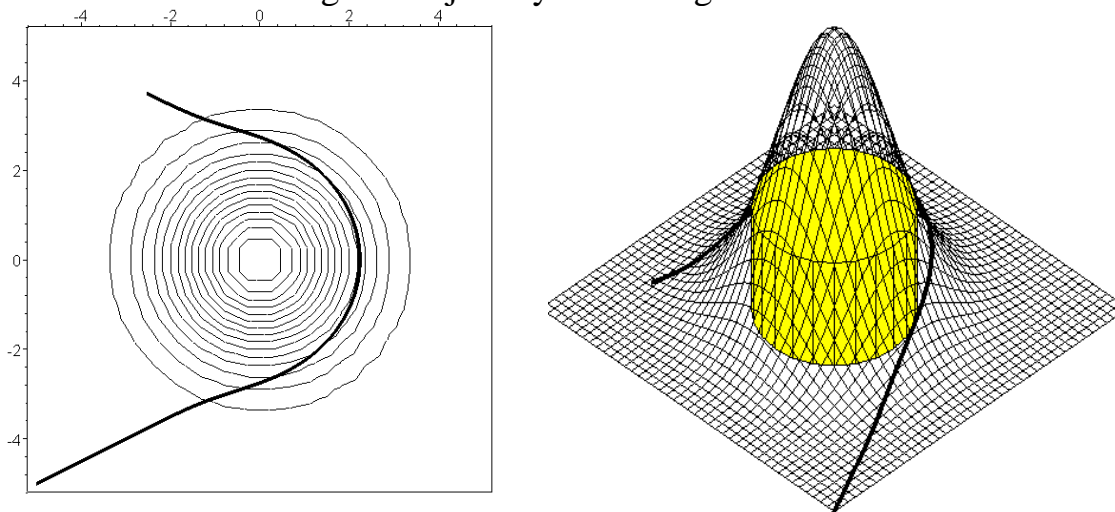


Fig. 6. Trajectory for setting $h = 3$.

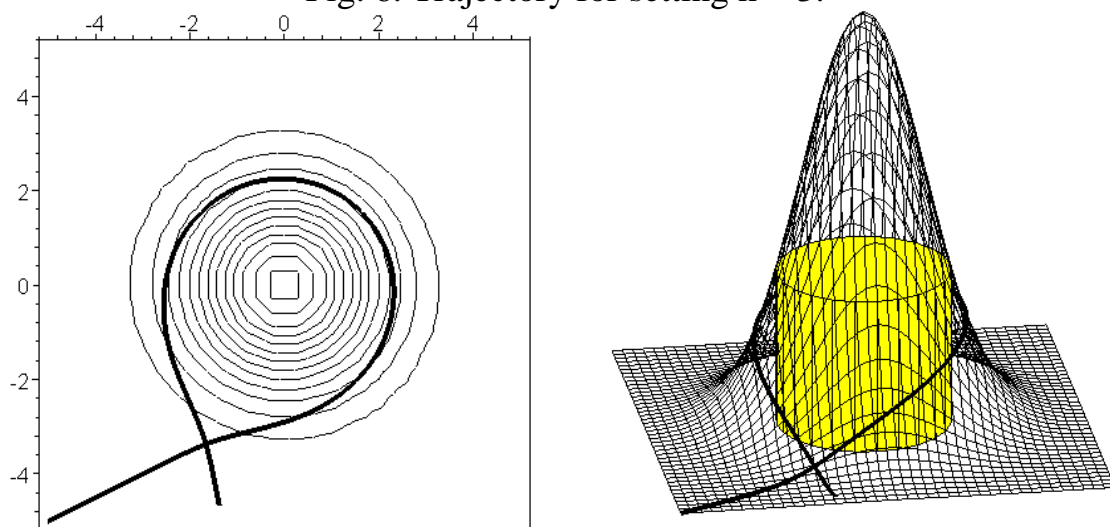


Fig. 7. Trajectory for setting $h = 4$.

The advantages of the method include the opportunity to "build" number of obstacles. To do this, using R-functions describe the combined obstacles.

Example. May need to define the path to bypass obstacles in the form of circles of radius and center coordinates respectively (0,0) and (0,6). To denote this:

$$f_1(u, v) = h \exp(1 - u^2 - v^2); \quad f_2(u, v) = h \exp(1 - u^2 - (v - 6)^2) \quad (8)$$

and K asked -dyz'yunktsiyu formula

$$F = (f_1(u, v) + f_2(u, v) + |f_1(u, v) - f_2(u, v)|) / 2, \quad (9)$$

or, in expanded form ($h = 2$):

$$F := e^{(1-u^2-v^2)} + e^{(1-u^2-(v-6)^2)} + \frac{1}{2} |2e^{(1-u^2-v^2)} - 2e^{(1-u^2-(v-6)^2)}|.$$

Fig. 8 shows a variant $h=2$ in computing to (-5, -5) - the starting point and the coordinates $\{Du_0 = 1,5; Dv_0 = 1\}$ vector direction of the initial movement.

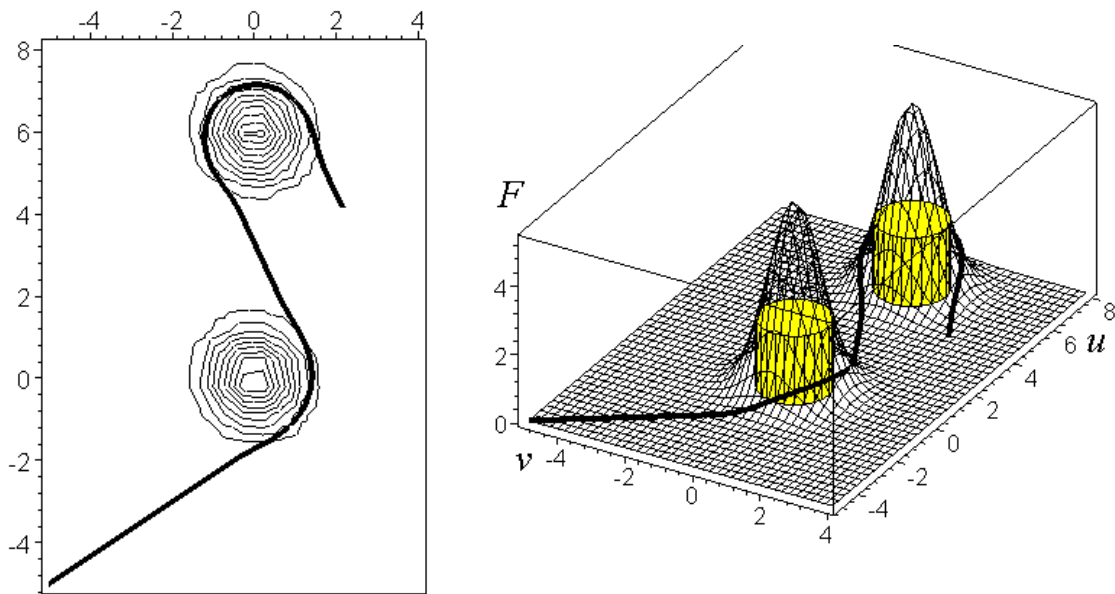


Fig. 8. Trajectory for setting $h = 2$.

Conclusion. The method description and construction of the trajectory of the plane on the work that has to get around the obstacle. It is essential that the method is based on using the geodesic line auxiliary (enveloping) surface. This allows you to vary the trajectory of work depending on the setting.

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