DETERMINING THE AREA OF THE SEGMENT BOUNDED BY ARC CURVE

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***Summary*. Suggested by means of a point-BN calculus solution of finding the square segment bounded by the arc of the curve, provided that the two points which form simplex located on this curve.**

***Keywords:* BN- calculus, area of segment, continuous point curve.**

Formulation of the problem. In-process [1] the calculation of area was first shown, by a limit flat reserved curve that is set by point equalization [2, 3]. Thus, a simplex at that a top was outside the reserved curve was elected, and other two tops that determine a simplex were elected among points that are in a middle a curve. Interesting will be a decision of task of being of corresponding area, when two (except a top) points that determine a simplex will be located on a curve.

Analysis of the recent researches. In-process [1] and to the real article the task of being of area of the segment limited to the arc of curve is examined first in a point BN-calculus development of that Melitopol school of the applied geometry engages in.

The wording of the purposes of the article. To work out a method for being of area of the segment, limited to the arc of the flat curve, set by point equalization in a simplex, a top of that is out of limits of curve, and two other points that determine a simplex - on her.

Main part. Let, in some global simplex, (fig.1) certain point equalization (1) curve of M.

Elect the local simplex of CAB, a top of what C is out of limits of curve of M and elected arbitrarily, and two other A and B - on the curve of M. Let point equalization of this curve be:

$$M = (A - C)\,p + (B - C)q + C, \tag{1}$$

where p i q – parameters that show a soba in an obvious or non-obvious form simple relation of three points and determine the form of curve.

For determination of points A and B But also In, that belong to the curve of M, it is necessary in relation to these points to untie the system of two equalizations (2):

$$\begin{cases} M_A = Ap_A - Cp_A + Bq_A - Cq_A + C \\ M_B = Ap_B - Cp_B + Bq_B - Cq_B + C. \end{cases} \tag{2}$$

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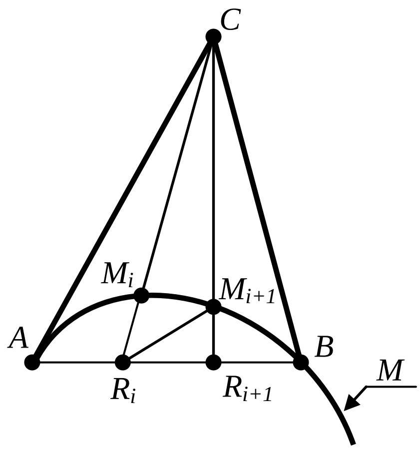

Fig. 1. A scheme is for determination of area
between the segment of AB and arc of AB of curve of M.

From where it is possible to write down:

$$A = B\frac{q_A}{p_A} + \frac{1}{p_A}(M_A - C(1 - p_A - q_A)), \tag{3}$$

if to accept $a = M_A - C(1 - p_A - q_A)$, then equalization (3) will look like :

$$A = B\frac{q_A}{p_A} + \frac{1}{p_A}a \cdot \tag{4}$$

Taking into account (4), it is possible to write down

$$M_B = B\frac{q_A p_B}{p_A} + \frac{p_B}{p_A}a + Bq_B + C(1 - p_B - q_B).$$

If to enter denotation: $b = \frac{q_A p_B}{p_A} + q_B$ and $c = M_B - a\frac{p_B}{p_A} - C(1 - p_B - q_B)$, then will define:

$$B = \frac{c}{b} = b_B \cdot \tag{5}$$

Taking into account (5), let us write (4):

$$A = a_A = \frac{1}{p_A}(b_{\mathbf{B}} q_A + a) \cdot \tag{6}$$

With taking (5) into account and (6) point equalization (1) will get a kind:

$$M = (a_A - C)p + (b_B - C)q + C. \tag{7}$$

Elect on a curve from (7) two arbitrary points M_i and M_{i+1} . Will write down point equalizations for these points that determine their coordinates :

$$M_i = (a_A - C)p_i + (b_B - C)q_i + C, \text{ where } p_i = p(t_i); \; q_i = q(t_i); \tag{8}$$

$$M_{i+1} = (a_A - C)p_{i+1} + (b_B - C)q_{i+1} + C, \text{ where } p_{i+1} = p(t_{i+1});$$
$$q_{i+1} = q(t_{i+1}). \quad (9)$$

Will define a point R_i from the simple relation of three points M_iCR_i :

$$M_iCR_i = r_i; \rightarrow \frac{M_i - R_i}{C - R_i} = r_i; \; R_i = \frac{M_i - Cr_i}{1 - r_i}, \text{ where } r_i = 1 - p_i - q_i. \quad (10)$$

Putting in (10) point equalization (8), will get a point R_i:

$$R_i = (a_A - C)\frac{p_i}{p_i + q_i} + (b_B - C)\frac{q_i}{p_i + q_i} + C. \quad (11)$$

By an analogical method, will define a point R_{i+1} from the simple relation of three points in a point form:

$$R_{i+1} = \frac{M_{i+1} - Cr_{i+1}}{1 - r_{i+1}}, \quad (12)$$

will find a point R_{i+1} from point equalization:

$$R_{i+1} = (a_A - C)\frac{p_{i+1}}{p_{i+1} + q_{i+1}} + (b_B - C)\frac{q_{i+1}}{p_{i+1} + q_{i+1}} + C. \quad (13)$$

The area of the sought after quadrangle $S(M_iR_iR_{i+1}M_{i+1})$ (fig.1) equals the sum of areas of two triangles $S(M_iR_iM_{i+1})$ and $S(M_{i+1}R_iR_{i+1})$, id est

$$S_{i,i+1} = \frac{ab\sin\gamma}{2(p_i + q_i)}\left(\begin{vmatrix} p_i & q_i & 1 \\ p_i & q_i & p_i + q_i \\ p_{i+1} & q_{i+1} & 1 \end{vmatrix} + \frac{1}{p_{i+1} + q_{i+1}}\begin{vmatrix} p_{i+1} & q_{i+1} & 1 \\ p_i & q_i & p_i + q_i \\ p_{i+1} & q_{i+1} & p_{i+1} + q_{i+1} \end{vmatrix}\right). \quad (14)$$

If to accept, that $\Delta_{i,i+1} = p_iq_{i+1} - p_{i+1}q_i$, then will get a formula for the calculation of area of quadrangle (fig.1):

$$S_{i,i+1} = \frac{ab\sin\gamma}{2}\left(\frac{r_{i+1} - r_i(p_{i+1} + q_{i+1})}{(p_i + q_i)(p_{i+1} + q_{i+1})}\Delta_{i,i+1}\right., \quad (15)$$

where $r_i = 1 - p_i - q_i$, a $r_{i+1} = 1 - p_{i+1} - q_{i+1}$.

Conclusions. Many tasks of the applied character get untied through the use of the areas limited to the arc of the crooked line, a that is why offer here method has the special value. It is necessary to notice that than less step between i and i+1 points, the area of triangle will be certain more precisely.

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