RECURRENCE FORMULAE OF A SINUSOID IN CREATION OF ONE-DIMENSIONAL GEOMETRIC IMAGES

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***Summary.* In the article we have studied a problem of a possibility of transition from a closed form to a recurrence form of a setting of numerical sequences of transcendental curves. A sinusoidal functional relationship has been investigated to use it for creation of one-dimensional geometrical images.**

***Keywords:* recurrent formulas, numerical sequences transcendental curves, sinusoid, discrete geometric modeling.**

Formulation of the problem. In the paper is noted, that most input data and conditions for solved applied problems, forms of presentation, processing and analyzing data on a computer have discrete character, although the most important theoretical and practical results creation of simulation methodologies for received continuous forms of input data.

Research recurrence formulas of numerical sequences ,as discrete analogue of continuous functional dependencies from the standpoint of their use in forming geometric images , is important because on their basis effective algorithms of transition from discrete presented image to its continuous analogue and vice versa can be created.

Formulation of Article purposes. The purpose of this article is to research opportunities of transition from closed to recurrent form of setting numerical sequences sinusoidal functional dependencies in order to further forming dimensional geometric image according to the numerical sequences.

Main part. Sinusoidal change of arbitrary magnitude is called harmonic oscillation. Examples can be any oscillatory processes starting from pendulum rocking and ending with sound waves (harmonic oscillation of air) -voltage fluctuation in the electric grid alternating current ,change current and voltage in oscillatory circuit and others. Therefore recurrent analogues of sinusoidal curves are interesting for research listed above processes.

The flat curve, given by the equation in rectangular coordinates is called sinusoid.

$$y = a + b\sin(cx + d) \ . \qquad (1)$$

Chart equation of the form $y = a + b\cos(cx + d)$ is often also called sinusoid. This chart is obtained from the sinusoidal displacement in $\frac{\pi}{2}$ a negative direction on abscissa (Fig. 1).

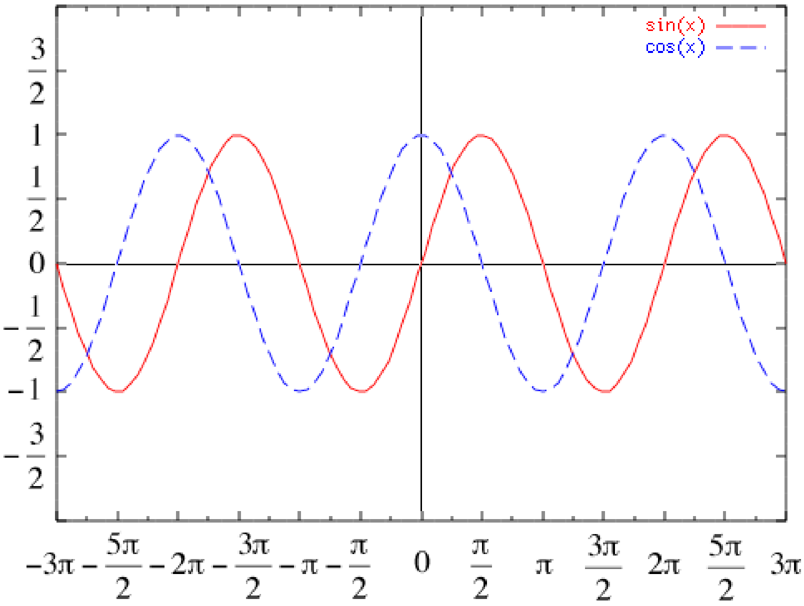

Fig. 1. Charts sinusoids.

The term "kosynusoyida" virtually is absent in the official literature as it is redundant.

If we replace argument discrete parameter in sinusoidal function (1), we have:

$$y_i = a + b \times sin(ci + d). \qquad (2)$$

From here:

$$b \times sin(ci + d) = y_i - a \ ; \quad sin(ci + d) = \frac{y_i}{b} - \frac{a}{b} \ .$$

In case of exemption from discrete parameter we obtain expressions for determination of adjacent ”nodes” :

$$y_{i+1} = a + bsin\left[c(i+1) + d\right] = a + bsin\left[(ci + d) + c\right] =$$

$$= a + b\left\{sin(ci + d)cosc + cos(ci + d)sinc\right\} =$$

$$= a + cosc \times (y_i - a) + b \times sinc \times \sqrt{1 - sin^2(ci + d)} =$$

$$= a + (y_i - a)cosc + b \times sinc \times \sqrt{1 - (\frac{y_i}{b} - \frac{a}{b})^2} =$$

$$= a + (y_i - a)cosc + \sqrt{b^2 - (y_i - a)^2} \times sinc \ . \qquad (3)$$

Similar to (3) we obtain:

$$y_{i-1} = a + (y_i - a)cosc - \sqrt{b^2 - (y_i - a)^2} \times sinc\ . \qquad (4)$$

By adding (3) and (4) we obtain the following recurrent formula of sequence:

$$\begin{gathered} y_{i+1} + y_{i-1} = 2a + 2(y_i - a)cosc \Rightarrow \\ \Rightarrow y_{i+1} = 2a + 2(y_i - a)cosc - y_{i-1} \end{gathered} \qquad (5)$$

Similar to (5) we obtain:

$$\begin{gathered} y_i = 2a + 2(y_{i-1} - a)cosc - y_{i-2} \Rightarrow \\ \Rightarrow y_i = 2a + 2y_{i-1}cosc - 2acosc - y_{i-2} \end{gathered} \qquad (6)$$

By subtracting (6) to (5) we obtain:

$$\begin{gathered} y_{i+1} - y_i = 2a + 2y_i cosc - 2acosc - y_{i-1} - \\ -2a - 2y_{i-1}cosc + 2acosc + y_{i-2} = 2(y_i - y_{i-1})cosc + (y_{i-2} - y_{i-1}) \end{gathered}$$

From here:

$$y_{i+1} = y_i + 2y_i cosc - 2y_{i-1}cosc - y_{i-1} + y_{i-2}\ ; \qquad (7)$$

$$y_{i+1} = y_i(1 + 2cosc) - y_{i-1}(1 + 2cosc) + y_{i-2}\ ; \qquad (8)$$

$$y_{i+1} = (y_i - y_{i-1})(1 + 2cosc) + y_{i-2}\ . \qquad (9)$$

Faithfulness derived recurrent formulas (7), (8), (9) the numerical sequence (2) can be easily checked by the relevant test examples with help of setting the specific parameters of the sequence.

Conclusion. Sinusoidal functional dependencies can be represented by different recurrent formulas of the endless number sequences ,that are discrete models of one-dimensional geometric images with a given number of “nodes”. Subsequent studies will be directed to study the possibilities of using the obtained recurrent analogue of sinusoid for the discrete geometric modeling images.

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