GRAPHIC-ANALYTICAL METHOD FOR SOLVING FUZZY MATRIX GAMES

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Summary. Graphic-analytical method for solving matrix games in which the payoff matrix elements - fuzzy number with unknown membership function considered in the article.

***Keywords* - payment matrix, fuzzy numbers, membership function, fuzzy straight, fuzzy geometry.**

Formulation of the problem. Every finite matrix game can be solved graphic-analytically (graphically) or its solution can be reduced to the solution of a pair of dual problems of linear programming [1]. However, in the case of matrix games in which elements of the payment matrix represent fuzzy numbers, the problem arises in the use for these games, the simplex method, associated with the division into fuzzy numbers, a carrier of which contains zero. This operation on fuzzy numbers is uncertain [2]. Graphic-analytical method is applicable only to games in which at least one of the players has two strategies. However, it illustrates well the content side of things in the game, clearly and graphically explains basic concepts of the theory of matrix games. In the cases of solution of fuzzy matrix games the problems of conducting operations on fuzzy numbers basically does not occur. The uncertainty in the solution of fuzzy matrix games by the graphic-analytical method can be described using the mathematical apparatus of theory of fuzzy sets and fuzzy geometry [3-5].

Analysis of recent researches. In the work[1], the traditional solution methods of solving clear and traditional analytical methods of solving fuzzy matrix games are considered. The work is dedicated to the consideration of games with fuzzy payment matrix [6]. The approach is to move from fuzzy problem to two clear linear programming problems. In the work [7] the solution of fuzzy matrix games with certain set of possible ends of the game, given with certain vagueness are discussed. The methods that give possibility to reduce the solution of the fuzzy problem to a clear are considered. The work is dedicated the consideration of possible technologies, analytical solutions of game theory with clear payment matrix [8]. To eliminate the drawbacks of the traditional method of solving the alternative procedure is proposed that based on the use of composite criteria that take into account the measure of closeness of the solutions obtained by modal, as well as the level of uncertainty in the resulting fuzzy value for the price of the game. This procedure reduces the original problem to a clear mathematical programming problem, solvable by known methods.

The wording of the purposes of the article. We propose consideration of the graphic-analytical method to solve fuzzy matrix games with using the mathematical apparatus of theory of fuzzy sets and fuzzy geometry.

Main part. Basic concepts of fuzzy set theory, fuzzy mappings and relations, the notion of fuzzy geometry objects and fuzzy geometric figures on the plane we assume the same as in [3-5].

Graphic-analytical method for solving fuzzy matrix games consists of two parts. First, in a matrix game is graphically revealed the qualitative features of the solution, than the complete fuzzy feature of the solution is analytical.

Graphic-analytical method is quite suitable for fuzzy matrix games is a dimension of $2\times n$ or $m\times 2$.

Let's consider at first the case when the game is set to a dimension $2\times n$ with a payment matrix

$$A=\begin{pmatrix} a_{11}a_{12}...a_{1n} \\ a_{21}a_{22}...a_{2n} \end{pmatrix}.$$

Let the elements of matrix A are Gaussian fuzzy numbers with membership functions

$$\mu(a_{ij})=\exp\left\{-\frac{(a_{ij}-a_{ij}^0)^2}{2\sigma_{ij}^2}\right\},$$

where a_{ij}^0 are modal values (the kernel) of fuzzy numbers, a_{ij}, σ_{ij} are the coefficients of concentration.

Let $(x_1,x_2)$is the fuzzy optimal strategy of player 1, (y_1,y_2) is fuzzy optimal strategy for player 2. Then, excluding the trivial case (the existence of fuzzy optimal pure strategy at least one of the players), we have:

$$x_1+x_2=1, x_1>0, x_2>0, y_1+y_2=1, y_1>0, y_2>0 \qquad (1)$$

Further, for convenience, the fuzzy points, fuzzy lines and fuzzy segments we will depict graphically to their modal values (cores).

We will choose a rectangular coordinate system and mark on the x-axis unit interval to represent fuzzy mixed strategies of player 1 (fig. 1).

At the ends of this piece we will restore two of the perpendiculars, on which we will defer the fuzzy player's wins when he uses the fuzzy and pure strategies A_2 and A_1.

Let player 2 chose a fuzzy strategy. Then when player 1 uses fuzzy pure strategy A_2 he gets fuzzy payoff a_{21} (corresponding fuzzy dot on the left perpendicular), and when he uses pure strategy of fuzzy A_1 – fuzzy payoff a_{11} (fuzzy dot to the right perpendicular).

Let player 2 chose a fuzzy strategy. Then when player 1 uses fuzzy pure strategy he gets fuzzy payoff (corresponding fuzzy dot on the left perpendicular), and when he uses pure strategy of fuzzy – fuzzy payoff (fuzzy dot to the right perpendicular). Combining these two fuzzy point of the fuzzy segment of fuzzy line, we get the dependence graph of the fuzzy player wins 1 $M(x,B_1)$from fuzzy mixed strategy *x*, assuming that player 2 uses fuzzy pure strategy B_1 (Fig. 1). Exactly the same fuzzy straight can be built for $B_1, B_2,..., B_n$.

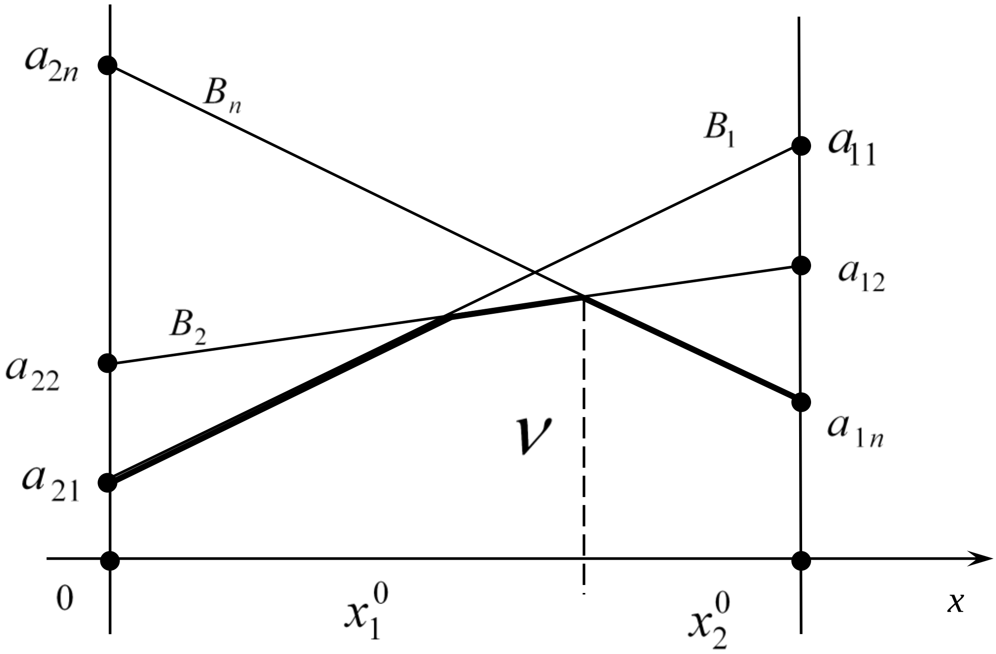

Fig. 1. Graph of fuzzy payoff of the player 1 mixed strategy

Next, we need for each fuzzy mixed strategy x, i.e. for each fuzzy point fuzzy unit segment on the x-axis, to find $\min\limits_{0 \le j \le n} M(x,B_j)$,that is the fuzzy lower bound sets and fuzzy lines. The fuzzy border is noted by fuzzy bold line (Fig. 1). The fuzzy point of the fuzzy segment, in which the fuzzy lower bound reaches a maximum, corresponds to the desired fuzzy mixed strategy x^0, maximum height gives the value of fuzzy lower prices of *x*.

Similarly, we can find the odd optimal mixed strategy of the player 2 and the fuzzy lower price of the game γ in an odd games $m \times 2$ with the only difference being that here we need to look not for the maximum of the odd lower bound but the minimum of odd upper bound.

According to the basic theorem of matrix games the solutions in a fuzzy in mixed strategies always exists $\alpha = \gamma = \nu$. Here ν is an odd game price.

And to clarify x_1^0 and x_2^0, y_1^0 and y_2^0 we need to find the fuzzy point of intersection of fuzzy lines that, in turn, reduces to solving an odd system:

$$\begin{cases} a_{11}x_1 + a_{21}x_2 = \nu, \\ a_{12}x_1 + a_{22}x_2 = \nu. \end{cases} \quad (2)$$

Solving the system of equations (2) and substituting $x_2 = 1 - x_1$, get

$$x_1 = \frac{a_{22} - a_{21}}{\Delta}, \quad x_2 = \frac{a_{11} - a_{12}}{\Delta} \quad (3)$$

Similarly we find

$$y_1 = \frac{a_{22} - a_{12}}{\Delta}, \quad y_2 = \frac{a_{11} - a_{21}}{\Delta} \quad (4)$$

Odd price of the game ν is the substitution of the found values x_1, x_2 in any of the equations system (2):

$$\nu = \frac{a_{11}a_{22} - a_{12}a_{21}}{\Delta} \quad (5)$$

In formulas (3) – (5) $\Delta = a_{11} + a_{22} - a_{12} - a_{21}$.

Since in this work the elements of payoff matrix are Gaussian fuzzy numbers to find fuzzy optimal strategies and the fuzzy price of the game according to the formulas (3) to (5) it is required to define the basic operations on these numbers. In the work [3] the rules of performing operations on Gaussian fuzzy numbers are presented (rules of summation and subtraction, rules multiplication and division). Let us formulate these rules.

1. In the summation of two fuzzy numbers A_x and A_y with membership functions are respectively equal to:

$$\mu_A(x) = exp\left\{ -\frac{(x - m_x)^2}{2\sigma_x^2} \right\}$$
$$\mu_A(y) = exp\left\{ -\frac{(y - m_y)^2}{2\sigma_y^2} \right\} \quad (6)$$

we will get the number B with the membership function:

$$\mu_B(z) = exp\left\{ -\frac{(z - m_z)^2}{2\sigma_z^2} \right\} \quad (7)$$

2. The difference of two Gaussian odd numbers with membership functions (6) are equal in number B with the membership function:

$$\mu_B(z) = \exp\left\{-\frac{(z-m_z)^2}{2\sigma_z^2}\right\},$$
$$m_z = m_x - m_y, \sigma_z^2 = \sigma_x^2 + \sigma_y^2. \tag{8}$$

3. When we are multiplying two odd numbers A_x and A_y with membership functions (6) we will receive a number *B* with the membership function:

$$\mu_B(z) = \exp\left\{-\frac{(z-m_z)^2}{2\sigma_z^2}\right\},$$
$$m_z = m_x \cdot m_y, \sigma_z^2 = \sigma_x^2 \cdot \sigma_e^2 \tag{9}$$

4. When we are dividing an odd number A_x by an odd number A_y with the membership functions (6) we will receive a number *B* with the membership function:

$$\mu_B(z) = exp\left\{-\frac{(z-\frac{m_x}{m_y})^2}{2\frac{\sigma_x^2 m_y^2 + \sigma_y^2 m_x^2}{m_y^2}}\right\}. \tag{10}$$

Conclusions. In this work, we propose consideration of the graphic-analytical method of solving matrix games in which the matrix elements of the matrix – the Gaussian odd numbers. First we consider the case, when we set the game with dimensions $2 \times n$. Case when you set the game with dimensions $m \times 2$ is treated similarly. Rules for performing basic algebraic operations on Gaussian fuzzy numbers are presented.

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