VALUE OF RECURRENT DEPENDENCE IN FORMATION OF DISCRETE CURVES USING SUPERPOSITION OF ONE-DIMENSIONAL POINT SETS

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Summary. In the article we have investigated some problems of the modeling of geometric images, using superposition of one-dimensional point sets, where forms of the curves are determined by a variation of superposition coefficients and a value of recurrent dependence, which is identical to the function of distribution of an outer forming load in the static geometric method.

Keywords: **geometric apparatus of superpositions, static-geometric method, numeric sequences, value of recurrent dependence.**

Formulation of the problem. Some of the properties, which has a discrete model line can be adapted to the model of the surface that is formed according to the same law, if this line is considered as part of the frame surface. Properties of discrete models of the surface can be obtained at the review of the respective properties of the model line.

Sagging thread, that evenly overloaded with in length, takes the form of the chain line, the same thread under uniform loading along the horizontal axis becomes already forms a parabola. When changing the type of load distribution of the thread is to control its form that matches one of the principles of proper-geometric way of constructing curves and contours [1].

The basis of the proper mathematical apparatus-geometric method is based on the solution quite bulky systems of linear equations, which complicates the process of computer implementation.

Analysis of recent researches. The issue of the expansion of either formative opportunities static -geometric way of using the mathematical apparatus of numeric sequences, which allows, in particular, to avoid the assembly of systems of linear equations in the formation of discrete images dedicated to work [2].

In the works [3, 4] of authors of this article the approach to the definition of discrete analogues of certain functional dependencies based on the geometric mean of the apparatus superpositions one-dimensional point sets are showed, which also allows you to create discrete images without compiling and solving the cumbersome systems of equations. Managing the form discretely presented curves (DPC) is performed by variation values of coefficients of superposition.

The wording of the purposes of the article. The purpose of this article is to study the possibilities of modeling the DPC based on one-dimensional superpositions of point sets, where control of the shape of the curves is also due to the magnitude of the recurrent addictions, that identical outer forming needs in proper-geometric way.

Main part. The term "the magnitude of the external forming load" are used if the geometric image is formed by the proper-geometric way as concentrated efforts in the key points include the presence of a balancing effort in the areas of the polyline.

In the formation of discrete images on the basis of the geometric mean of the apparatus superpositions use the terms "magnitude, recurrent addictions, that will identical to the value of the external load.

Formula

$$y_i = k_1 y_{i-1} + k_2 y_{i+1}$$

will identical over-differential three-point dependence

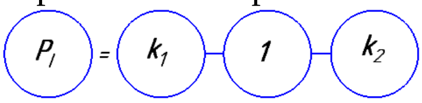
$$2y_i = 1y_{i-1} + 1y_{i+1},$$

therefore, the recursive dependencies that will be the prototype of forming external load to form a discrete analogue of the polynomial of the 2nd degree on the basis of the composition of the set of nodal points can be written as:

$$P_i = y_i - k_1 y_{i-1} - k_2 y_{i+1}$$

where P is a discrete value of the recurrent dependence.

Or in the form of a computational template:



Provided $k_1 + k_2 = 1$ we have:

$$P_i = y_i - k_1 y_{i-1} - \; 1 - k_1 \; y_{i+1}$$

$$k_1 \; y_{i-1} - y_{i+1} \; = y_i - y_{i+1} - P_i$$

From here, the famous discrete size recurrent dependencies can be determined coefficients superposition formulas:

$$k_1 = \frac{y_i - y_{i+1} - P_i}{y_{i-1} - y_{i+1}}$$

$$k_2 = 1 - k_1$$

So, as

$$y_{i-1} - y_{i+1} \; = \; y_{i-1} - y_i \; + \; y_i - y_{i+1}$$

the discrete value of recurrent dependency is determined by the formula:

$$P_i = \Delta_i - k_1 \; \Delta_{i-1} + \Delta_i$$

and the dependence of the magnitude of the coefficient of the superposition k1 from the magnitude of the recurrent addictions will look like:

$$k_1 = \frac{P_i - \Delta_i}{\Delta_{i-1} + \Delta_i}$$

Because over difference of the third order is formed as the difference between the two over differences of second order, then magnitude recurrent dependencies is evenly distributed for the formation of the discrete analogue of polynomial the 3rd degree based on superpositions of the given nodal points will look like:

$$P_i = y_i - y_{i+1} + k_1 \; y_i - y_{i-1} \; + k_2 \; y_{i+2} - y_{i+1}$$

Provided $k_1 + k_2 = 1$ we have:

$$P_i = y_i - y_{i+1} + k_1 \; y_i - y_{i-1} \; + \; 1 - k_1 \; y_{i+2} - y_{i+1}$$

$$P_i = y_i - 2y_{i+1} + y_{i+2} + k_1 \; y_i - y_{i-1} + y_{i+1} - y_{i+2}$$

From here:

$$k_1 = \frac{P_i - y_i + 2y_{i+1} - y_{i+2}}{y_i - y_{i-1} + y_{i+1} - y_{i+2}}$$

You can also write:

$$P_i = \Delta^2_{i+1} - k_1 \; \Delta_i + \Delta_{i+1}$$

and,

$$k_1 = \frac{P_i - \Delta^2_{i+1}}{\Delta_i + \Delta_{i+1}}$$

Recurrent amount evenly according to form the discrete analogue of polynomial n-th degree based on superpositions preset nodal points will look like:

$$P_i = \Delta_i^{n-2} + k_1 \Delta_{i-1}^{n-2} + k_2 \Delta_{i+1}^{n-2}$$

or,

$$P_i = \Delta_i^{n-2} - \Delta_{i+1}^{n-2} + k_1 \; \Delta_{i+1}^{n-2} - \Delta_{i-1}^{n-2}$$

or,

$$P_i = k_1 \; \Delta_{i+1}^{n-1} - \Delta_i^{n-1} \; - \Delta_{i+1}^{n-1}$$

The value of *k1* is determined by the formula:

$$k_1 = \frac{P_i - \Delta_i^{n-2} + \Delta_{i+1}^{n-2}}{\Delta_{i+1}^{n-2} - \Delta_{i-1}^{n-2}} = \frac{P_i + \Delta_{i+1}^{n-2} - \Delta_i^{n-2}}{\Delta_{i+1}^{n-2} - \Delta_i^{n-2} + \Delta_i^{n-2} - \Delta_{i-1}^{n-2}} =$$

$$= \frac{P_i + \Delta_{i+1}^{n-1}}{\Delta_{i+1}^{n-1} + \Delta_i^{n-1}}$$

Statement. The coordinates of any point of the numerical sequence of the n-th order can be defined as superposition of the coordinates of two arbitrary points of the sequence with a known size recurrent dependency.

For arbitrary values of *p, p1, p2* relationship between coefficients of superposition and largest recurrent dependencies can be determined from the system of equations:

$$\begin{cases} P_{i+p} = y_{i+p} - k_1 y_{i+p_1} - k_2 y_{i+p_2} \\ k_1 + k_2 = 1 \end{cases} \qquad (1)$$

The system (1) contains 2 equations and three unknowns – *Pi + p, k1* and *k2,* so she has lots of solutions with two basic unknown and one free. For example, free can act – *Pi + p*.

Expressions that define the correlation coefficients of superposition and recurrent addictions will have the form:

$$k_1 = \frac{P_{i+p} - y_{i+p} + y_{i+p_2}}{y_{i+p_2} - y_{i+p_1}} \qquad k_2 = \frac{P_{i+p} - y_{i+p} + y_{i+p_1}}{y_{i+p_1} - y_{i+p_2}}$$

$$P_{i+p} = k_1\left(y_{i+p_2} - y_{i+p_1}\right) + y_{i+p} - y_{i+p_2},$$

$$P_{i+p} = k_2\left(y_{i+p_1} - y_{i+p_2}\right) + y_{i+p} - y_{i+p_1}$$

Hence, a value y_{i+p} is determined by the formulas:

$$y_{i+p} = P_{i+p} - k_1\left(y_{i+p_2} - y_{i+p_1}\right) + y_{i+p_2},$$

$$y_{i+p} = P_{i+p} - k_2\left(y_{i+p_1} - y_{i+p_2}\right) + y_{i+p_1}$$

Thus the famous largest recurrent according to the coordinates of any point of the numerical sequence of the n-th order you can calculate as superposition of the coordinates of two arbitrary points of the given sequence.

Conclusions. In the article the results of studies are given, that allow you to model geometric images based on geometric apparatus superpositions due to varying values of coefficients of superposition and largest recurrent addictions, that identical outer forming needs in proper-geometric way. It is proved that the coordinates of any point of the numerical sequence of the n-th order can be defined as ssuperposition of the coordinates of two arbitrary points of the sequence with a known size of recurrent dependency.

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