УДК 515.2

GEOMETRIC MODELING OF SGM SURFACES WITH INVERSION TRANSFORMATION

S. Kovalev, S. Botvinovska, A. Zolotova

Summary. The paper considers the way of surfaces geometric modeling by the static-geometric method using spatial inversion transformation. An example of modeling of a discrete carcass surface with desired properties on an arbitrary reference circuits provided.

The algorithm of formation of architectural membranes with preserving of such property as a rounded shape is proposed. The inversion usage greatly simplifies the solution of the problem.

The proposed method of transporting properties of the prototype surface shape to the simulated surface will allow the designer or architect to formally receive the digital frame surface, which will correspond to the original data.

Keywords: **geometric modeling, inversion transformation, the properties of surfaces, digital frame, static-geometric method, shaping architectural membranes.**

Statement of the problem. For covering large spans in construction practice found a wide application of spatial design. The use of a variety of architectural membranes architectural structures significantly increases the economic, aesthetic and architectural-artistic expressiveness of buildings and individual premises in general. For today an actual problem of applied geometry remains the issue of improving the various methods of shaping surfaces of complex shapes with predetermined conditions and a given reference loop.

The transfer of properties known surfaces on the surface, is modeled, requires from the machine discrete geometry great flexibility, the possibility of using simple methods and advanced algorithms to solve practical problems while maintaining the original conditions.

Discrete point-skeletons of various curved surfaces found wide application in those cases when it is impossible to obtain the analytical equation of the surface at the prescribed conditions and to an arbitrarily given reference loop.

Analysis of recent researches and publications. In [1] was considered the method of forming discrete frames surfaces with given initial conditions in the processes of design-design design of curved structures in architecture. An example of the formation of discrete frame surface according to the specified parameters, which preserves compositional properties of the surface of the prototype.

The perfection of the spherical form since ancient times attracted the attention of architects and scientists who tried to explain the harmony of the universe [2]. From different sources it is known that the spherical surface has a number of advantages over other surfaces, which are widely used in modern architecture. First of all, the main advantage is in the natural form. Of all objects that have the same surface area, the sphere limits the maximum volume. Conversely, the sphere has the smallest surface area of all objects to restrict the same volume. That is why the body of spherical shape are very common in nature. For example, a drop of water in free fall takes a spherical shape due to minimization of surface area under the Earth's gravitational pull. That is why the naturalness of this form attracts modern architects

Geometric shapes of various objects of design and architecture are a combination of simple geometric shapes with curved surfaces. In the modern architecture of high-tech style is often used designs with curved outlines. The originality of each object is achieved due to special properties of the surface is modeled, the geometry of its control circuit and the various additional conditions for the establishment of load-bearing structures, conditions on the contour [3, 4].

In [5] was described algorithm for constructing the surface as an insult or a simple plane surface in a non-linear transformation space. What was listed three main advantages of this method: "...first, it is possible to study the properties of the design surface as a whole (due to the one-to-one correspondence between surface points and their prototypes). Thus, having the device convert and knowing the properties of the prototype, you can set the basic properties of the constructed image. Second, it simplifies the computation of the equation coefficients of the constructed surface defined by a number of conditions, as some of them involved in the task of unit conversion (and the remaining part in the task of the prototype). Thirdly, it is possible to control the shape (because of the design properties the surface properties of the prototype, the unit conversion and the position of the prototype in the machine of transformation, whose parameters are available to control...". This approach was the basis for this work. The work is dedicated to spatial transformation of inversion and its use in the formation of discrete frames of the surfaces with desired properties.

It is known that changing the equation of the surface F(x, y, z)=0 variables x, y, z by the formulas of the transformation: $x=f_1(x_1,y_1,z_1)$, $y=f_2(x_1,y_1,z_1)$, $z=f_3(x_1,y_1,z_1)$ it is possible to establish a correspondence between points on the surface of the prototype and the points of the new surface $F(x_1, y_1, z_1)=0$, i, thus, from one surface to another. Thus, with enough simple, well-studied surface properties of which are known, by using geometric transformations to obtain discrete frame of the new surface, which will maintain the surface properties of the prototype.

The article goals. The aim of this work is to form a discrete skeleton of the surface is rounded, for arbitrarily given reference contour with inversion symmetry in space.

Generally it is known that inversion is a special type of transformation of points in the plane or in space. The practical value of the inversion is that often such a transformation allows us to reduce the solution to geometric problems with circles or spheres to solve their tasks with straight or planes, that is, to tasks that would normally have a much easier solution. From [6] it is known that the transform function inversion is a rational function of 2-th order. Having the device conversion and taking the prototype of the spherical surface, it is necessary to model the discrete frame surfaces on round. The reference contour which is formed, is set on a cylinder surface that is adjacent to one of the spheres of the apparatus of inversion and coaxial with the field.

The main part. It is known [6], the transformation of inversion is a particular case of quadratic transformations. In three-dimensional space, the formulas of inversion will be:

$$x = \frac{r^2 x'}{(x')^2 + (y')^2 + (z')^2},$$

$$y = \frac{r^2 y'}{(x')^2 + (y')^2 + (z')^2}, \qquad (1)$$

$$z = \frac{r^2 z'}{(x')^2 + (y')^2 + (z')^2},$$

where r is the radius of the base sphere S double points of the inversion (Fig.1).

Use the property of inversion symmetry in which any plane not passing through the center of inversion goes into a sphere which will pass through the center of inversion. Then, any plane Γ, which intersects the base sphere corresponds to the sphere passing through the center Of inversion and the circle of the cross section of the base of the sphere with S square Γ. The set of points between two intersecting

squares $\boldsymbol{\Gamma}$ and $\boldsymbol{D}$ goes into the set of points between the spheres $\Gamma'i\Delta'$. In Fig.1 cross section of these zones shaded.

If between the planes $\boldsymbol{\Gamma}$ and $\boldsymbol{D}$ to place some limited surface, it will move into a new surface, all points of which are placed between the spheres Γ' and Δ'.
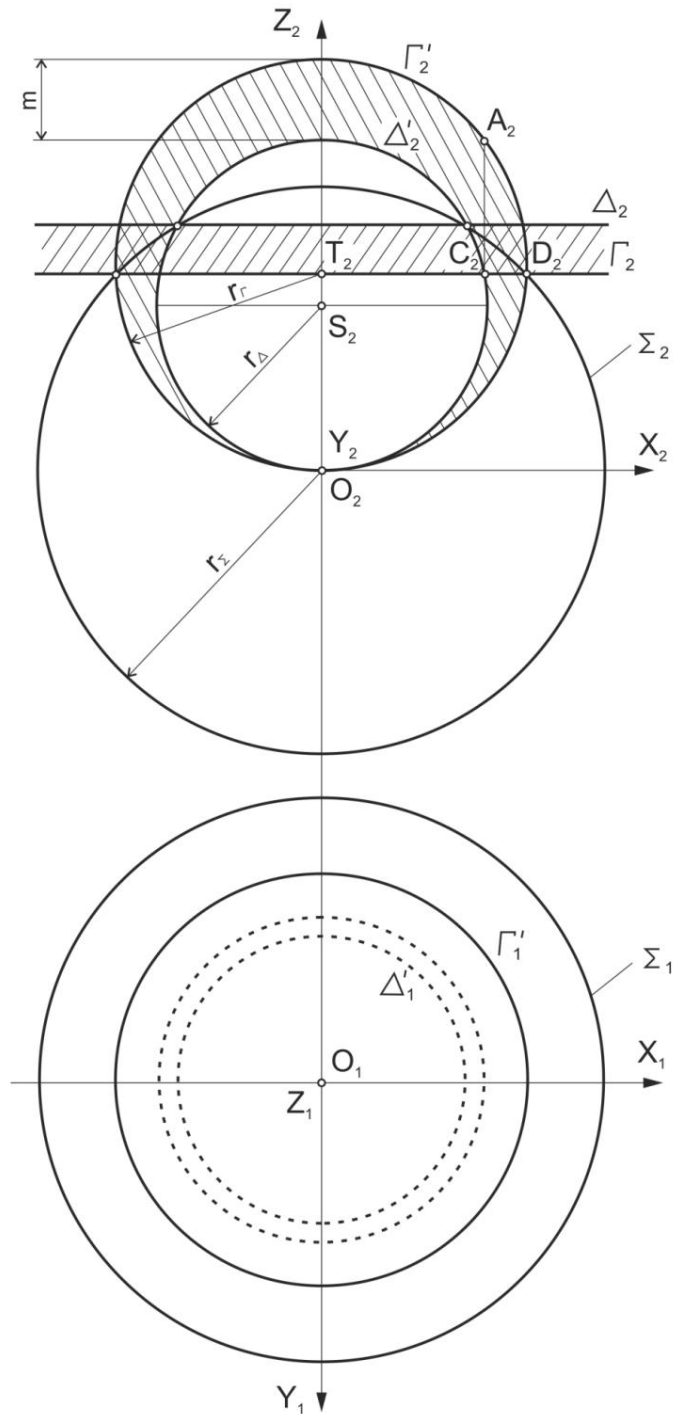

Fig. 1. Conversion scheme of the
inversion in two projections

With the reduction in the distance between the planes G and D form

the transformed surface will be closer to spherical, and the maximum thickness (m) zone between Γ' *and* Δ' the spheres is:

$$m = \frac{r^2(z_\Delta - z_\Gamma)}{z_\Delta z_\Gamma}, \tag{2}$$

where Z_Δ, Z_Γ – this is the height of the most distal inversion of the points belonging to the spheres Δ' *ma* Γ'.

The smaller thickness area (m), the more geometric the shape of the surface, which is simulated, is closer to the field.

The maximum height of the reference circuit can be provided due to the high (A) and lower (C) points a kennel at the zone boundaries between the spheres Δ' *and* Γ'. It should be noted also that the surface that is being modeled will not be compiled. Special points will only stay on the circuit. The original data (the nodes of the reference circuit), which are placed in the area between the spheres by the formula (1) are transferred to the layer between the planes Γ and D , where a static-geometric method (SGM) selectable frame stretched mesh with no external load. Such a surface will be similar to the soapy film. Then, using the inverse transform on the transform formulas (3), which are fully reversible (1) obtain the desired surface with a given reference loop

$$x' = \frac{r^2 x}{x^2 + y^2 + z^2}, y' = \frac{r^2 y}{x^2 + y^2 + z^2}, z' = \frac{r^2 z}{x^2 + y^2 + z^2}. \tag{3}$$

The resulting surface should be close to the field. Example 1. Set the field Δ', $r_\Delta = 6.00$ (рис.1). The height of the highest point of the circui. A, $h_A = 2.00$. For a basic contour which is placed in zone (m) to select the arch, built of arcs of parabolas (Fig.2), which are deposited on the cylindrical surface (ω) that is adjacent to the field and is coaxial with the sphere. The base radius of the cylindrical surface $r_\omega = r_\Delta$.

This circuit is selectable in order to obtain a surface as close as possible to the curved surface of the sphere. For the center of inversion take a point O. Construct the device of inversion, as described above. For this, we write the equation of the sphere Γ', passing through the point a and find its radius:

$$r_\Gamma = \frac{r_\Delta^2 + (r_\Delta + h)}{2(r_\Delta + h)}, \tag{4}$$

where h – the height of the point A; r_Δ – the radius of the given sphere Δ'.

According to the formula (4) we get $r_\Gamma = 6.25$. Next step, find the radius of the base sphere S which passes through the point D, on the sphere Γ':

$$r_\Sigma = \pm\sqrt{2r_\Delta \cdot r_\Gamma} \ . \tag{5}$$

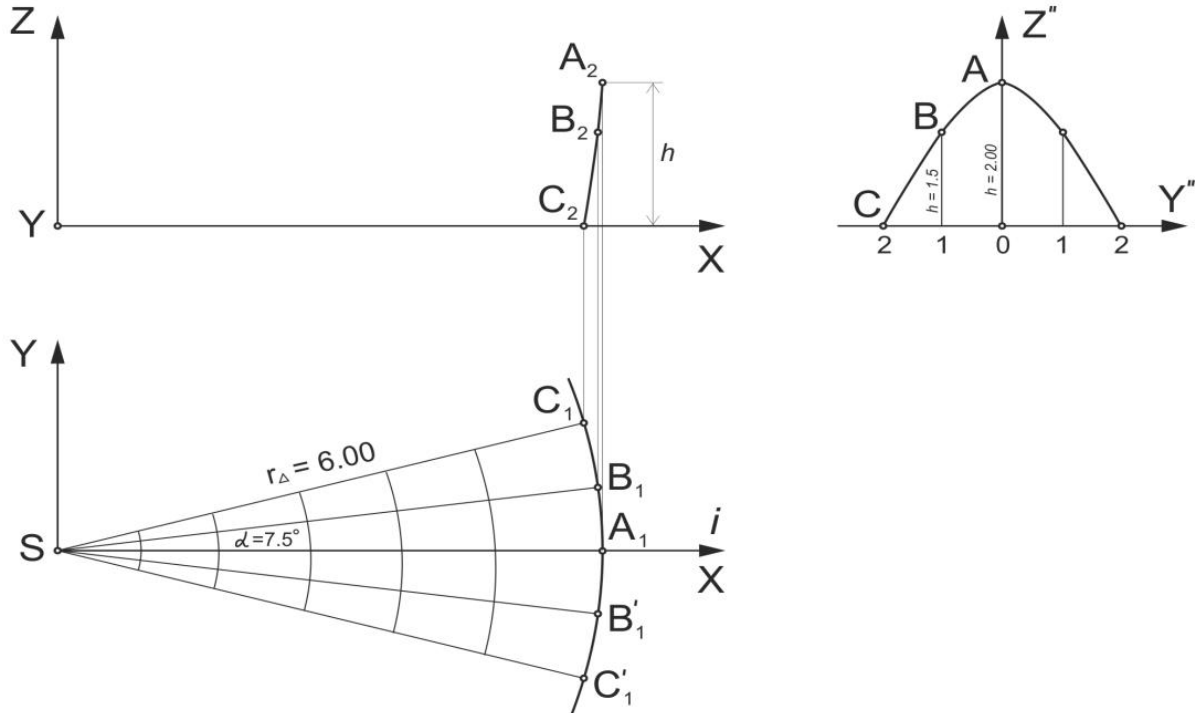According to the formula (5) we get $r_\Sigma = 8.6603$.

Fig.2. Scheme of reference of the contour of the surface

From Fig.2 shows that the coordinates of the point *A* (x_A=6.00; y_A=0.00; z_A=2.00) known, and the coordinates of the points B and C calculated according to the formulas:

$$x_B = r_\Delta \cos\alpha, y_B = r_\Delta \sin\alpha, x_C = r_\Delta \cos 2\alpha, y_C = r_\Delta \sin 2\alpha. \tag{6}$$

According to the formulas (6), we obtain: the coordinates of the point $B(x_B$=5.94; y_B=0.78; z_B=7.50), the coordinates of the point $C(x_C$=5.79; y_C=1.55; z_C=6.00).

According to the formulas (1) transformation of inversion and radius of the base sphere *S* ($r_\Sigma = 8.6603$)find the coordinates of images of the surface.

Selectable frame stretched mesh will be in the layer between planes $\boldsymbol{\Gamma - D}$. For meshing make up the system of equilibrium equations of the nodes (7) at the SGM formotorola without external load taking into account the symmetry of the output than would be secured the balance of the nodes in the plan.

Plan discrete frame surface in the layer between planes $\boldsymbol{\Gamma - D}$. shown in Fig.3.

53

$$x_{i-1,j} + x_{i+1,j} - 4x_{i,j} + x_{i,j-1} + x_{i,j+1} = 0;$$
$$y_{i-1,j} + y_{i+1,j} - 4y_{i,j} + y_{i,j-1} + y_{i,j+1} = 0;$$
$$z_{i-1,j} + z_{i+1,j} - 4z_{i,j} + z_{i,j-1} + z_{i,j+1} = 0; \qquad (7)$$
$$x_{i,j+1} = 0.5y_{i,j-1} + 0.866x_{i,j-1}; \; y_{i,j+1} = 0.5x_{i,j-1} - 0.866y_{i,j-1};$$
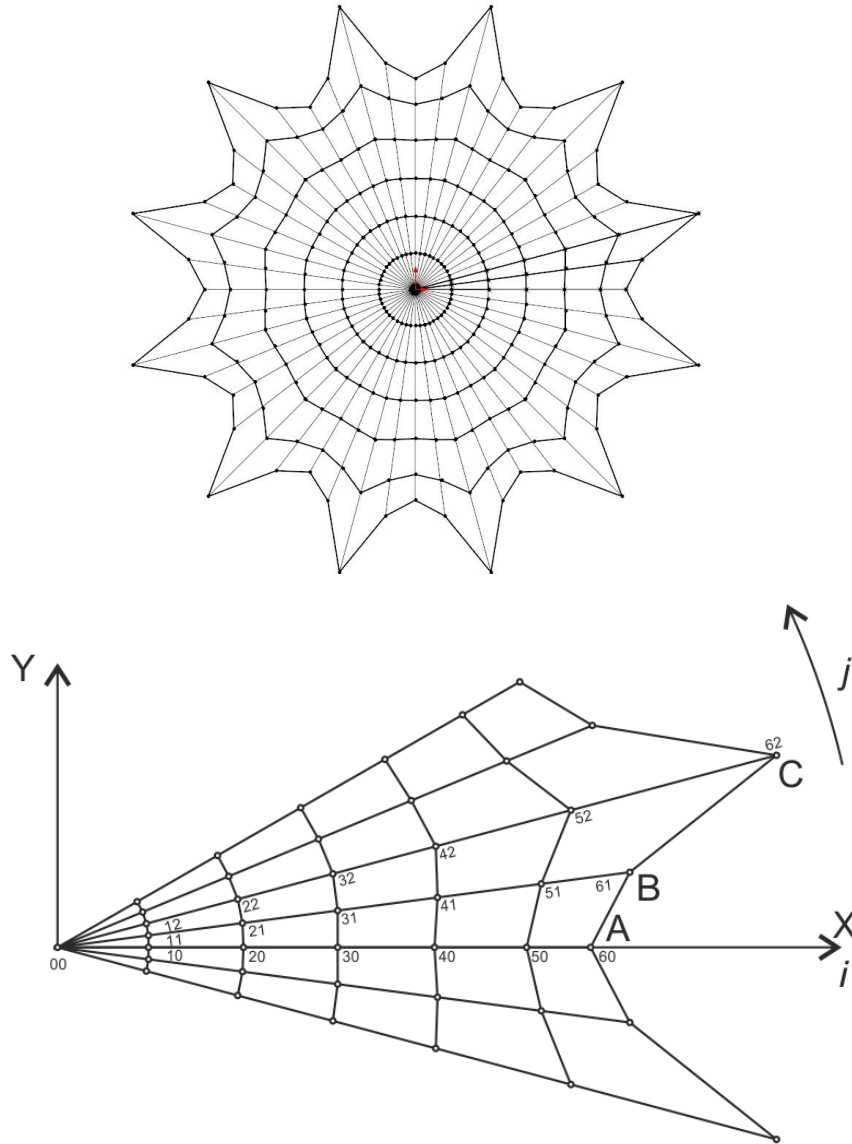$$48z_{i+1,j} - 48z_{0,0} = 0.$$

Fig. 3. The plan of the surface in the layer between the planes Γ – D
a) General view of the plan;
b) enlarged fragment of the 1/16 grid plan

Found coordinates of all points of the discrete grid are recalculated by the formula (3). The results of the calculations are presented in table 1. The results of all the coordinates of discrete grid generated using the SGM and using the inverse transform inversion, built discretely-presented surface (Fig. 4).

Table 1

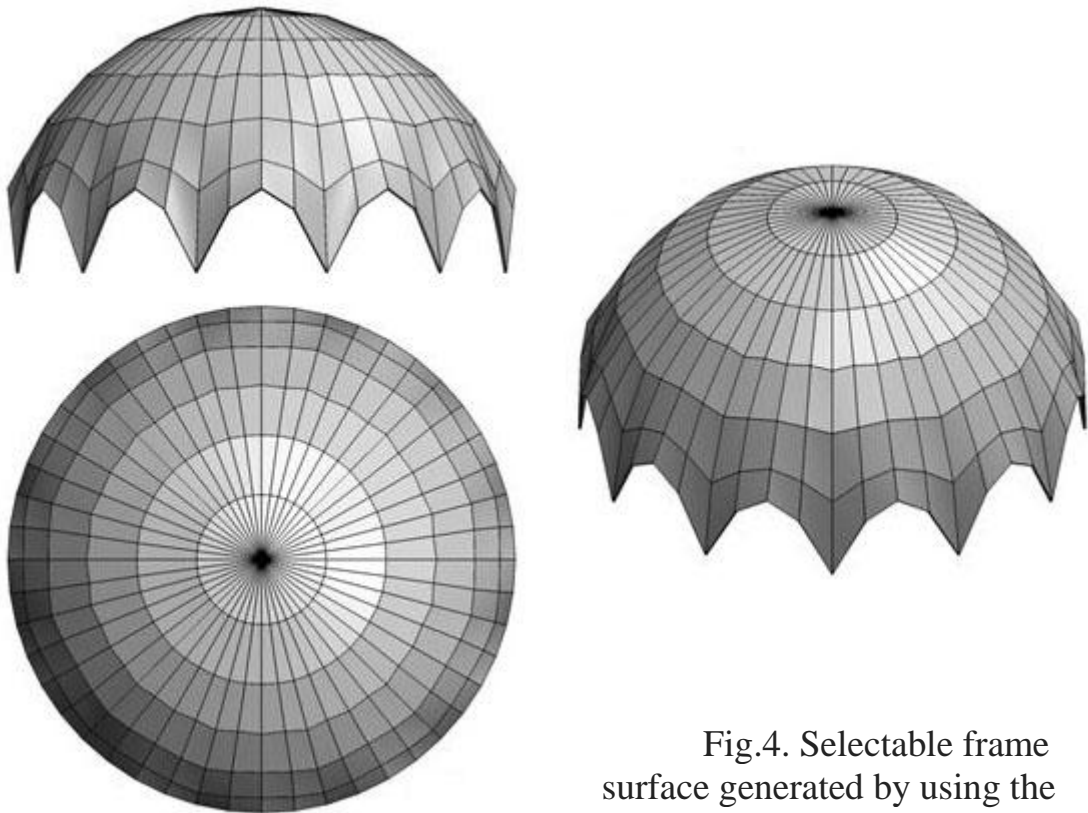| i=0	i=1	i=2	i=3	i=4	i=5	i=6	
x y z	1,4203 0.5867 12.0786	2.7317 1.1226 11.5155	3.8457 1.5768 10.6516	4.7076 1.90304 9.5836	5.2741 2.1288 8.4488	5.5433 2.2962 7.5001	j=3
	1.4869 0.39806 12.0775	2.8625 0.7661 11.5096	4.0381 1.0794 10.6240	4.9602 1.3211 9.4726	5.5274 14.8110 7.9229	5.7956 1.5529 6.000	j=2
	1.5234 0.2018 12.0786	2.9289 0.3903 11.5155	4.0677 0.5593 10.6975	5.0283 0.70561 9.5837	5.6253 0.7915 8.4391	5.9487 0.7833 7.5000	j=1
0.0000 0.0000 12.2743	1.5344 0.0000 12.0794	2.9476 0.0000 11.5207	4.1385 0.0000 10.6734	5.0409 0.0000 9.6598	5.6361 0.0000 8.6731	6.000 0.000 8.000	j=0

Fig.4. Selectable frame surface generated by using the spatial inversion

Conclusions. The proposed method of transferring the features of the surface geometry of the simulated prototype on the surface allows the designer or architect formally to discrete frame surface that corresponds to the initial conditions.

Similarly, you can use any other transformations that have the features to account for the properties of a form at design-design and architecture.

Literature
1. Ковальов С.Н. Геометричне моделювання поверхонь із заданими властивостями в дизайні та архітектурі / С.Н. Ковальов, С.И. Ботвіновська, А.В. Золотова // Управління розвитком складних систем. – 2016. – № 26.
2. Goggle. Вікіпедія [Електронний ресурс]: Режим доступу: www/URL: http://ru.wikipedia.org/wiki/Сфера/ - 06.05.2016 – Загл. з екрану.
3. Скопенко, В.А. Тентовая архитектура: «оригинальный эпилог» [Текст] / В.А. Скопенко // Академический вестник УралНИИпроектРААСН–ГРНТИ:67 – Строительство и архітектура, 2010. – № 40. – С. 89-95.
4. Удлер, Е.М. Проектирование тентових оболочек [Текст] / Е.М. Удлер, Е.Тостов // CAD master. – М., 2001. – № 1. – С. 43-47.
5. Казарян, Н.Л. Конструирование поверхностей на основе нелинейного преобразования пространства [Текст] / Н.Л. Казарян, В.В. Геворкян // Изв. НАН ОА и ГИУА. Сер. ТН., 2002. – Т.LV. – № 2. – С. 282-285.
6. Савелов, А.А. Плоские криве. Систематика, свойства, применение. (Справочное руководство) / А.А. Савелов. – М.: Государственное издательство физико-математической литературы, 1960. – 289 с.