# THIRD-DEGREE SPLINE WITH CONTROL POINTS OF THE CURVE INCIDENTNYMI

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***Summary. Splines are smooth but flexible curves, with great practical importance during constructing curvilinear forms and graphing. A method of constructing a third degree Lagrange polynomial-based spline with the first order of smoothness on points that incidental (belongs) a curve is proposed in this article.***

***Keywords:*** **polynomial in the form of Lagrange polynomial curve segment, the control points, the curve points are incident.**

***Formulation of the problem.*** For modelling smooth curves, surfaces and solids in constructing engineering is implemented a way of presenting vector-parametric curves in the form of Ferguson and Bezier or Bernstein-using rational vector-parametric functions. But, if you need to work only with points on the curve, these methods are not quite convenient because frame middle point in Bernstein form-Béziers don't lie on the newly specified bitmap frame, and in the final form is not on the curve. Sometimes it is not very convenient to work with curves in the form of Ferguson from having to calculate derivatives at anchor points. Invited to view vector-parametric curve in the form in which the frame will belong primarily to the specified bitmap row and lie on the desired curve.

***Analysis of recent research.*** In the [1-10] provides ways to describe cubic splines. In [3.10] describes the spline polynomial segments based on Lagrangian method.

***The article goals.*** Proposed to view vector-parametric curve in the form in which the frame will belong primarily to the specified bitmap row and lie on the desired curve. Develop this idea further to get cubic splines. For a study of this way of presenting the take for the beginning of polynomial curves of the third, fourth and fifth degrees, applying the Lagrange interpolation formula.

***Main part***. In [8, 9] it is shown that to obtain the formula for the polynomial with the desired properties apply Lagrange's interpolation polynomial according to y=y(x).

Assign parameter $\mathbf{u = (x-x_0)/(x_N-x_0)}$.

$$y=\sum_{i=0}^{N}[y_i\prod_{\substack{j=0\\ j\neq i}}^{N}\frac{(u-u_j)}{(u_i-u_j)}]. \tag{1}$$

If you take the uniform location of points **u0 =0, u =1, u =i/N,** the formula (1) will be:

$\mathbf{u_0 =0, u_N=1, u_i =i/N}$, то формула (1) будет иметь вид:

$$y = \sum_{i=0}^{N} [y_i \prod_{\substack{J=0 \\ j \neq i}}^{N} \frac{(Nu - j)}{(i - j)}]. \tag{2}$$

Thus, formulas (1) and (2) determine a polynomial curve with points belonging to the curve.

Substitute specific values of the parameter **u.** Assign at these points the value of the parameter **u: u =0, u =1/3, u =2/3, u =1**, which will correspond to a uniform arrangement of points. Will get:

$$y = y_0 + (-\frac{11}{2} y_0 + 9y_1 - \frac{9}{2} y_2 + y_3)u + (9y_0 - \frac{45}{2} y_1 + 18y_2 - \frac{9}{2} y_3)u^2 +$$
$$+(-\frac{9}{2} y_0 + \frac{27}{2} y_1 - \frac{27}{2} y_2 + \frac{9}{2} y_3)u^3. \tag{3}$$

Curve (3), after the reduction of the coefficients, written as:

$$y = \frac{9}{2}[y_0(1-u)(\frac{2}{3}-u)(\frac{1}{3}-u) + 3y_1(1-u)(\frac{2}{3}-u)u + 3y_2(1-u)(u-\frac{1}{3})u +$$
$$+y_3(u-\frac{2}{3})(u-\frac{1}{3})u]. \tag{4}$$

Thus, all four points lie on a curve in the limits **u =0, u =1.** In this case, assigned to a specific parameter value **u** at each point that correspond to a uniform location.

**Third-degree spline with control points, incident curve**

*Local spline third-order polynomial (defect 2)*

The question arises of joining cubic curves with control points, incident curve in the simulation (the design) smooth spline curve. Take the first derivative of (3) in the x direction. Will get:

$$y'_x = y'_u u'_x = y'_u (\frac{x - x_0}{x_3 - x_0})'_x = \{(-5.5y_0 + 9y_1 - 4.5y_2 + y_3) +$$
$$+ (18y_0 - 45y_1 + 36y_2 - 9y_3)u - 13.5(y_0 - 3y_1 + 3y_2 - y_3)u^2\} \frac{1}{h}, \tag{5}$$
$$h = x_3 - x_0.$$

Consider the intersection of two cubic segment $\mathbf{S^{(0)}}$**,** $\mathbf{S^{(1)}}$ [8].

In the previous segment $\mathbf{S^{(0)}}$ at **u=1** (point $\mathbf{3^{(0)}}$) will get derivative:

$$y'^{(0)}_{x(u=1)} = \{-y_0^{(0)} + 4.5y_1^{(0)} - 9y_2^{(0)} + 5.5y_3^{(0)}\} \frac{1}{h^{(0)}}. \tag{6}$$

In the next segment $\mathbf{S^{(1)}}$ при **u=0** (точка $\mathbf{0^{(1)}}$) we have derivative:

$$y'^{(1)}_{x(u=0)} = \{-5.5y_0^{(1)} + 9y_1^{(1)} - 4.5y_2^{(1)} + y_3^{(1)}\} \frac{1}{h^{(1)}}. \tag{7}$$

Try to equate (6) to (7). If replace the upper index **0, 1** to **i-1, i**, will get the system:

$$\frac{1}{h^{(i-1)}}[4.5y_1^{(i-1)}-9y_2^{(i-1)}]+\frac{1}{h^{(i)}}[4.5y_2^{(i)}-9y_1^{(i)}]=$$
$$=\frac{1}{h^{(i-1)}}[Y_0^{(i-1)}-5.5Y_3^{(i-1)}]+\frac{1}{h^{(i)}}[Y_3^{(i)}-5.5Y_0^{(i)}],\ i=1,...,N-1. \qquad (8)$$

In the right part (8) $\mathbf{Y_0^{(i-1)}, Y_3^{(i-1)}= Y_0^{(i)}, Y_3}^{(i-1)}$ – this set of nodal points of the interpolated curve. In the left part four unknown point y1(i-1), y2(i-1), y1(i) y2(i). If you ask any three points, the fourth will be determined from (8). These two segments will be joined at the point Y3(i-1)≡Y0(i) in such a way that the tangent to them will be the only one that is docked will be from the first order smoothness.

*The algorithm for designing local cubic spline with a first order smoothness.*

Based on the analysis of formula (8) can propose the algorithm for designing local spline of the third degree with the smoothness of the first order.

Imagine you are given N+1 points of 0, 1,..., N. On the section 0-1 define two intermediate points, and another on the site 1-2. On the basis of (8) will be determined the first two segments of the curve. Further on the area 2-3 need to specify only one point. The second will be determined from the formula (8). Thus we define and segments 2-3. The algorithm for segment 3-4 is repeated similarly to the area 2-3, etc. Therefore, additional points at each site, starting from the second, to determine recurrent dependence:

$$y_2^{(i)}=\frac{h^{(i)}}{h^{(i-1)}}[2y_2^{(i-1)}-y_1^{(i-1)}+\frac{2}{9}Y_0^{(i-1)}-\frac{11}{9}Y_3^{(i-1)}]+$$
$$+2y_1^{(i)}+\frac{2}{9}Y_3^{(i)}-\frac{11}{9}Y_0^{(i)},$$
$$h^{(i-1)}=x^{(i)}-x^{(i-1)},\ h^{(i)}=x^{(i+1)}-x^{(i)},$$
$$i=1,...,N-1. \qquad (9)$$

The algorithm can be applied to the two ends. It is implemented in the system of Auto CAD with Auto LISP language [9]. Test case submitted on Pic.1.

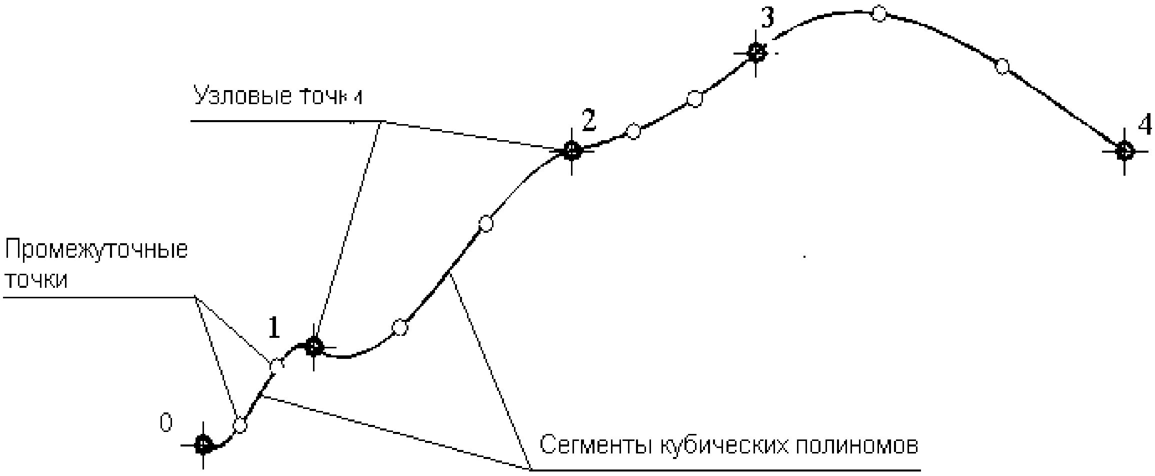


Рис.1. Test example of a cubic spline
with first order smoothness

**Conclusions.** The proposed representation of the vector-parametric curve in the form in which frame the point will belong to the initially specified point to a number and lie on the desired curve. It is proposed to develop this idea further to obtain the cubic splines and splines of higher degrees, particularly the fourth and fifth degree. To study this method of representation is taken, for the beginning of the polynomial curves of third degree, using the Lagrange formula for the interpolation polynomial.

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