GEOMETRIC SCHEMATIC MODELING OF PROSTHETIC FOOT

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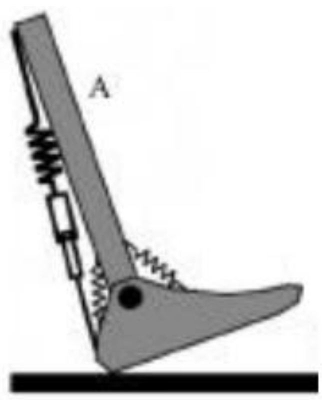
Summary. The proposed scheme of the artificial foot with the springs and the method of calculating values of coefficients of rigidity depending on the angles and lengths of design elements, and values of masses concentrated in the nodes of the structure.

***Key words*: prosthetic foot with springs, Lagrangian, Lagrange's equations of the second kind, the program Maple, the numerical method Runge-Kutty.**

Formulation of the problem. To create an anthropometric prosthetic limbs necessary for scientific computing, including geometric modeling in time steps of prosthetic products. In this work is analyzed the scheme of a geometric model of the prosthetic foot. There is a significant amount of development of these implants [1-5], which are different mechanical schemes and electrical engineering. The most convenient are the prostheses in the form of mechanical structures where it is possible to take into account the individual anthropological data of the user.

Analysis of recent research. The most common are mechanical prosthetic foot with a built-in titanium adapter, the elastic properties of which are provided with S-shaped element. To enhance the effect, in the form of "Shoe" enter special foam material. For user settings such prosthesis is reduced [1] to the selection of S-shaped titanium elements (which, for the average user is high).

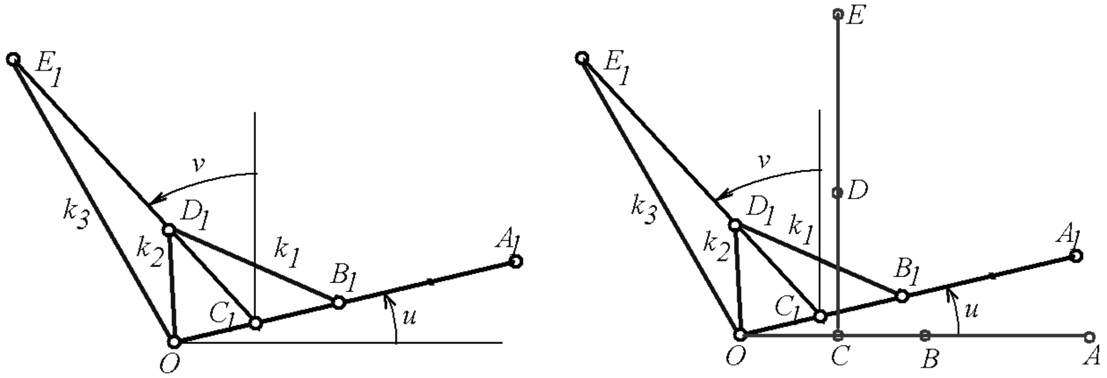
More technologically advanced will be prosthetic foot with springs. To customize the anthropometric parameters of this type of prosthesis it is necessary to have a set of springs. In [2,3] calculations given the idea of the scheme of the prosthetic foot with spring elements (Fig. 1). In [4,5] consider more complex schemes of mechanical prosthetic foot with springs. Calculated the proportions of the elements of the prosthesis taking into account the constant stiffness of the springs. From the results of work [1-5] it follows that the implementation of the anthropometric parameters of mechanical prostheses necessary be able to calculate the stiffness of springs interconnected depending on the parameters of the



Pic. 1 - The scheme of the prosthesis with springs elements [2,3] scheme design. This is an open question calculate the mechanical model of the prosthetic foot with spring elements.

The article goals. To develop the outline design of the artificial foot with the springs and method of calculation of the values of their stiffness depending on the angles and lengths of design elements, and values of masses concentrated in the nodes of the structure.

Main part. The main part. In Fig. 2A shows the proposed scheme of the prosthetic foot with spring elements. Here adopted designation: B1D1, OD1, OE1 – spring stiffness, respectively, k1, k2, and k3. The lengths of linear elements: OA1 = d1; OB1 = d1/2; OC1 = d1/4; C1D1 = d2; C1E1 = d3. Two angular parameters u and v are considered as generalized coordinates. At the point C1, the concentrated mass m1, and the A1 point – mass m2.



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Pic. 2. The scheme of the prosthetic foot with spring elements.

To determine the dynamics of motion of the prosthetic foot is necessary to determine the kinetic and potential energy of the system.

The formula to describe the kinetic energy has the form:

$$K = \frac{m_1\,(0.5\cdot OA)^2\,\dot{u}^2}{2} + \frac{m_1 OA^2 \dot{u}^2}{6} + \frac{m_2\,(0.5\cdot CE)^2\,\dot{v}^2}{2} + \frac{m_2 CE^2 \dot{v}^2}{6}. \quad (1)$$

To describe the potential energy use Fig.2, b, which, compared to Fig. 2, and made addition to the "stable" position of the system. Because of this it is not difficult to calculate the effect of spring constants on the deformation of the whole system according to the angles u and v as generalized parameters.

The formula to describe the potential energy has the form:

$$P = P_1 + P_2 + P_{12}, \quad (2)$$

де $P_1 = -0.5 m_1 g \cdot OA \cdot \sin u$; $P_2 = -0.5 m_2 g\,(OA \sin u + OE \sin v)$;

$$P_{12}=\frac{k_1\left(DB_1-DB\right)^2}{2}+\frac{k_2\left(OD_1-OD\right)^2}{2}+\frac{k_3\left(OE_1-OE\right)^2}{2}.$$

Тут

$$DB=\sqrt{(OC-OB)^2+CD^2};\ OD=\sqrt{OC^2+CD^2};\ OE=\sqrt{OC^2+CE^2};$$

$$DB_1=\sqrt{\left(OC\cos u-CD\sin v-OB\cos u\right)^2+\left(OC\sin u+CD\cos v-OB\sin u\right)^2};$$

$$OD_1=\sqrt{(OC cosu-CD sinv)^2+(OC sinu+CD cosv)^2};$$

$$OE_1=\sqrt{\left(OC\cos u-CE\sin v\right)^2+\left(OC\sin u+CE\cos v\right)^2}.$$

With the help of Lagrangian L=K–P [8] to determine in advance a given point in time the relative position of circuit elements of the prosthesis was composed of a system of Lagrange equations of the second kind. The solution of the system is made using the numerical method Runge-Kutty with the initial conditions u(0)=u0, u□(0)=Du0, v(0)=v0, v□(0)=Dv0.

We present the calculation of the stiffness coefficient k3 of the spring OE1 (correlated with k1 and k2) depending on the other constant circuit parameters of the prosthesis. We assume that by the choice of the values of k3 the movement of the circuit elements of the prosthesis should not happen randomly.

We choose the initial values of the generalized coordinates u0=-Pi/12; Du0 = 0; v0 = Pi/2 + Pi/20; Dv0 = 0, and the values of the parameters (all in units):

d1 = 0.35 – cut’s length OA1; d2 = 0.30 – length cut C1D1;

d3 = 0.75 – length of the segment C1E1;

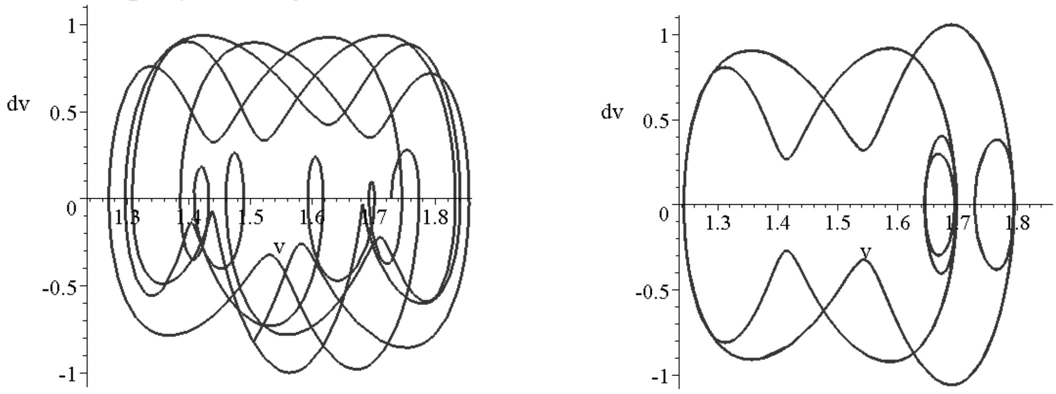
m1 = 70 - weight at the point C1; m2 = 0.4 is the mass at point A1;

k1 = 50 - the stiffness coefficient of the spring B1D1;

k2 = 50, the stiffness coefficient of the spring OD1;

Approximately solve the system of Lagrange equations of second kind with the given initial values of generalized coordinates and build in the phase space {v, Dv, t} the set of points belonging to integral curve. After combining consecutive points by line segments will receive an approximate image of the integral curve. This image will depend on the specific values of the "control" parameter (in our case, k3). When random values of k3 in the phase space {v, Dv, t} formed "confusing" integral curve, its projection on the phase plane {v, Dv} are also "confused" phase trajectory (Pic. 3, a).

In the case of changing the values of k3 should be changed and the character of phase trajectories. At a certain (critical) value of k3 the nature of the phase trajectories of change on a qualitative level – it will turn into a "natural" curve (Fig. 3, b). Dynamics will be observed if the optical effect of prompting on sharpness" confusion phase trajectories in the plane {v, Dv}. Thanks to this analogy [6,7] finding critical values of parameters *mentioned projection focus*.



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Pic. 3. Phase trajectories in the plane {v, Dv} if k1=50 k2= 50:
a) for any values of k3; b) the value of k3=110,2

In this example, with k1=50, k2= 50, the critical value of the stiffness coefficient of the spring will OE1 k3 = 110,2. For comparison, at Pic. 4, and the phase trajectory, if k1=100; k2= 100, the value of k3=of 157.2. In Fig. 4, b shows the same with k1=150 and k2=150 for values of k3=165,6.

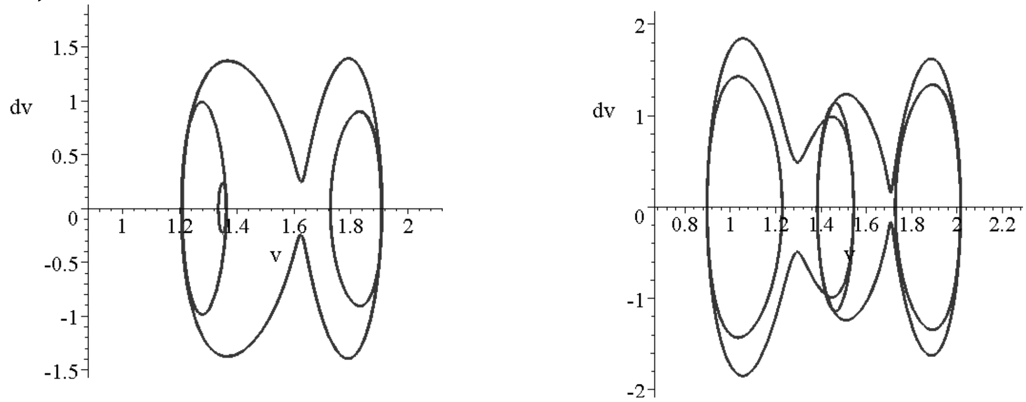

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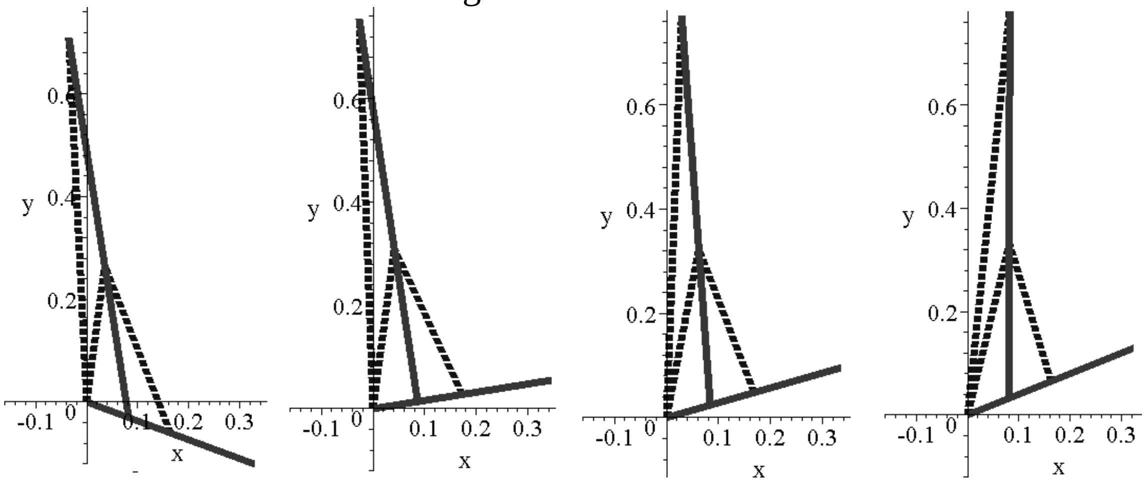
Рис. 4 - Phase trajectories in the plane {v, Dv}
a) if k1=100; k2= 100; k3=of 157.2;
b) if k1=150; 2= 150; k3=165,6

Computer experiments showed that for k1=200, k2= 200 it is not possible to prevent the chaotic movement of the circuit elements of the prosthesis by selecting the value of the parameter k3. In addition, three of the above options should prefer the option with the parameters k1=100; k2= 100 and k3=of 157.2 (Fig. 4, a). This is due to the minimal area of phase trajectory is estimated by the number of pixels that make it up. In Fig. 5 shows a few frames of the animation motion model of the foot at certain points of time t for the values of the coefficients of stiffness k1=100; k2=100 and k3=of 157.2. With an animated film you can clearly see that based on the value k3=of 157.2 in the process of calculations of angles of u(t) and v(t) in the solution of the system of Lagrange equations of the second kind allows to negation movement of circuit elements.



t = 0 t = 0,5 t = 0,6 t = 0,8

Pic. 5. Frames in the motion model of the foot at time t
for the coefficients of stiffness k1=100; k2= 100 and k3=of 157.2

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Integral curves in the phase space {u, Du, t} will be "winding" on a cylindrical surface with generator parallel to the axis Ot. Therefore, the phase trajectories in the plane {u, Du} is stationary for an arbitrary value of the control parameter here is not considered.

Conclusions. Given method allows to calculate the mechanical model of the foot with spring elements by defining mutually acceptable values of the coefficients of stiffness of the springs depending on the angles and lengths of design elements, and values of masses concentrated in the nodes. Further studies will be aimed at setting acceptable ranges of parameters.

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