AUTOMATION OF THE TRANSITION FROM RECTANGULAR TO ISOMETRIC MESH ON SURFACES OF REVOLUTION

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Summary. It is developed a computer model of automated retask of surface of rotation isometric co-ordinate lines of symbol algebra of Maple.

***Key words:* surfaces of revolve, the coefficients of the first quadratic forms, isometric coordinate lines.**

Raising of problem. Basic property of isometric (too exactly, that isothermal) surfaces is that elementary cells them co-ordinate lines have a form of squares. Formation and research of isometric surfaces stipulates realization enough difficult differential and integral calculations. Therefore the issue of the day is development of corresponding computer tool that would allow to take off labour intensive analytical expositions with the aim of expansion of list of isometric surfaces.

Analysis of the last researches and publications. The usage of isometric nets was shown at the reflection of flat inscriptions on curvilinear forms [2] with maintenance of their conformalness (corners between lines).

Formulation of aims of the article. To work out a computer model in the environment of symbol algebra of Maple [1] constructions are on the surfaces of rotation of isometric co-ordinate lines.

Main body. Self-reactance equalization of surface of rotation:

$$\boldsymbol{R}(u,v)=\boldsymbol{R}[\varphi\ v\ \ \cos\ u\ ,\ \varphi\ v\ \ \sin\ u\ ,\psi\ v\ ], \qquad (1)$$

where: $\varphi\ v\ ,\psi\ v$ - self-reactance equalizations of flat curve (meridian);

$u\in\ 0;2\pi\ ,v\in R$ – independent parameters.

The first quadratic form of surface of rotation (1) looks like:

$$ds^2=\varphi\ v\ ^2du^2+\quad \frac{d}{dv}\varphi\ v\quad ^2+\ \frac{d}{dv}\psi\ v\quad ^2\quad dv^2, \qquad (2)$$

or:

$$ds^2=\varphi\ v\ ^2\quad du^2+\frac{\frac{d}{dv}\varphi\ v\quad ^2+\frac{d}{dv}\psi\ v\quad ^2}{\varphi\ v\ ^2}dv^2\quad . \qquad (3)$$

In order that a surface of rotation was isometric, it is necessary to pass to the first quadratic form in a next kind:

$$ds^2=g\ v\quad du^2+dw^2\ . \qquad (4)$$

For this purpose it is necessary to execute next transformations. Enter:

$$dw^2 = \frac{\left(\frac{d}{dv}\varphi(v)\right)^2 + \left(\frac{d}{dv}\psi(v)\right)^2}{\varphi(v)^2} dv^2, \quad (5)$$

from where:

$$w = \int \frac{\sqrt{\left(\frac{d}{dv}\varphi(v)\right)^2 + \left(\frac{d}{dv}\psi(v)\right)^2}}{\varphi(v)} dv. \quad (6)$$

It could not be always integrated the got expression (6), especially to get equalization $w = w(v)$. If it is succeeded, then to express the variable of v from equalization of w=w (v) is not always possible. The computer model of linear algorithm of analytical transformations (fig.1) was created for the different surfaces of revolve (1) with corresponding self-reactance set formative $[\varphi(v), \psi(v)]$.

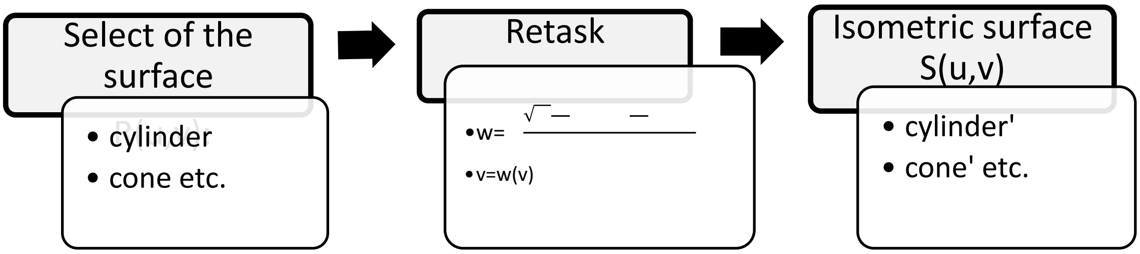


Fig.1. A sequence of passing to the isometric surface of revolve

The results of calculable experiments are grouped in a table.1, where it is given the name of surface $\boldsymbol{R}(u,v)$, self-reactance equalizations of formative $[\varphi(v), \psi(v)]$ surfaces, its the first quadratic form, self-reactance equalization of isometric surface, its the first quadratic form, image of surfaces $\boldsymbol{R}(u,v)$ i $\boldsymbol{S}(u,v)$.

At the construction of images of surfaces it is necessary to set both the parameters of form and the parameters of co-ordinate lines: u_o, u_n, v_o, v_n – initial and eventual value according to u and $v -$ co-ordinate lines; n_u, n_v – mount of square cells along u, v – co-ordinate lines of net. It is necessary to adhere of the condition, that a size of increase of both parameters of u and v was identical $ud = vd$. For the certain meaning u_o, u_n, n_u the size of increase of parameter u is:

$$ud = \frac{u_n - u_o}{n_u}, \quad (8)$$

from where the value of parameter of v is determined from expression:

$$v = v_o + vd \cdot j, \quad (9)$$

where $j = 0..n_v$ – sequence number of square cell along v- the co-ordinate line of isometric net.

Parameters of surfaces' forms are given in the table 1, were received $a = 2$ and $b = 4$.

Table 1

Equalization and image of isometric surfaces of rotation

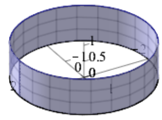
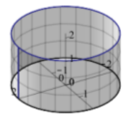
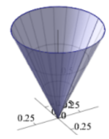
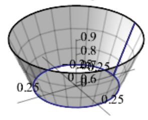
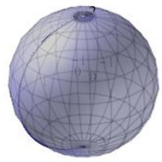
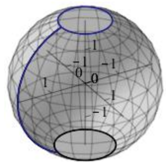
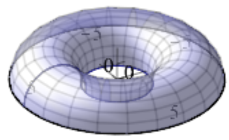
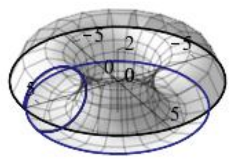
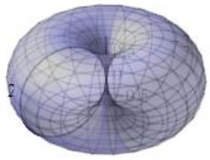
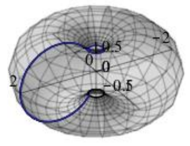
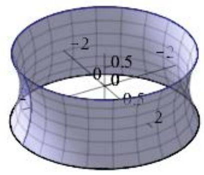
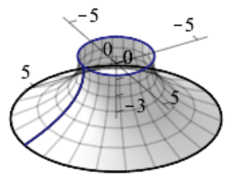
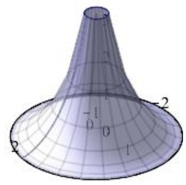
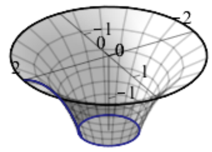
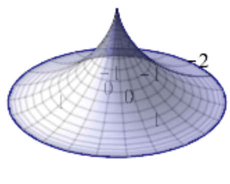
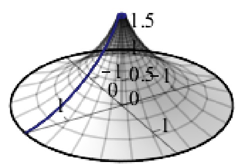
Cylinder	$\varphi(v) = a,\ \psi(v) = v$, where a - radius $\boldsymbol{R}(a\cos(u), a\sin(u), v)$ $ds^2 = a^2 du^2 + dv^2$ $\Downarrow$ $\boldsymbol{S}(a\cos(u), a\sin(u), a v)$ $ds^2 = a^2 (du^2 + dv^2)$	 
Cone	$\varphi(v) = \cos(a) v,\ \psi(v) = \sin(a) v$, where $a -$ angle of slope $\boldsymbol{R}(\cos(a) v\cos(u), \cos(a) v\sin(u), \sin(a) v)$ $ds^2 = \cos(a)^2 v^2 du^2 + dv^2$ $\Downarrow$ $\boldsymbol{S}(\cos(a) e^{\cos(a)}\cos(u), \cos(a) e^{\cos(a)}\sin(u), \sin(a) e^{\cos(a) v})$ $ds^2 = \cos(a)^2 e^{2\cos(a) v} (du^2 + dv^2)$	 
Sphere	$\varphi(v) = a\cos(v),\ \psi(v) = a\sin(v)$, де a - радіус $\boldsymbol{R}(a\cos(v)\cos(u), a\cos(v)\sin(u), a\sin(v))$ $ds^2 = a^2 v^2 du^2 + (a^2 + a^2) dv^2$ $\Downarrow$ $\boldsymbol{S}\left(\frac{2a e^v}{e^{2v}+1}\cos(u), \frac{2a e^v}{e^{2v}+1}\sin(u), \frac{a(e^{2v}-2)}{(e^{2v}+1)^2}\right)$ $ds^2 = \frac{4a^2 e^{2v}}{(e^{2v}+1)^2}(du^2 + dv^2)$	 
Top $b>a$	$\varphi(v) = b + a\cos(v),\ \psi(v) = a\sin(v)$ $\boldsymbol{R}((b + a\cos(v))\cos(u), (b + a\cos(v))\sin(u), a\sin(v))$ $ds^2 = (a\cos(v) + b)^2 du^2 + a^2 dv^2$ $\Downarrow$ $\boldsymbol{S}$ $\left((b + a\cos(2atan(\frac{\tanh(\frac{v\sqrt{a^2-b^2}}{2a})}{\sqrt{a^2-b^2}})))\cos(u),\right.$ $(b + a\cos(2atan(\frac{\tanh(\frac{v\sqrt{a^2-b^2}}{2a})}{\sqrt{a^2-b^2}})))\sin(u),$ $\left.a\sin(2atan(\frac{\tanh(\frac{v\sqrt{a^2-b^2}}{2a})}{\sqrt{a^2-b^2}}))\right)$	 

Table1 (continue)

<table>
<tr><td>Top $a = b = 1$</td><td>$\varphi(v) = 1 + \cos(v),\ \psi(v) = \sin(v)$
$\boldsymbol{R}((1 + \cos(v))\cos(u),\ (1 + \cos(v))\sin(u),\ \sin(v))$
$ds^2 = (\cos(v) + 1)^2\, du^2 + dv^2$
$\Downarrow$
$\boldsymbol{S}((1 + \cos(2\,\mathrm{atan}(v)))\cos(u),\ (1 + \cos(2\,\mathrm{atan}(v)))\sin(u),\ \sin(2\,\mathrm{atan}(v)))$
$ds^2 = \frac{1}{(v^2 + 1)^2}\, du^2 + dv^2$</td><td></td></tr>
<tr><td>Catenoid</td><td>$\varphi(v) = \sqrt{a^2 + v^2},\ \psi(v) = a\,\arcsin\left(\frac{v}{a}\right)$
$\boldsymbol{R}\left(\sqrt{a^2 + v^2}\cos(u),\ \sqrt{a^2 + v^2}\sin(u),\ a\,\arcsin\left(\frac{v}{a}\right)\right)$
$ds^2 = (a^2 + v^2)\, du^2 + dv^2$
$\Downarrow$
$\boldsymbol{S}\left(\frac{2a\,e^v\cos(u)}{e^{2v} + 1},\ \frac{2a\,e^v\sin(u)}{e^{2v} + 1},\ \frac{a\,(e^{2v} - 2)}{(e^{2v} + 1)^2}\right)$
$ds^2 = \frac{a^4 e^{-2v} + 2a^2 + e^{2v}}{4}\, du^2 + dv^2$</td><td></td></tr>
<tr><td>Pseudo sphere</td><td>$\varphi(v) = a\sin(v),\ \psi(v) = a\left(\cos(v) + \ln\left(\tan\left(\frac{v}{2}\right)\right)\right)$
$\boldsymbol{R}\left(a\sin(v)\cos(u),\ a\sin(v)\sin(u),\ a\left(\cos(v) + \ln\left(\tan\left(\frac{v}{2}\right)\right)\right)\right)$
$ds^2 = \frac{a^2\left((\cos(v)^2 - 1)^2\, du^2 + \cos(v)^2\, dv^2\right)}{\sin(v)^2}$
$\Downarrow$
$\boldsymbol{S}\left[\frac{a\cos(v)}{v},\ \frac{a\sin(v)}{v},\ \frac{a\left(\ln\left(\tan\left(\frac{1}{\sqrt{v^2 - 1}}\right)\right)v + \sqrt{v^2 - 1}\right)}{v}\right]$</td><td></td></tr>
<tr><td>Astroid</td><td>$\varphi(v) = a\cos\left(\frac{v}{4}\right)^3,\ \psi(v) = a\sin\left(\frac{v}{4}\right)^3$
$\boldsymbol{R}\left(a\cos\left(\frac{v}{4}\right)^3\cos(u),\ a\cos\left(\frac{v}{4}\right)^3\sin(u),\ a\sin\left(\frac{v}{4}\right)^3\right)$
$ds^2 = \frac{a^2\cos\left(\frac{v}{4}\right)^2\left(16\cos\left(\frac{v}{4}\right)^4 du^2 - 9\left(\cos\left(\frac{v}{4}\right)^4 - 1\right) dv^2\right)}{16}$
$\Downarrow$
$\boldsymbol{S}\left(\frac{27a\cos(u)}{(v - 3)^3},\ \frac{27a\sin(u)}{(v - 3)^3},\ \frac{729a\cos(u)}{(v - 3)^3}\right)$
$ds^2 = \frac{729a^2}{(v - 3)^6}\, du^2 + dv^2$</td><td></td></tr>
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Some equalizations are from a table.1 are given in the research [2]. Their comparison shows that systems of computer algebra, in particular Maple [1], not always can be automatically erect the analytical transformations to the most simple kind. In particular, equalization of torus with isometric co-ordinate lines with the parameters of form of a=b=1 looks like :

$$S\ u,v\ = S\ \frac{1}{v^2+1}cos\ u\ , \frac{1}{v^2+1}\ sin\ u\ , -\frac{v}{v^2+1}\ , \quad (9)$$

Which is different with the given table 1, as a system of Maple was not able to simplify expression $1 + cos\ 2\,atan\ v$ for more simple.

Conclusions. The worked out software in the environment of symbol algebra of Maple allows to automatize difficult analytical transformations for forming and analysis of isometric surfaces of revolve. For some surfaces automatically completed analytical transformations will not be the simplest, that needs to additional simplification. For the same values of independent u and v parameters of co-ordinate lines, their kind for some surfaces of revolve is differ, for example, on a cone, catenoid, pseudo sphere.

Literature:

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