MODELING OF SURFACE ROTATION ON PERSPECTIVE IMAGES

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Summary. In current article, authors performed a structural and parametric analysis of formation of revolution surfaces specified with axis and a wrapping cone, offered shaping algorithms created with use of computer tech.

Keywords: **surface of revolution, contours line, wrapping cone, computer-modeling technologies.**

Formulation of the problem. The role of the contour line is crucial to adequate perception of objects of three-dimensional virtual space on perspective images. But in the modern graphic systems of computer modeling there aren't any means of shaping the surfaces for such case. The design of different objects will be really effective from the side of aesthetics, if the graphic forming of their surfaces is realized on perspective images using the desired contour lines.

Analysis of recent researches. The building of surface rotation with perspectiveoutlines has been analyzed in this work[1]. A certain algorithm solves the problem, but has several disadvantages. First of all, to build each point of the meridian one should build the cone section. This algorithm may be used for modeling the surfaces with curved axis, but for surfaces of rotation it's necessary to use another algorithms.

Formulation of article purposes. It is necessary to analyze the problems and develop the most convenient algorithms of its solution that will satisfy the various options of initial terms and give the possibility for broad conclusions.

Main part. Modeling of design objects by their perspective lines is based on the outlines of objects and their properties of wrapping cones. Here are some of them [2], with obvious beneficial conclusions:

• The arbitrary surface of rotation of wrapping cone touches the spatial curve. In the rotation of second order the wrapping cone touches the flat curve of the second order.

• Each plane passing through the axis of rotation is the surface plane of symmetry. And with them, passing through the point of view S is the plane of symmetry of wrapping cone. All figures are marked as Δ. Perspective outline is a section of a wrapping cone by a picture plane. If the plane Δ is perpendicular to its poicture, perspective projection Δ ' will be the axis of symmetry of the line shape. This case will be called trivial. Otherwise, the user can set only half of the outline on one side of Δ '.

• In a certain picture of perspective outlines on one side of the line Δ ', wrapping cone is fully defined, because its other half can be constructed as symmetrical comparatively to the plane Δ.

• Each line with a straight set of twoparametric plane Δ can be chosen as the axis of surface rotation. Perspective outlines of all of these surfaces are the same, but the image of boundary circles (if there are any) will be different.

The outline can be given on a picture that is aligned with the plane of the screen by one of the methods that are available in graphical system of modeling. But we must remember that the problem of the outline is followed: at each point that will be used for building it's necessary to arrange the position of a tangent line. This is a separate problem that is solved in several ways, but it is not the subject of this article.

We show the rightfulness of such a claim.

Let's consider: the rotation of axis L is given; the point of view S does not belong to L; K crosses the plane Δ (S, L); the outline is clearly displayed on the projection axis on the picture; at each point of the line there is one tangent. Under these conditions, the surface of rotation is defined clearly.

But let's prove the following: if the rotation of axis of the surface is given, each point of contour lines with defined tangent sets one circle in a plane that is perpendicular to that axis.

The construction scheme is given in Fig. 1. It is supposed that on the line $p \subset K$ r⊂K there is a discrete set of points with tangents in them. The pair $\{P'_i, l_i\}$ and direct SP'_i form the plane G_i, which is tangent to the wrapping cone and the surface of rotation that is modeled. Axis L has a set of twoparametric circles in planes that are perpendicular to it. One-parameter set of one plane is tangent to $G_{i.}$ Points of contact belong to the straight, which is the intersection of the plane $G_{i.}$ and the plane that is carried through the axis L. Fig. 1 is a direct A_iB_i. The desired point must belong to a straight SP'_i line and A_iB_i. On the Fig. 1 there is the point of intersection P_i.

Thus, each pair $\{P'_i, l_i\}$ defines one circle of surface rotation. In outline p there is = a one-parameter set of pairs $\{P'_i, l_i\}$, because it specifies the surface rotation. This statement is proved.

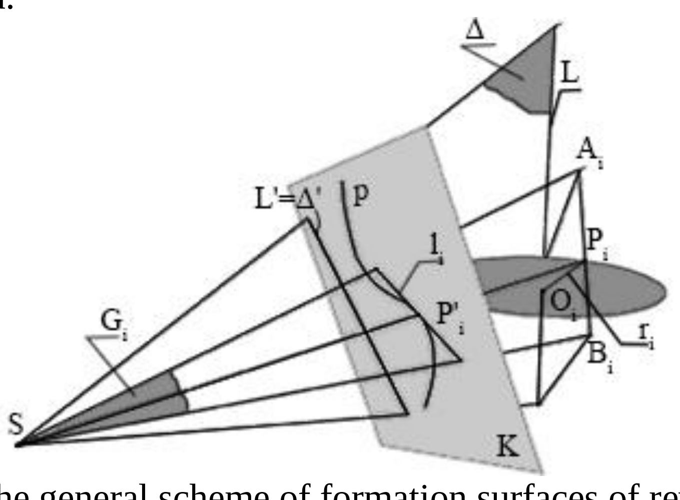

Fig.1. The general scheme of formation surfaces of revolution

The appearance of singular points on p must be investigated separately. But beforehand one can say: inflection points and points of the back of the first kind is not contrary to the statement; breaking point can cause a fracture surface and its appearance requires additional information for its elimination. Other features – points of the back of the second kind, selfcrossing and selftouch are contrary to the statement.

The need of the nontrivial system takes place in designing the objects, which consists of several surfaces with different planes of symmetry. In the case of the surface of the rotation their axis are diverhgent. This need arises in the static case, mainly, on the photos of real scenes or while preparing a virtual brochure. If one observes the scene in a real space, he explores the objects. This changes the position of pictures - they are perpendicular to the planes Δ.

Users of the system should provide both possibilities.

The point of view and contour line on any picture generates a wrapping cone. After its definition, from the geometric point of view, the position of the picture has no value and can be changed so that it is a trivial case. For this purpose new picture *W* should be perpendicular to the plane of Δ (Fig. 2).

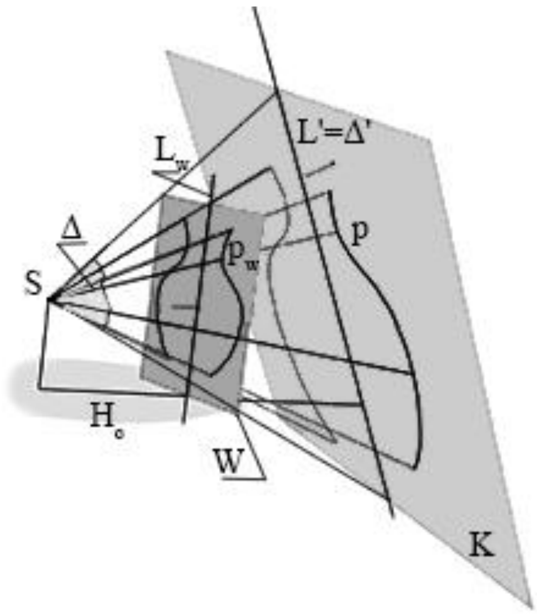

Fig.2. Bringing the perspective machine to the trivial case

The plane $W \perp \Delta$ has two free parameters, but their values are not significant and can be chosen free. For example, $W \| L$, or even coincide with it. Picture *W* has at least three applications:

1. It allows to symmatry the line p that is set on one side of Δ. For this purpose its projection on W is displayed to the axis L_W and turns back to the plane K.
2. Changing the position of picture, the user can see how will the contour lines look like, if the observer watches at the separate object.
3. All algorithms of solving the problems can be focused on a trivial case.

Everything said above is true not only for surfaces with a given axis of rotation, but also for any surfaces, planes of symmetry which pass through the point.

The scheme of modeling in bringing the apparatus of projection to the trivial case is given in Fig. 3. This plane *W* is parallel to the axis *L*.

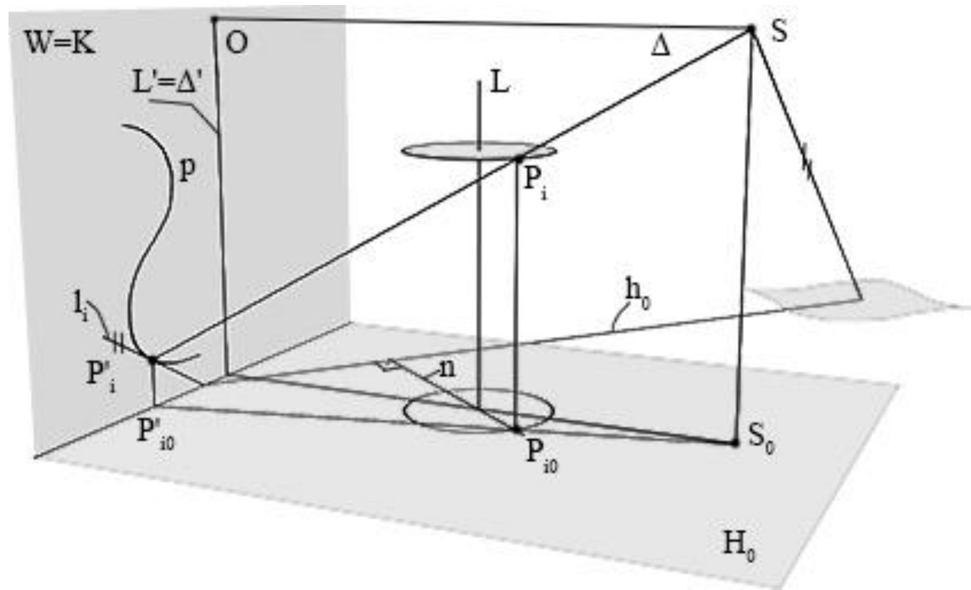

Fig. 3. Modeling of surface rotation in the trivial case

Let the plane H_0 to be an arbitrary plane that is perpendicular to the tangent plane L. Tangent plane G_i is defined as in the general scheme (Figure 1). Direct h_0 is the intersection of the plane with the plane H_0. Then circles at the point of contact to G_i have a tanget parallel to h_0. From the bases of axis we put perpendicular n to direct h_0. Its intersection with a straight $P'_{i0}S_0$. determines the point P_{i0} and radius of the desired circle. Then with the help of the back projection we find a point P_i on the line P'_iS, and the center of the circle on the line l at the same height.

Doing this procedure gradually $\{P'_il_i\}$ we get the set $\{P_i\}$, which defines the desired surface meridian of rotation.

Fig. 4 illustrated the technology of graphic computer modeling. Spatial axis of rotation was determined from preliminary from design considerations. On the picture given projection of axis and the user represent the desired shape of the surface of rotation (Figure 4 a).

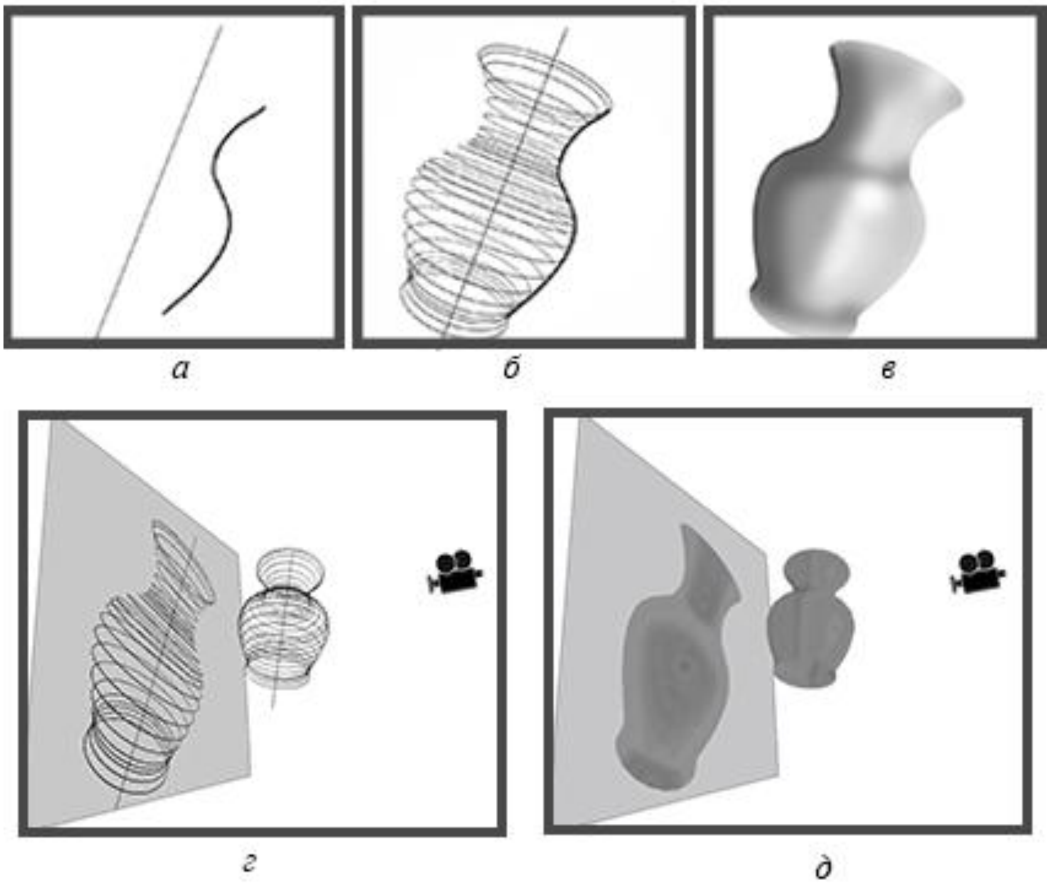

Figure 4. Sample of surface modeling for a given contour line

This information is sufficient to construct the surface. Figure 4b shows a frame of circles. Figure 1 shows that the surface of contour coincides with a previously set contour line, and Figure 4c shows a realistic image. Further it shows from which point of view the construction and location of the axis of rotation in the frame (Fig. 4d) and tinted (Fig.4d) images is carried out.

Conclusions. The properties and algorithms are given as baseline for modeling surfaces at difficult initial conditions, such as contour lines granted in full and the position of the axis should be defined. In addition, the shape of the surface of the rotation can be the outline of a set of surfaces, the plane of symmetry passes through the point of view.

Literature

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