ABOUT PROPERTIES OF THE OPERATION OF THE INTERNAL CONDENSATION IN FEM

H. Ya. Tuluchenko, N. Starun, T. Malomuzh

Summary. In this paper we prove the statement that the task of the minimizing the value of stiffness matrix trace for triangular finite elements Lagrange-type with third and higher orders and the task of the constructing harmonic bases for the same elements are equivalent tasks on the set of the functions which are obtained from the standard basis with the help of the internal condensation operation.

Keywords: **finite element method, operation of the internal condensation, trace of stiffness matrix.**

Formulation of the problem. Investigation of the effect of the stiffness trace the accuracy of the resulting solutions in finite element method (FEM) and study the usefulness of bases consisting of harmonic functions are carried out separately. The transition from standard bases for harmonic accompanied by a decrease in stiffness trace. But the results of a comparative analysis of solutions to both problems available in the literature were not found.

Analysis of recent research and publications. The question of building harmonious triangular bases for CE investigated in [1-3]. A feature of these studies is that they are considered CE and basis functions are given in the form of trigonometric series. The feasibility of using the value of trace element stiffness approximation for predicting properties of the base has in [4-5].

Formulation of article purposes. To prove the assertion that the use of the operation of internal condensation to basic functions lagrangian triangular CE of the third and higher order leads to the construction of the same basis in the task of minimizing the trace of hardness in the problem of building basis, consisting of harmonic functions (or functions, least evading harmonic).

Main part. Let triangular CE has n nodes on the sides and m internal nodes. We denote by N basis functions that are associated with external nodes on the sides, and the M - basic functions that are associated with internal nodes. Note that the expression of each basic functions associated with the internal node is composed of factors that match the triangle equation:

$$M_j = g_1(x;y)\,g_2(x;y)\,g_3(x;y)\,f_j(x;y), \qquad (1)$$

where $g_p(x;y)=0$ $(p=\overline{1;3})$ – the triangle equation; $f_j(x;y)$ – a power polynom.

As a result of the operation of internal condensation obtain new basis NC, whose functions are set expressions:

$$NC_i = N_i + \sum_{j=1}^{m} \alpha_{i,j} M_j\,, \tag{2}$$

where $\alpha_{i,j}$ – seeking parameters; $i=\overline{1;n}$; $j=\overline{1;m}$; for every *j* the equality $\sum_{i=1}^{n} \alpha_{i,j} = 1$.

Trace stiffness provided the identity matrix coefficients elastic medium condensed basis is calculated by the formula [4]:

$$trace(K^{(e)}) = \sum_{i=1}^{n} \iint_{\Omega} \left(\left(\frac{\partial NC_i}{\partial x} \right)^2 + \left(\frac{\partial NC_i}{\partial y} \right)^2 \right) dxdy. \tag{3}$$

Thus, the structure chosen for basic functions (2) the best basis for this is the one for which

$$T = trace(K^{(e)}) \to \min. \tag{4}$$

Functional (4) is the sum of positive terms, because it is the minimum value of the sum of the minimum values of each term, that is the task of minimizing the trace of hardness divided into n independent optimization problems for each of the basic function NC_i, $i=\overline{1;n}$:

$$T_i = \iint_{\Omega} \left(\left(\frac{\partial NC_i}{\partial x} \right)^2 + \left(\frac{\partial NC_i}{\partial y} \right)^2 \right) dxdy \to \min,\ i=\overline{1;n}. \tag{5}$$

To address them we substitute the expression of the desired basic functions NC_i (2) functionals of the form (5):

$$T_i = \iint_{\Omega} \left(\left(\frac{\partial NC_i}{\partial x} \right)^2 + \left(\frac{\partial NC_i}{\partial y} \right)^2 \right) dxdy =$$

$$= \iint_{\Omega} \left(\left(\frac{\partial}{\partial x} N_i \right)^2 + \left(\frac{\partial}{\partial y} N_i \right)^2 \right) dxdy +$$

$$+ \iint_{\Omega} \left(\left(\frac{\partial}{\partial x} \sum_{j=1}^{m} \alpha_{i,j} M_j \right)^2 + \left(\frac{\partial}{\partial y} \sum_{j=1}^{m} \alpha_{i,j} M_j \right)^2 \right) dxdy +$$

$$+ 2 \cdot \iint_{\Omega} \left(\frac{\partial}{\partial x} N_i \cdot \frac{\partial}{\partial x} \sum_{j=1}^{m} \alpha_{i,j} M_j \right) dxdy + 2 \cdot \iint_{\Omega} \left(\frac{\partial}{\partial y} N_i \cdot \frac{\partial}{\partial y} \sum_{j=1}^{m} \alpha_{i,j} M_j \right) dxdy.$$

Calculation of double integrals of the last two terms of the formula (6) using the method of integration by parts, taking into account the structure of the basic functions (1) leads to the expression:

$$\iint_{\Omega}\left(\frac{\partial}{\partial x}N_i\cdot\frac{\partial}{\partial x}\sum_{j=1}^{m}\alpha_{i,j}M_j\right)dxdy=-\iint_{\Omega}\frac{\partial^2 N_i}{\partial x^2}\cdot\sum_{j=1}^{m}\alpha_{i,j}M_j dxdy;$$

$$\iint_{\Omega}\left(\frac{\partial}{\partial y}N_i\cdot\frac{\partial}{\partial y}\sum_{j=1}^{m}\alpha_{i,j}M_j\right)dxdy=-\iint_{\Omega}\frac{\partial^2 N_i}{\partial y^2}\cdot\sum_{j=1}^{m}\alpha_{i,j}M_j dxdy.$$

Where have that

$$\iint_{\Omega}\left(\frac{\partial}{\partial x}N_i\cdot\frac{\partial}{\partial x}\sum_{j=1}^{m}\alpha_{i,j}M_j\right)dxdy+\iint_{\Omega}\left(\frac{\partial}{\partial y}N_i\cdot\frac{\partial}{\partial y}\sum_{j=1}^{m}\alpha_{i,j}M_j\right)dxdy=$$

$$=-\iint_{\Omega}\left(\frac{\partial^2 N_i}{\partial x^2}+\frac{\partial^2 N_i}{\partial y^2}\right)\cdot\sum_{j=1}^{m}\alpha_{i,j}M_j dxdy$$

Thus, the functional formula (6) finally look like:

$$T_i=\iint_{\Omega}\left(\left(\frac{\partial}{\partial x}N_i\right)^2+\left(\frac{\partial}{\partial y}N_i\right)^2\right)dxdy+$$

$$+\iint_{\Omega}\left(\left(\frac{\partial}{\partial x}\sum_{j=1}^{m}\alpha_{i,j}M_j\right)^2+\left(\frac{\partial}{\partial y}\sum_{j=1}^{m}\alpha_{i,j}M_j\right)^2\right)dxdy-$$

$$-2\cdot\iint_{\Omega}\left(\frac{\partial^2 N_i}{\partial x^2}+\frac{\partial^2 N_i}{\partial y^2}\right)\cdot\sum_{j=1}^{m}\alpha_{i,j}M_j dxdy.$$

Conclusions. Functionals T_i (7) different from the functional, leading to the construction of harmonic functions Ritz method, the presence of the first application $\iint_{\Omega}\left(\left(\frac{\partial}{\partial x}N_i\right)^2+\left(\frac{\partial}{\partial y}N_i\right)^2\right)dxdy$.

These functional minimization problem solved with the help of equations in which partial derivatives $\alpha_{i,j}$ of functional M_j for unknown weights features zero. The term, which is different these functionals contains unknown factors $\alpha_{i,j}$. Thus, the partial derivatives of it by coefficients equal to zero $\alpha_{i,j}$. Thus, in dealing with both functional optimization problems we get to the same system of linear equations that has a unique solution, and therefore, called the problem lead to the construction of the same basis.

Separately we note that in case of failure to obtain harmonic basis functions in the application operation of internal condensation (as is the case for example for triangular CE of order [6]), the problem of minimizing the functional of the Ritz method leads to the construction of basic functions that the least deviate from harmonic normal metric L_2 [7].

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