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SIMULATION OF ERMITOV'S SPLINE OF 5-TH ORDER BY POINTS WITH A GIVEN LAW CURVATURE

Y. Badayev, I. Gannoshina

Proposed definition of ermìtov's spline of 5-th order for the given points and law curvates in these points.

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Formulation of the problem. In designing the contours of machines and units that operate in a moving medium (aircraft, cars, etc.) it is important to outline objectives for a given change in curvature of the law. You must have an analytical device to solve this problem.

Analysis of recent research and publications. In [2-6] offers interactive design methods contours with a given shape and curvature, but does not allow you to predict results in early planning.

The wording of article purposes. The article is an analytical output device modeling curved contours on a predetermined curvature change the law, which is important for the design contours of planes, cars, ships, and so on.

Main part. Put the following tasks:

A given point on the plane row:

$$\Delta : x_i, y_i, i=0,1, \dots , n,$$

and at each point given curvature K_i.

It is known from [1] for the curvature of the curve $y = f(x)$ determined by the formula:

$$K^2 = \frac{y''^2}{(1+y'^2)^3} \tag{1}$$

As shown in equation (1) to set the curvature at a given point, you should at least ask first derivative y_i'. For a given first derivative y_i' and curvature K_i determined by the value of the second derivative

$$y_i''^2 = K_i^2 (1 + y_i'^2)^3 \quad \text{або} \quad \pm y_i'' = \pm K(1 + y_i'^2)^3 . \tag{2}$$

Thus the task turns to the next.

Given the number of point of the first and second derivatives of them

$$\Delta : x_i, y_i, y_i', y_i'', i=0, 1, \dots , n.$$

We model the curve which is constructed from segments of polynomials docked in areas $i \div i+1$. That must find a polynomial curve which at tracts $i \div i+1$ through the points i , $i+1$ and has raised these points in derivatives $y_i^{'}, y_i^{''}, y_{i+1}^{'}, y_{i+1}^{''}$.

First prove the following theorem.

In order polynomial segment passing through two given points 0 and 1 and had raised these points in the first and second derivatives, requires that a log was at least the 5th degree.

Such a system is determined polynomial 6-linear equations:

$$\begin{aligned} y(x_0) &= y_0 \\ y^{'}(x_0) &= y_0^{'} \\ y^{''}(x_0) &= y_0^{''} \\ y(x_1) &= y_1 \\ y^{'}(x_1) &= y_1^{'} \\ y^{''}(x_1) &= y_1^{''}. \end{aligned}$$

The system with 6-linear equations solved and has a solution only if the number of unknowns at least 6 and no more. Polynomial $y = f(x)$ has 6 coefficients only at 5 degrees.

Theorem proved.

So we look for a polynomial of 5th degree.

$$y = a + bx + cx^2 + dx^3 + ex^4 + fx^5 \quad . \tag{3}$$

Derivatives will be equal to:

$$y^{'} = b + 2cx + 3dx^2 + 4ex^3 + 5fx^4, \tag{4}$$
$$y^{''} = 2c + 6dx + 12ex^2 + 20fx^3. \tag{5}$$

Substituting in (3), (4), (5) the coordinates points $0(x_0, y_0)$ and $1(x_1, y_1)$ and derivatives at these points $y_0^{'}$, $y_0^{''}$, $y_1^{'}$, $y_1^{''}$. We have the following system of 6-linear equations

$$\begin{pmatrix} 1 & x_0 & x_0^2 & x_0^3 & x_0^4 & x_0^5 \\ 0 & 1 & 2x_0 & 3x_0^2 & 4x_0^3 & 5x_0^4 \\ 0 & 0 & 2 & 6x_0 & 12x_0^2 & 25x_0^3 \\ 1 & x_1 & x_1^2 & x_1^3 & x_1^4 & x_1^5 \\ 0 & 1 & 2x_1 & 3x_1^2 & 4x_1^3 & 5x_1^4 \\ 0 & 0 & 2 & 6x_1 & 12x_1^2 & 25x_1^3 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \\ e \\ f \end{pmatrix} = \begin{pmatrix} y_0 \\ y_0^{'} \\ y_0^{''} \\ y_1 \\ y_1^{'} \\ y_1^{''} \end{pmatrix} . \tag{6}$$

Solving this system gives us the coefficients a, b, c, d, e, f, which determine the polynomial (3) that meets the challenge.

Application polynomial is not very convenient also necessary to solve a system of 6-linear equations.

Therefore, to solve this problem, take Bezier curve segment of the 5th degree [2].

$$r = r_0(1-t)^5 + 5r_1(1-t)^4 t + 10r_2(1-t)^3 t^2 + 10r_3(1-t)^2 t^3 + 5r_4(1-t)t^4 + r_5 t^5. \tag{7}$$

Reconstruct (7) in the formula:

$$r = a + bt + ct^2 + dt^3 + et^4 + ft^5. \quad (8)$$

At point $r_0(x_0, y_0)$ t=0.

Get:

$$a = r_0\ ; \quad b = 5(r_1 - r_0); \quad (9)$$

$$c = 10\ \ r_0 - 2r_1 + r_2\ \ ; \qquad d = 10(r_3 + r_1 - r_0 - 3r_2); \quad (10)$$

$$e = 5\ \ r_0 - 4r_1 + 6r_2 - 4r_3 - r_4\ \ ; \quad (11)$$

$$f = \ -r_0 + 5r_1 - 10r_2 + 10r_3 + 5r_4 + r_5\ ; \quad (12)$$

$$r' = b = 5(r_1 - r_0)\ ; \quad (13)$$

$$r'' = 2c = 20\ \ r_0 - 2r_1 + r_2\ \ . \quad (14)$$

When given r_0' і r_0'' із (13) and (14) will have

$$r_1 = r_0 + \frac{r_0'}{5}; \quad (15)$$

$$r_2 = 2r_1 - r_0 + \frac{r_0''}{20}. \quad (16)$$

At point r_5 (t=1) we have the following equation:

$$r_4 = r_5 - \frac{r_5'}{5}\ ; \quad (17)$$

$$r_3 = r_5 + 2r_4 + \frac{r_5''}{20}. \quad (18)$$

Thus formula (13) - (18) completely define the segment Bezier 5th degree which passes through two points r_0 і r_5 with predetermined first and second derivatives of them.

Conclusions. The article is the result of analytical outline design for a given change in curvature of the law. Further studies will be conducted to develop management proektuyemoho form curved contours.

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