

THICKENING DISCRETELY GIVEN CURVE USING THE ACCOMPANYING TRIANGLE

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Proposed way of supporting the empowerment of triangles by developing an algorithm that allows the thickening point series with breakpoints, turnover, gap and so on.

Keywords: discrete condensed form interpolated (DPC), thickening, point equation, triangle, simplex, BN-point calculation.

Formulation of the problem. The basis of this research unit began work Naydysha V.M., Balyuby I.G., Vereschahy V.M., Naydysha A. [1-4]. For discrete submitted curve (KDP) and the cover band dyfproektsiy Triangle [5] have the same nature origin, equally well used in discrete variational geometric modeling. This article discusses the use of supporting triangles (UE) using methods BN-point calculation for thickening duodenum. This issue was developed in [6], the difference between our study is another representation of the parameters that uses a different geometry of numbers.

Analysis of recent research and publications. In [1.6] and this paper the problem of condensation curve discrete submitted using the accompanying triangles seen for the first time in many a point-BN, which deals with the development of Melitopol school of applied geometry.

Forming the article purposes. Enhance method of supporting triangles by developing an algorithm that will thicken point series with break point, turnover, etc. rupture.

Main part. For research first consider the geometric scheme of general form, which is shown in Fig. 1. simplex overall appearance should be understood that none of the points A, B, C , determining simplex CAB , does not coincide with the points D_i , defining discrete submitted curve: $D_{i-1}, D_i, D_{i+1}, D_{i+2}$.

For the solution of the problem is necessary from the duodenum to the first four points, then zrushtys one point and consider the following four points $D_i, D_{i+1}, D_{i+2}, D_{i+3}$. This process must be repeated until you get to the last point, determining the duodenum.

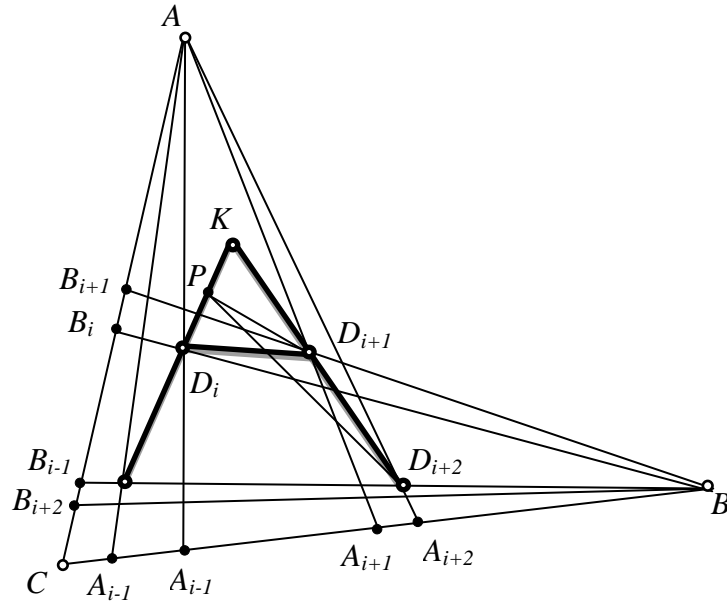


Fig. 1. Scheme of condensation on the overall appearance

In the proposed scheme (aig. 1) point KDP $D_{i-1}, D_i, D_{i+1}, D_{i+2}$ connected straight segments for clarity, no semantic load they carry. Coordinates all points $A, B, C, D_{i-1}, D_i, D_{i+1}, D_{i+2}$, as shown in fig. 1 are known and some are defined in the global coordinate system (not shown in figure 1, because you can take any). Since the point A, B, C identify local simplex CAB , then the other four points D_i is expressed through the points simplex. Spot equation that meet this requirement are as follows:

$$D=(A-C)p+(B-C)q+C. \quad (1)$$

In [6] as parameters p, i, q considered related triangles, the paper suggests to use related segments.

$$\text{For point } D_{i-1} : p_{i-1} = \frac{D_{i-1}A_{i-1}}{AA_{i-1}}; q_{i-1} = \frac{D_{i-1}B_{i-1}}{BB_{i-1}};$$

$$\text{for point } D_i : p_i = \frac{D_iA_i}{AA_i}; q_{i-1} = \frac{D_iB_i}{BB_i};$$

$$\text{for point } D_{i+1} : p_{i+1} = \frac{D_{i+1}A_{i+1}}{AA_{i+1}}; q_{i+1} = \frac{D_{i+1}B_{i+1}}{BB_{i+1}};$$

$$\text{for point } D_{i+2} : p_{i+2} = \frac{D_{i+2}A_{i+2}}{AA_{i+2}}; q_{i+2} = \frac{D_{i+2}B_{i+2}}{BB_{i+2}}.$$

From (1) and taking into account certain parameters that can record point equations for the mentioned points:

$$D_{i-1} = (A-C)p_{i-1} + (B-C)q_{i-1} + C; \quad (2)$$

$$D_i = (A-C)p_i + (B-C)q_i + C; \quad (3)$$

$$D_{i+1} = (A-C)p_{i+1} + (B-C)q_{i+1} + C; \quad (4)$$

$$D_{i+2} = (A - C)p_{i+2} + (B - C)q_{i+2} + C. \quad (5)$$

With four points (2) - (5) can build one point thickening $D_{i+0.5}$ in the triangle $\Delta D_i D_{i+1} K$. The first two points of the triangle D_i та D_{i+1} defined in simplex CAB (3) and (4). It is necessary to determine the last vertex of the triangle - point K . We find it as a direct result of crossing $D_{i-1} D_i$ та $D_{i+1} D_{i+2}$. To do this on the line $D_{i-1} D_i$ introduce changeable point P , point equation which takes the form (fig. 1):

$$P = (D_{i-1} - D_i)u + D_i. \quad (6)$$

By moving the point P a straight line $D_{i-1} D_i$ by changing the parameter u , it must choose a value to the area of a triangle $S_{D_{i+1} D_{i+2} P}$ опинилась рівною нулю, або іншими словами, щоб три точки P, D_{i+1}, D_{i+2} were aligned. By the condition of collinearity to point P owned direct $D_{i+1} D_{i+2}$ necessary in order determinant that reflects point line equation, zero. Putting this determinant, but first point in equation (6) point substitute equation (2) and (3) determining the point D_i та D_{i-1} .

$$P = (A - C)uq_{i-1} + (B - C)uq_{i-1} + Cu - (A - C)up_{i-1} - (B - C)uq_i - Cu + (A - C)p_i + (B - C)q_i + C = (A - C)(up_{i-1} - up_i + p_i) + (B - C)(uq_{i-1} - up_i + q_i) + C.$$

In the final version, convenient for further action are:

$$P = (A - C)(u(p_{i-1} - p_i) + p_i) + (B - C)(u(q_{i-1} - q_i) + q_i) + C. \quad (7)$$

Thus the determinant with which to form point line equation $D_{i+1} D_{i+2}$, which will be the point P , draw up the parameters of point equations (7) (4) and (5). The first row of the determinant variables point P , the second - a constant point settings D_{i+1} , in the third - D_{i+2} , we obtain:

$$\begin{vmatrix} u(p_{i-1} - p_i) + p_i & u(q_{i-1} - q_i) + q_i & 1 \\ p_{i+1} & q_{i+1} & 1 \\ p_{i+2} & q_{i+2} & 1 \end{vmatrix} = 0. \quad (8)$$

We expand

$$\text{it: } u(p_{i-1} - p_i)q_{i+1} + p_i q_{i+1} + u(q_{i-1} - q_i)p_{i+2} + q_i p_{i+2} + p_{i+1} q_{i+2} - u(p_{i-1} p_i)q_{i+2} - p_i q_{i+2} - u(q_{i-1} - q_i)p_{i+1} - q_i p_{i+1} - p_{i+2} q_{i+1} = 0.$$

From define the parameters:

$$u = \frac{p_{i+2} q_{i+1} + q_i p_{i+1} + q_i p_{i+2} - p_{i+1} q_{i+2} - q_i p_{i+2} - p_i q_{i+1}}{(p_{i-1} - p_i)q_{i+1} + (q_{i-1} - q_i)p_{i+2} - (p_{i-1} - p_i)q_{i+2} - (q_{i-1} - q_i)p_{i+1}}. \quad (9)$$

If the number of (9) equate to d , we obtain:

$$d = \frac{p_{i+2}(q_{i+1} - q_i) + p_{i+1}(q_i - q_{i+2}) + p_i(q_{i+2} - q_{i+1})}{(p_{i-1} - p_i)(q_{i+1} - q_{i+2}) + (q_{i-1} - q_i)(p_{i+2} - p_{i+1})}. \quad (10)$$

Substitute numbers d from (10) to (7), obtain a desired point K :

$$K = P = (A - C)(d(p_{i-1} - p_i) + p_i) + (B - C)(d(q_{i-1} - q_i) + q_i) \quad (11)$$

In triangle $\Delta D_i D_{i+1} K$, bring that its image in fig. 1 separate figure 2, calculate the point of thickening $D_{i+0,5}$. In [6] proved many possible choices thickening point, in our view, the simplest is the calculation of thickening as the center of the triangle $\Delta D_i D_{i+1} K$

$$D_{i+0,5} = \frac{D_i + D_{i+1} + K}{3}. \quad (12)$$

Formulas thickening point coordinates in the global coordinate system will be as follows:

$$\begin{aligned} x_{i+0,5} &= \frac{x_i + x_{i+1} + x_k}{3}, \\ y_{i+0,5} &= \frac{y_i + y_{i+1} + y_k}{3}, \\ z_{i+0,5} &= \frac{z_i + z_{i+1} + z_k}{3}. \end{aligned} \quad (13)$$

Thus, the above method of construction was considered a point thickening $D_{i+0,5}$ using formal geometric machine BN-point calculation. To build the next point thickening should zrushtyts one step and repeat the entire algorithm for the next four points and so on around a point number duodenum.

When performing the process of thickening of the duodenal every step of the simplex elected general view (points that define simplex, do not

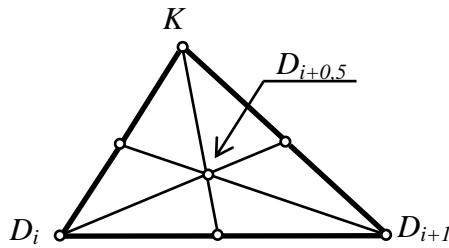


Рис. 2. Визначення точки загушення $D_{i+0,5}$

coincide with the points of the curve) with respect to the reference points point range, resulting formulas proved cumbersome and calculation formulas for these resources increases costs. Simplex general form can be called nulchastynnym.

This one will be called simplex simplex, top or one of the points that define it is the same point duodenum (fig. 3).

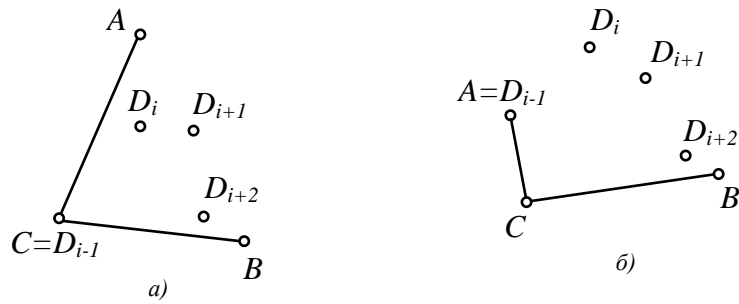


Fig. 3. Examples of one-simplex:
 a) – top $C=D_{i-1}$; б) point simplex $A=D_{i-1}$;
 в) point simplex $B=D_{i-1}$

It should be noted that these examples a), b), c) point simplex A, B, C as shown reflect the views D_{i-1} , but it is not necessarily possible falling of their singing other points KDP – D_i, D_{i+1}, D_{i+2}

Dvohchastynnym simplex simplex will be called when the two points that define it is the same in different variations with points duodenum (fig. 4).

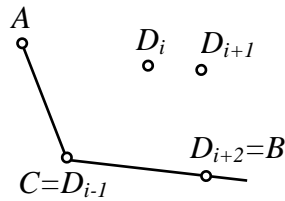


Fig. 4. Example dvohchastyn simplex

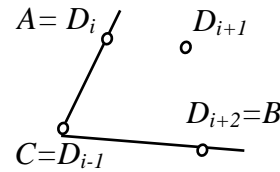


Fig. 5. Three-part simplex

Three-part simplex simplex will be called when all three points that define it is the same variations in butcher with points duodenum (fig. 5).

We show how simplified compared using simplex general form, accounts for one-, two-, and three-part Simplex.

For example, consider the one-simplex, shown in Fig. 3a), whose top simplex - point C coincides with the point D_{i-1} discrete submitted curve.

Because of this, spot equation (2) will look like:

$$D_{i-1} = C. \quad (14)$$

Then point equation (6) will look like:

$$P = Cu - (A - C)up_i - (B - C)uq_i - Cu + (A - C)p_i + (B - C)q_i + C = (A - C)p_i(1 - u) + (B - C)q_i(1 - u) + C. \quad (15)$$

Key (8) takes the form:

$$\begin{vmatrix} 1-u & 1-u & 1 \\ p_{i+1} & q_{i+1} & 1 \\ p_{i+2} & q_{i+2} & 1 \end{vmatrix} = 0. \quad (16)$$

We expand this caller and remove the parameter value u :

$$\begin{aligned} (1-u)q_{i+1} + (1-u)p_{i+2} + p_{i+1}q_{i+2} - p_{i+2}q_{i+1} - (1-u)q_{i+2} - (1-u)p_{i+1} &= 0 \\ (1-u)(q_{i+1} + p_{i+2} - q_{i+2} - p_{i+1}) &= p_{i+2}q_{i+1} - p_{i+1}q_{i+2} \\ u &= \frac{p_{i+1}q_{i+2} - p_{i+2}q_{i+1} + p_{i+2} + q_{i+1} - q_{i+2} - p_{i+1}}{q_{i+1} + p_{i+2} - q_{i+2} - p_{i+1}} = d. \end{aligned} \quad (17)$$

As you can see, for the calculation of d (10) requires fifteen operations and from (17) - Ten, which simplifies the calculations compared using simplex overall appearance.

Substituting (17) to (15), we obtain the point K :

$$K = P + (A - C)(1 - d)p_i + (B - C)(1 - d)q_i + C. \quad (18)$$

Consider dvohchastynnyy simplex, shown in figure 4, in which its top point $C = D_{i-1}$, and point $B = D_{i+2}$. In this case, the point of (2) and (5) will be respectively form:

$$D_{i-1} = C; D_{i+2} = B. \quad (19)$$

At that point, equation (15) will remain unchanged, and the determinant (16) takes the form:

$$\begin{vmatrix} 1-u & 1-u & 1 \\ p_{i+1} & q_{i+1} & 1 \\ 0 & 1 & 0 \end{vmatrix} = 0. \quad (20)$$

Whence have:

$$p_{i+1} - 1 + u = 0; u = 1 - p_{i+1} = d. \quad (21)$$

When calculating d using one-simplex had 10 operations, using dvohchastynnoho simplex - one that significantly reduces the cost of computing and other resources. In this case, the point of (18) to calculate the point K the shape remains the same, but the value d from (21).

And finally, consider the three-part simplex, shown in Figure 5, which all point simplex reflect the views of the original duodenum, ie, in this case, to build each point thickened with a shift one step - elected a new simplex in points duodenum. We have:

$$C = D_{i-1}, A = D_i, B = D_{i+2}. \quad (22)$$

Given (22) is written (6):

$$P = (C - A)u + A = Cu + Au + A = A(1 - u) + Cu. \quad (23)$$

For points P, D_{i+1}, D_{i+2} , including (4), (22) and (23), will make the caller through which we gain corresponding parameter values u .

$$\begin{vmatrix} 1-u & 0 & u \\ p_{i+1} & q_{i+1} & 1 \\ 0 & 1 & 0 \end{vmatrix} = 0 \text{ where } up_{i+1} - 1 + u = 0; u(p_{i+1} + 1) = 1;$$

$$u = \frac{1}{p_{i+1} + 1} = d. \quad (24)$$

Value d from (24) substituting in (23), we obtain the required point K :

$$KP = A(1-d) + Cd. \quad (25)$$

Calculation of terms K (25) using a three-part simplex, compared with (18) and (11) are the least expensive. Therefore, if the available opportunities to save resources should be used for thickening of the duodenum, three-part Simplex.

In all of the above, cases peak point simplex elected C , the other two – A and B formed simplex. Where peak point is chosen simplex A or point B , should be in all the earlier point equations and formulas to make, according to [1], the necessary changes.

Conclusions. Summing up, it should be noted that the method of supporting triangles can be used to KDP, with break points, points back, the inflection point for the KDP, I'm locked or unlocked. The availability of such opportunities is an important way to build a topographical contour surface submitted formalized geometric model or in digitized form.

Literature

1. Балюба И.Г. Точечное исчисление [учебное пособие] / И.Г. Балюба, В.М. Надыш; под ред. Верещаги В.М. – Мелітополь: Изд-во МГПУ ім. Б.Хмельницького, 2015. – 234 с.
2. Найдиш В.М. Дискретна інтерполяція / В.М. Найдиш. – Мелітополь: Люкс, 2007. – 250 с.
3. Найдиш В.М. Дискретне диференціювання / В.М. Найдиш, В.М. Верещага, А.В. Найдиш. – Мелітополь: Люкс, 2007. – 114с.
4. Найдиш В.М. Основи прикладної дискретної геометрії / В.М. Найдиш, В.М. Верещага, А.В. Найдиш, В.М. Малкіна. – Мелітополь: Люкс, 2007. – 193с.
5. Верещага В.М. О поле дифпроекции эмпирической кривой / В.М. Верещага // В кн.: «Начертательная геометрия и черчение» (межвузовский сборник). – Алма-Ата, 1979. – С.63-66.
6. Бездітний А.О. Варіативне дискретне геометричне моделювання на основі геометричних співвідношень у точковому численні Балюби-Найдиша: дис. ... канд. техн. наук: 05.01.01 / Андрій Олександрович Бездітний; Таврійський держ. агротехнол. ун-т. – Мелітополь, 2012. – 155 с.