

PRESENTATION OF EXPERIMENTAL DATA IN THE FORM OF THE RESPONSE SURFACE USING INTERPOLATION POLYNOMIALS

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The method of presenting the results of multifactorial experiment in the form of the response surface using interpolation polynomials. As polynomials can be used Chebyshev polynomials, Lagrange or Newton. Developed the program in algorithmic language C for processing the results of the experiment based on one and two factors. As an example, the surface response, which determines the dependence of the magnetic flux in the gap of the heat of the wind energy Converter from its construction.

Keywords: algorithmic language, multiple-factor experiment, processing of the experiment, the Lagrange polynomial, the software product, the surface response.

Formulation of the problem. Mathematical methods have found wide application for the processing of experiments in many fields of science and technology [1-3]. In a one-factor experiment, when the influence of the factor "x" on the output value "y" is studied, the results obtained are presented in the form of table. 1.

Table 1

Form for presenting experimental data

The factor x	x ₀	x ₁	x ₂	...	x _n
Output y	y ₀	y ₁	y ₂	...	y _n

One of the ways of mathematical processing of experimental data is related to finding the response function $f(x)$, Which can be represented using the regression equation [4]:

$$f(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n . \tag{1}$$

The coefficients of equation (1) are found by the method of least squares, and also using polynomials or splines. Many studies study the influence of one or more factors on a certain parameter [4-5]. In this case, the mathematical treatment consists in finding the response surface, the definition of which is encountered with analytical methods with great

difficulties, and sometimes it is practically impossible. Therefore, carrying out research in this direction is of practical interest.

Analysis of recent research and publications. The use of the regression equation (1) in a single-factor experiment is in many cases limited to polynomials of the first or second degree [4], [5]. If the experimental investigations are carried out with a sufficiently high accuracy, then the problem of constructing the polynomial (1) can be reduced to known methods, which are used in the interpolation of functions. We consider one of these methods based on the use of the Lagrange polynomial. We require that the coordinates of the nodes of the polynomial are given by the pairs of values from Table 1. 1. In the one-dimensional case, the polynomial of the n-th power has the form [2]:

$$P_n(x) = \sum_{k=0}^n y_k \prod_{\substack{j=0 \\ j \neq k}}^n \frac{x - x_j}{x_k - x_j} \quad (2)$$

The Lagrange polynomial in the case of interpolation of a function of two variables $f(x,y)$ has the form [1]:

$$L(x,y) = \sum_{n=0}^N \sum_{m=0}^M f_{nm} l_{nm}(x,y), \quad l_{nm}(x_n, y_m) = 1. \quad (3)$$

The basis polynomials in the formula (3) are defined by the formula:

$$l_{nm}(x,y) = \prod_{\substack{i=0, j=0 \\ i \neq n, j \neq m}}^{i=N, j=M} \frac{(x - x_i)(y - y_j)}{(x_n - x_j)(y_m - y_j)}$$

The maximum degree of a polynomial $L(x,y)$ not more $n \times m$. A priori $L(x_n, y_m) = f(x_n, y_m)$. It is assumed that the value of the function $f(x_n, y_m)$ at the nodal points x_n, y_m it is known. Indexing of coordinates x_n, y_m , Where the indices vary in intervals $0 \leq m \leq M, 0 \leq n \leq N$, is presented in table. 2.

At first glance, it may seem that, when going over to a two-factor experiment, the use of the polynomial (3) completely solves the problem of constructing the response surface. However, this is not the case. Formula (4) is valid for the case of placement of nodal points in accordance with the selected rectangular grid, which can be ensured only in a planned experiment. But in the case of experiment planning, it is more appropriate to use the theory, the foundations of which are laid down by Box and Wilkson [4]. Therefore, the question of effective methods for constructing the response surface in the case of the so-called passive experiment remains open.

Table 2

Indexing the coordinates of node points in the Lagrange polynomial (3) with two variables.

The left index refers to the factor x, the right index refers to the factor y				
0, 0	0, 1	0, 2	...	0, N
1, 0	1, 1	1, 2	...	1, N
2, 0	2, 1	2, 2	...	2, N
...
M, 0	M, 1	M, 2	...	M, N

Formation of article objectives. The purpose of this article is to develop a numerical algorithm for constructing the response surface to describe the results of a single-factor and multifactor experiment using the one-dimensional Lagrange polynomial (2).

Main part. One-factor experiment. Let the results of a single-factor experiment be presented in the form of tab. 1. The Lagrange formula (2) is valid for an arbitrary distribution of nodal points and, consequently, its application does not involve planning an experiment. To investigate the accuracy of the Lagrange polynomials, we created the *Lagrang1(x)* program in the algorithmic C language. Test tests of the program were carried out for the power function e^x on the interval $[0,2]$ using a polynomial of fifth degree with nodes at points $x_0=0, x_1=0.4, x_2=0.8, \dots, x_5=2.0$. In table. 3 shows the polynomial values computed using the program *Lagrang1(x)* in several points in increments 0.333333 , and the exact values of the functions known for these points e^x .

Table 3

Test results of the program *Lagrang1(x)* polynomial

X	0.000	0.333	0.667	1.000	1.333	1.667
e^x	1.00000	1.39561	1.94773	2.71828	3.79367	5.29449
$P_5(x)$	1.00000	1.39568	1.94767	2.71834	3.79360	5.29457

From table. 2 that the relative error of calculations is 10^{-5} . With an increase in the degree of the polynomial to 10-30, the relative error decreases to 10^{-17} .

1. **Multifactorial experiment.** Let us formulate the solution of the problem of constructing the response surface at an arbitrary point in a multifactorial space without any restriction on the number of variables using the program *Lagrang1(x)* for one variable.

At the first stage, consider the method of constructing the response surface in the case of two variables. We assume that the values of the interpolated function $f(x,y)$ At the nodal points with the coordinates x_n, y_m are known and equal $f(x_n, y_m)$. It is required to find the value $f(x,y)$ at an arbitrary point with coordinates (x_u, y_u) , which satisfy the condition $x_m \leq x_u \leq x_{m+1}, y_n \leq y_u \leq y_{n+1}$. The calculation algorithm can be represented in the following form. Calling the program *Lagrang1(x)* for building M one-dimensional Lagrange polynomials $L_0(x), L_1(x), L_2(x), \dots, L_M(x)$ for various constant values of y , namely: the first of them $L_0(x)$ corresponds to a fixed value $y = y_0$, the second $L_1(x)$ – fixed value $y = y_1$, etc., the last $L_M(x)$ – fixed value $y = y_M$. From what has been said it is clear that one-dimensional polynomials $L_i(x)$ Are constructed for nodal points, the coordinates of which are given in table. 4.

Table 4

Coordinates of nodes for constructing polynomials
 $L_0(x), L_1(x), L_2(x), \dots, L_M(x)$

Node number n	0	1	2	...	N
Values x_n	x_0	x_1	x_2	...	x_N
Values $f(x_n, y_0)$ for $L_0(x)$	$f(x_0, y_0)$	$f(x_1, y_0)$	$f(x_2, y_0)$...	$f(x_N, y_0)$
...
Values $f(x_n, y_m)$ for $L_m(x)$	$f(x_0, y_m)$	$f(x_1, y_m)$	$f(x_2, y_m)$...	$f(x_N, y_m)$
...
Values $f(x_n, y_M)$ for $L_M(x)$	$f(x_0, y_M)$	$f(x_1, y_M)$	$f(x_2, y_M)$...	$f(x_N, y_M)$

Knowing polynomials $L_m(x), 0 \leq m \leq M$, calculate their values $L_m(x_u)$ at point $x=x_u$ at different values y_m And the results are recorded in table. 5.

Table 5

Values of polynomials $L_m(x_u)$ at different values y_m .

Node number m	0	1	2	...	M
Values y_m	y_0	y_1	y_2	...	y_M
Values $L_m(x_u)$	$L_0(x_u)$	$L_1(x_u)$	$L_2(x_u)$...	$L_M(x_u)$

Based on the data of table. 5 we construct a one-dimensional polynomial $L(y)$ by nodal points $y_0, y_1, y_2, \dots, y_M$, where function $f(x_u, y)$ is, respectively, $L_0(x_u), L_1(x_u), L_2(x_u), \dots, L_M(x_u)$. In constructing $L(y)$ use the program *Lagrang1(x)*. Next, we calculate the value $L(y)$ at point y_u . The value obtained is the desired value of the function $f(x_u, y_u)$. For calculating a program has been created *Lagrang2(x,y)*.

The algorithm for processing an experiment with a large number of factors looks like this. Let the function $f(x,y,z)$ set at the nodal points (x_n, y_m, z_k) , where the integer k is within $0 \leq k \leq K$, and is $f(x_n, y_m, z_k)$. It is required to find the corresponding three-dimensional polynomial $L(x,y,z)$ and use it to determine the value $f(x,y,z)$ at an arbitrary point in the quotient space whose coordinates (x_u, y_u, z_u) belong to segments $x_m \leq x_u \leq x_{m+1}, y_n \leq y_u \leq y_{n+1}, z_k \leq z_u \leq z_{k+1}$. We fix a point z_u and consider for it a two-dimensional surface $f(x,y,z_u)$. Using the program *Lagrang2(x,y)* we can construct the corresponding two-dimensional polynomial $L(x,y)$ and calculate its value at the point (x_u, y_u) , which, obviously, is the desired value of a three-dimensional polynomial $L(x_u, y_u, z_u)$. By induction, it is easy to show that finding a polynomial in any number of variables s can be reduced to constructing a polynomial of degree $s-1$.

2. *Example of processing the results of an experiment.* The paper [6] contains data on the dependence of the magnetic flux in the wind energy converter on heat from the displacement of the teeth x and the gap size y . In Table. 6 shows the results of processing these experiments using the program *Lagrang2(x,y)*.

Table 6

Calculated dependences of the magnetic flux F

From the movement of the teeth of the armature and the gap using the function *Lagrang2(x,y)*

Зазор, Z	Перемещение, X							
	0	5	10	15	20	25	30	35
0.3000	23.2000	23.0000	22.5000	21.5000	20.5000	19.6000	19.0000	18.8000
0.2500	22.0000	21.7000	21.0000	20.0000	19.0000	17.9000	17.1000	16.8000
0.2000	20.9000	20.6000	19.5000	18.5000	17.1000	15.8000	14.9000	14.5000
0.1500	20.1000	19.5000	18.2000	16.9000	15.3000	13.7000	12.5000	11.9000
0.1000	19.0000	18.3000	16.7000	14.9000	13.0000	11.0000	9.3000	9.0000
0.0800	18.6000	17.8000	15.8000	13.8000	11.8000	9.7000	8.0000	7.6000
0.0600	17.8000	16.9000	15.0000	12.9000	10.5000	8.3000	6.4000	5.8000
0.0400	17.4000	16.2000	13.8000	11.5000	9.1000	6.9000	5.1000	4.3000

During the calculations, the coordinates of the node points for the movement were taken with a step of 5 mm from $x_0 = 0$ to $x_7 = 30$ mm. The value of the gaps y varied from $y_0 = 1.31$ to $y_7 = 10.5$ mm in increments of 1.31 mm. The values of the gaps in table 6 are given in relative units $\delta = y / 10.5$. The results of the calculations made it possible to analyze the influence of the gap and displacement on the magnetic flux and to choose the optimal design of the magnetic circuit.

Conclusions. A method is proposed for constructing a multidimensional response surface using the Lagrange polynomial of one variable. In C, programs have been developed $Lagrang1(x)$ and $Lagrang2(x,y)$, Which allow us to calculate the value of a function of one and two variables at an arbitrary point in a multifactorial space. Program $Lagrang1(x)$ Realizes calculations in the case of one variable by formula (2). Program $Lagrang2(x,y)$ Is intended for calculating the value of a function of two variables $f(x,y)$. As an example, a mathematical treatment of the experiments was carried out [6], where the dependence of the magnetic flux on the motion of the armature teeth and the gap in the magnetic wind power converter was investigated. The data obtained can be used to construct response surfaces in multivariate experiments using other polynomials, for example, Newton or Chebyshev.

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