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# CONSTRUCTION OF THE MINIMAL SURFACE USING ISOTROPIC CURVED, LYING ON THE ROTATIONAL SURFACE OF THE ASTROID

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***The paper considers an analytical description of the minimal surfaces with isotropic curves that lie on the surface of rotation of the astroid, referred to the isometric grid of the coordinate lines.***

***Key words: minimal surface, isometric grid of the coordinate lines, linear element of a surface, astroid, isotropic curve.***

***Formulation of the problem.*** Design and analytical description of minimal surfaces is an important issue continuous geometric modeling. By minimal surfaces results in a geometric problem: find a surface that passes through a closed curve and has the smallest area [1].

The analysis of scientific papers showed that there are three areas of current research analytical description of minimal surfaces, creating powerful new methods in the calculus of variations that allow to prove the regularity of minimal surfaces in multidimensional cases [2]; solve practical problems of designing architectural membrane surfaces [3, 4] and the development of efficient numerical methods for solving differential equations in partial derivatives, which leads to the problem of analytical description of minimal surfaces [4, 5]; Continuous development of methods for constructing the frame of minimal surfaces by means of complex variable [6, 7]. This work is dedicated to the implementation of analytical methods descriptions minimal surfaces using isotropic curves that lie on the surfaces of rotation assigned to coordinate isometric grid lines.

***Analysis of recent research and publications.*** Analytical description of continuous frame minimal surfaces associated with finding the parametric equations of isotropic curves zero length. [1] The construction of minimal surfaces using Bezier curves isotropic implemented dissertation [6]. In the thesis [7] found in some cases methods of constructing isotropic spatial curves Weierstrass and Schwarz formulas [1]. Therefore, expansion origin isotropic curves using complex variable is essential to solve the problem of designing a continuous frame of minimal surfaces.

***The wording of article purposes.*** Find analytical description of surface rotation astroid, referred to the isometric grid coordinate lines and curves of isotropic behind its surface. Based on these curves to build isotropic minimal surface and connected to their minimum surface.

***Main part.*** Consider the surface of revolution, parametric equations which have the appearance of:

$$X(\tau;v)=\varphi(\tau)\cos v;\quad Y(\tau;v)=\varphi(\tau)\sin v;\quad Z(\tau;v)=\psi(\tau)\qquad(1)$$

де $\varphi=\varphi(\tau);\ \psi=\psi(\tau)$ – parametric equations meridian surface of revolution.

In [8] an algorithm for finding parametric equations meridian surface of revolution, in which the surface will be referred to the isometric grid coordinate lines. Transition from orthogonal to the isometric grid coordinates is performed by introducing a new variable $\tau$ associated with variable follows [8]:

$$t=\int\frac{\sqrt{(\varphi'_\tau)^2+(\psi'_\tau)^2}}{\varphi}\,d\tau.\qquad(2)$$

Consider astroid surface rotation that Parametric equations:

$$X(\tau;v)=r\cos^3\tau\cdot\cos v;\quad Y(\tau;v)=r\cos^3\tau\cdot\sin v;$$
$$Z(\tau;v)=r\sin^3\tau,\qquad(3)$$

where $r>0$ – setting astroid; $\tau\in[0;\,2\pi);\ v\in[0;\,2\pi)$.

Having determined according to (2), the transition to the isometric grid coordinate lines $\tau(t)=\arccos\left(\frac{3}{t}\right)$, obtain a parametric equation astroid surface rotation, referred to the isometric grid coordinate lines:

$$X(t;v)=\frac{27r}{t^3}\cdot\cos v;\quad Y(t;v)=\frac{27r}{t^3}\cdot\sin v;\ Z(t;v)=r\left(1-\frac{9}{t^2}\right)^{\frac{3}{2}}.\qquad(4)$$

Factorization of linear expression element surface (4) that determines the length of any curve that lies on the surface:

$$ds^2=\frac{729r^2}{t^6}\cdot(dv-i\cdot dt)(dv+i\cdot dt)\qquad(5)$$

where $i$ – imaginary unit. Equating to zero the right side of the last equality, we get after integration:

$$v=i\cdot t+C\quad \text{або}\quad v=-i\cdot t+C,\qquad(6)$$

where $C$ – arbitrary constants of integration.

When substituting expression $v=i\cdot t+C$ in equation (4) for each value obtain parametric equations isotropic imaginary curve that lies on the surface of the rotation astroid:

$$x(t)=\frac{27r}{t^3}\cdot\cos(i\cdot t+C);\ y(t)=\frac{27r}{t^3}\cdot\sin(i\cdot t+C);\ z(t)=r\left(1-\frac{9}{t^2}\right)^{\frac{3}{2}}.\qquad(7)$$

Take for functions (7) replacement: $t=u+i\cdot v$. Separating real and imaginary parts, we obtain the minimal surface equation (C - arbitrary constant):

$$X(u,v)=\frac{27r\left[u(u^2-3v^2)\cos(C-v)\,\mathrm{ch}(u)-v(3u^2-v^2)\sin(C-v)\,\mathrm{sh}(u)\right]}{(u^2+v^2)^3};$$
$$Y(u,v)=\frac{27r\left[u(u^2-3v^2)\sin(C-v)\,\mathrm{ch}(u)+v(3u^2-v^2)\cos(C-v)\,\mathrm{sh}(u)\right]}{(u^2+v^2)^3}; \qquad (8)$$
$$Z(u,v)=r\left[\frac{324u^2v^2}{(u^2+v^2)^4}+\left(1-\frac{9(u^2-v^2)}{(u^2+v^2)^2}\right)^2\right]^{\frac{3}{4}}\cos\left(\frac{3}{2}\mathrm{arctg}\frac{18uv}{(u^2+v^2)^2-9(u^2-v^2)}\right).$$

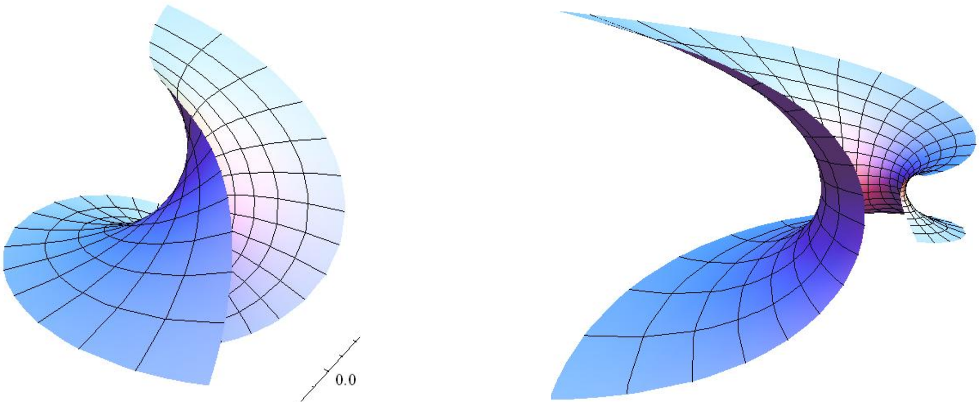
and the associated minimal surface:

$$X^*(u,v)=\frac{-27r\left[u(u^2-3v^2)\sin(C-v)\,\mathrm{sh}(u)+v(3u^2-v^2)\cos(C-v)\,\mathrm{ch}(u)\right]}{(u^2+v^2)^3};$$
$$Y^*(u,v)=\frac{27r\left[u(u^2-3v^2)\cos(C-v)\,\mathrm{sh}(u)-v(3u^2-v^2)\sin(C-v)\,\mathrm{ch}(u)\right]}{(u^2+v^2)^3}; \qquad (9)$$
$$Z^*(u,v)=r\left[\frac{324u^2v^2}{(u^2+v^2)^4}+\left(1-\frac{9(u^2-v^2)}{(u^2+v^2)^2}\right)^2\right]^{\frac{3}{4}}\sin\left(\frac{3}{2}\mathrm{arctg}\frac{18uv}{(u^2+v^2)^2-9(u^2-v^2)}\right).$$

Figure 1 (a, b) shows a minimum and compartments attached minimal surface built on equations (8) and (9) in accordance with $C=0;\ u\in[-2;...2];\ v\in[0;...12]$.

Expression linear surface element (4) can be decomposed into factors as:

$$ds^2=\frac{729r^2}{t^6}\cdot(dt-i\cdot dv)(dt+i\cdot dv). \qquad (10)$$




*a* *b*

Fig. 1. Bays minimal surfaces:
a) minimal surface compartment, built by equations (8);
b) compartment attached surface built on equations (9)

Substituting $t = i \cdot v + C$ or $t = -i \cdot v + C$, obtained from (10), the parametric equation of the surface (4), we obtain the equation of two other families imaginary isotropic curves. For each found using isotropic curves can build minimal surfaces and are attached to them, which are characterized by common features and common metric properties curvature surface.

***Conclusions.*** On the surface astroid rotation, referred to the isometric grid coordinate lines for each value can construct four families isotropic curves, every curve and put in correspondence minimal surface and attached to it. The resulting minimal surface and connected surface with common minimum metric properties and general properties of surface curvature.

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