GEOMETRIC MODELING OF FLUCTUATIONS

SPATIAL SPRING PENDULUM

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The method of choice of values of parameters is examined for the receipt of unchaotic trajectories of vibrations of load of spatial spring pendulums.

Keywords: a spatial spring pendulum, equalization of Lagrange 2th family, trajectory of moving of load.

Formulation of the problem. Mathematical spatial spring pendulums are versatile models for studying processes that are described by a certain class of differential equations [1-3]. These pendulums are treated as an example of two linear systems, and nonlinear parametrically related. It is shown that the frequency ratio 2:1 these systems there is a complete energy transfer of angular fluctuations in the energy vertical and back.

The possibility of such phenomena must be considered in the calculation of various structures (suspension bridges, cable-stayed-beam system, cableways, power lines, different space tether system for maintenance of facilities, flexible hoses, various antennas, etc.). A spatial model of a spring pendulum is used in structural mechanics to analyze the conditions under which the effects of loss of dynamic stability of a supersonic aircraft, high speed ships are manifested [1,2].

Analysis of recent researches. There are a considerable number of publications devoted to mathematical spatial spring pendulum [1-3]. To illustrate the solutions of these equations you should be able to build a spatial shape of movement trajectories (center) of the load spring pendulums [3].

Then, by analogy, the junction can be used in a similar sense to the task. Therefore, these studies would complement the development of the method a graphical representation of the trajectories of the oscillations of the load as a result of solving differential equations to describe them with the aim of identifying among them unchaotic trajectories.

The wording of the article purposes. To develop a graphical computer method for selecting parameter values to ensure unchaotic trajectories of the fluctuations of the load of spatial spring pendulums.

Main part. To describe the dynamics of oscillations of a spring pendulum in Cartesian coordinate *Ouvw* we use [3] the system of equalization of Lagrange 2th family:

$$\frac{d^2}{dt^2}\mathrm{u}(t) = -\frac{k\left(\sqrt{\mathrm{u}(t)^2+\mathrm{v}(t)^2+\mathrm{w}(t)^2}-L0\right)\mathrm{u}(t)}{m\sqrt{\mathrm{u}(t)^2+\mathrm{v}(t)^2+\mathrm{w}(t)^2}}$$

$$\frac{d^2}{dt^2}\mathrm{v}(t) = -\frac{k\left(\sqrt{\mathrm{u}(t)^2+\mathrm{v}(t)^2+\mathrm{w}(t)^2}-L0\right)\mathrm{v}(t)}{m\sqrt{\mathrm{u}(t)^2+\mathrm{v}(t)^2+\mathrm{w}(t)^2}} \qquad (1)$$

$$\frac{d^2}{dt^2}\mathrm{w}(t) = -\frac{k\left(\sqrt{\mathrm{u}(t)^2+\mathrm{v}(t)^2+\mathrm{w}(t)^2}-L0\right)\mathrm{w}(t)}{m\sqrt{\mathrm{u}(t)^2+\mathrm{v}(t)^2+\mathrm{w}(t)^2}} - g$$

Here the following designations are used: u(t), v(t) and w(t) – the coordinates (center) of the load of a spring pendulum at time t ; L0 is the length of the spring in unloaded condition; k is the stiffness coefficient of the spring; m is the mass of the cargo; g = 9.81. The attachment point of the pendulum is at the origin.

We will solve the system of differential equations by Runge-Kutt's numerically method with conditions: u0, v0 and w0 – the initial coordinates of the load in "zero" time; du0, dv0, and dw0 is the initial speed of the pendulum in the directions of the respective coordinate.

To determine the parameter values u0, v0, w0, du0, dv0, and dw0, which would ensure unchaotic spatial trajectory of the pendulum load, we apply the method of projection focusing [4]. For this numerical method with selected (for example) the initial conditions u0 = 1; du0 = 0; v0 = 0; w0 : = 1.1; dw0 = 0 and given parameter values k = 9; m = 1 and L0 = 1 we solve the system of equations (1) and build the image of an integral curve in the phase space.

First we will construct the image in the phase space {u, Du, t} depending on a certain value in "command" parameter. As a "Manager" you can select any task parameter (e.g., dv0), provided that all other parameter values are fixed. When random values of the parameters in the phase space {u, Du, t} than formed "confusing" integral curve (Fig. 1, a). We will design it on the phase plane {u, Du}, where we also see a "confused" phase trajectory.

In the case of a change of values in "command" parameter must be changed and the character of phase trajectories. At some critical value the character of phase trajectories will change on a qualitative level – it will turn into a "natural" curve (Fig. 1, b). On the phase plane will be observed if the optical effect of "prompting on sharpness" confusion phase trajectories.

Thanks to this analogy, the proposed finding critical values of parameters is referred to as focus projection [4].

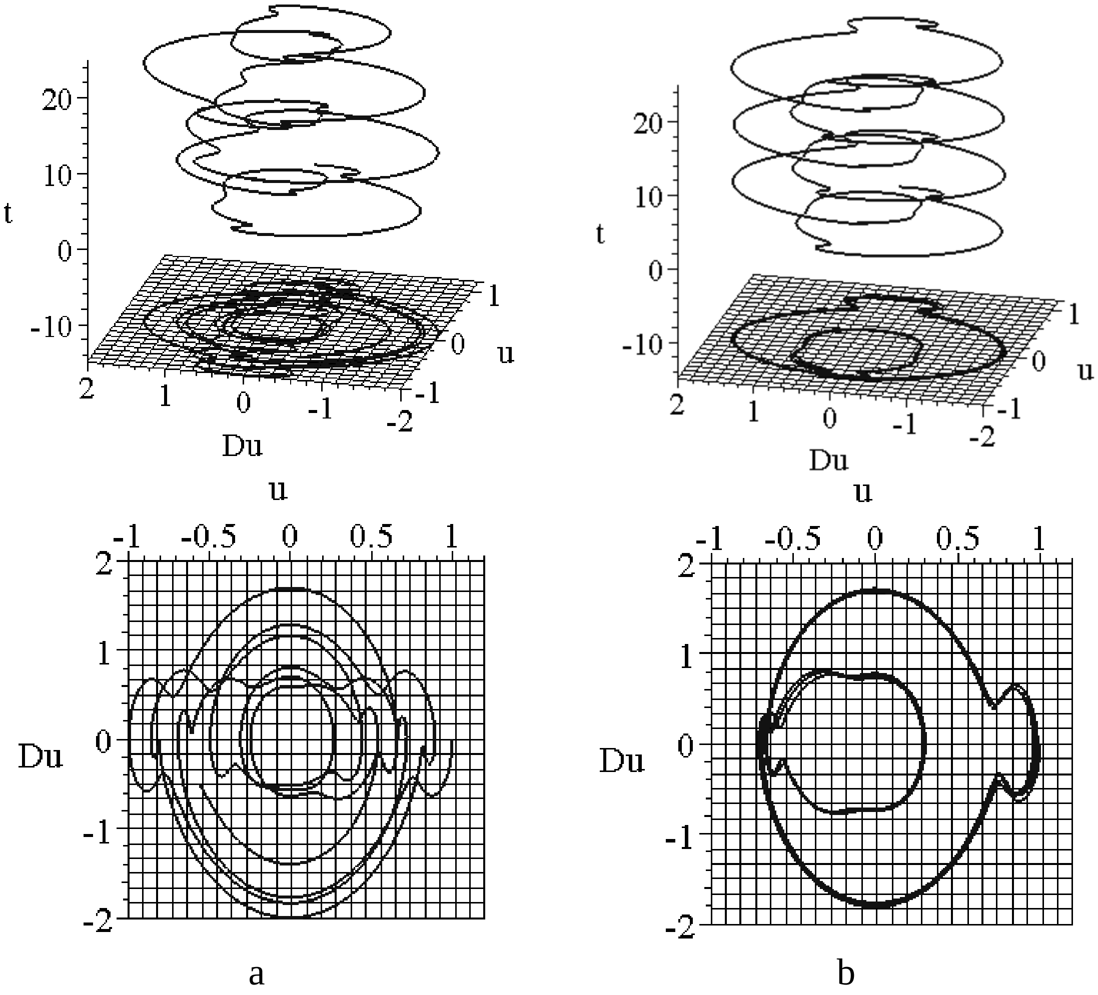


Fig. 1 – Phase trajectories as projections of integral curves:

a) for an arbitrary parameter value (dv0 = 0,48);

b) for the critical value of the parameter dv0 = 0,587

All this takes place to construct the image of an integral curve in the phase space {v, Dv, t}. In Fig. 2, a the phase trajectories is depicted as projections of integral curves for random values of dv0 = 0,48, and Fig. 2, b for the critical values of the "control" parameter dv0 = 0,587.

Considering the value of the parameter dv0 = 0,587 in the process of solving the system of equations (1) entails computing the coordinates of points in space {u,v,w} (Fig. 3), which must be located in unchaotic trajectory (or close to it).

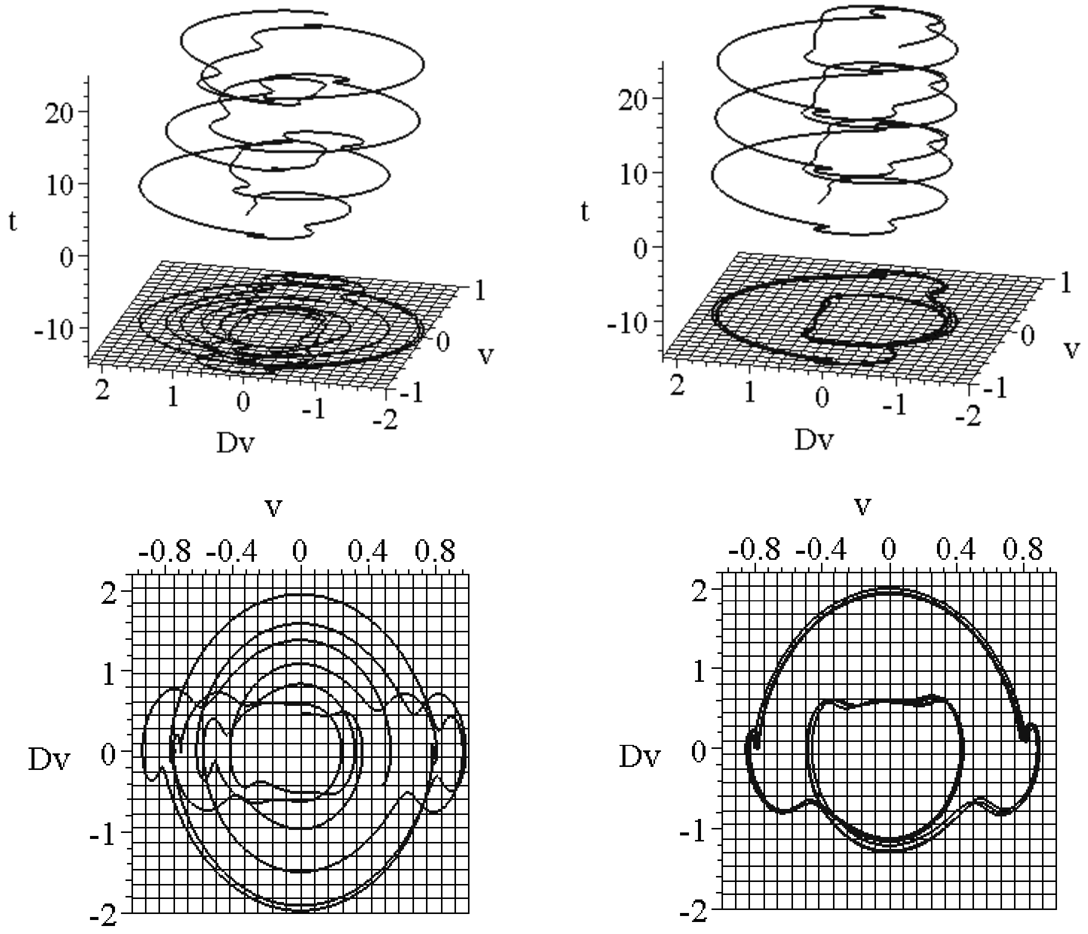
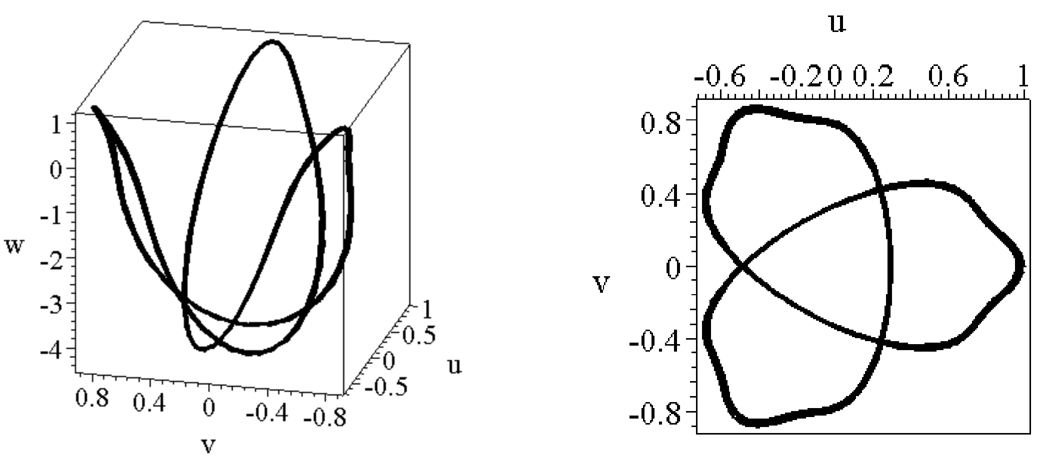


Fig. 2 – Phase trajectories as projections of integral curves:

a) for an arbitrary parameter value (dv0 = 0,48);

b) for the critical value of the parameter dv0 = 0,587



a

B

Fig. 3. The spatial trajectory of the movement of cargo (a)

and its projection on the plane Ouv (b)

Conclusions. The developed method allows to select parameter values to obtain the unchaotic trajectories of oscillations of a spring pendulum load. Further studies will be related to the choice of parameters to ensure the desired shape of the trajectory.

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