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MODELING ONE-DIMENSIONAL DISCRETE GEOMETRIC IMAGE BY SUPERPOSITIONS OF POINT SETS ON THE BASIS OF TWO AND SINGLE ADJUSTED IMAGES

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In the article the method of discrete modeling of curves is considered. This method is based on superpositions of two curves and on superpositions of nodal points of one curve. All these curves were formed by the static-geometric method. The proposed method allows modeling balanced discrete structures, which were formed on adjusted contour nodes and pass through given nodal points without making up and solving any system of equations.

Key words: one-dimensional geometric images, discrete models of curves, superposition of point sets, superposition coefficients, static-geometric method.

Formulation of the problem. One of the important directions of discrete geometry is the static-geometric method (SGM) [1], created on the basis of static interpretation of the classical method of finite difference. Its simplicity and practicality are manifested in the constructivity and visibility of the process of shaping the geometric image of a certain continuous object under the influence of external loading, taking into account the given conditions. The discrete image itself is an idealized model of a grid, whose ligaments are non-hardening threads, and external loads are perceived by the nodes of this grid. The convenience of HSM is also the ability to interpret not only objects that are topologically close to core or mesh structures, but also any physical processes that involve taking into account the interaction between their individual components arbitrarily located in space.

However, the main disadvantage of SDM is the need to build and solve bulky systems of linear equations. This disadvantage can be eliminated due to the use of a geometric superposition device. Therefore, the task of this study is to improve this method by involving a geometric apparatus of superpositions of point sets for the formation of one-dimensional geometric images.

Analysis of recent research and publications. In [2], the problems of the construction of discrete surfaces of surfaces by functional addition were studied on the basis of two pre-calculated static-geometric frameworks method. However, the verification of grids obtained as a result of the superposition of the initial grids with different coefficients of stretching of the joints showed that the resulting grid is not balanced with a given external load on the knot, that is, the results of such superpositions were not accurate but approximate.

In [3] the notion of a superposition apparatus of sets in applied geometry is defined. A number of properties have been proved, which allowed us to draw conclusions about the promising depth of a comprehensive study of the superposition apparatus.

In the papers [4-7] of the authors of this article, approaches to the determination of discrete analogues of certain functional dependences on the basis of a geometric apparatus of superpositions of one-dimensional point sets are shown, which allows to form discrete images without compiling and solving cumbersome systems of equations. The management of the form of discretely represented curves is carried out by varying the magnitudes of the superposition coefficients.

Formulating the goals of the article. The purpose of this article is to study the method of simulation of a one-dimensional geometric image in the form of a discretely presented curve with the use of a geometric apparatus of superpositions of one-dimensional point sets on the basis of one curve formed by a static geometric method.

Main part. Let us first consider the example of the formation of a discrete curve model based on the superpositions of two curves fixed in two given nodes constructed by a static-geometric method (Fig. 1).

The discrete values of the first curve are formed according to the initial data $y_{A_1} = 8$, $y_{A_3} = 8$, $P_i = -1$.

The system of equations for determining the ordinates of the points to be sought will look [1]:

$$\begin{cases} 8 - 2y_{A_0^1} + y_{A_2} = 1; \\ y_{A_0^1} - 2y_{A_2} + y_{A_0^2} = 1; \\ y_{A_2} - 2y_{A_0^2} + 8 = 1. \end{cases}$$

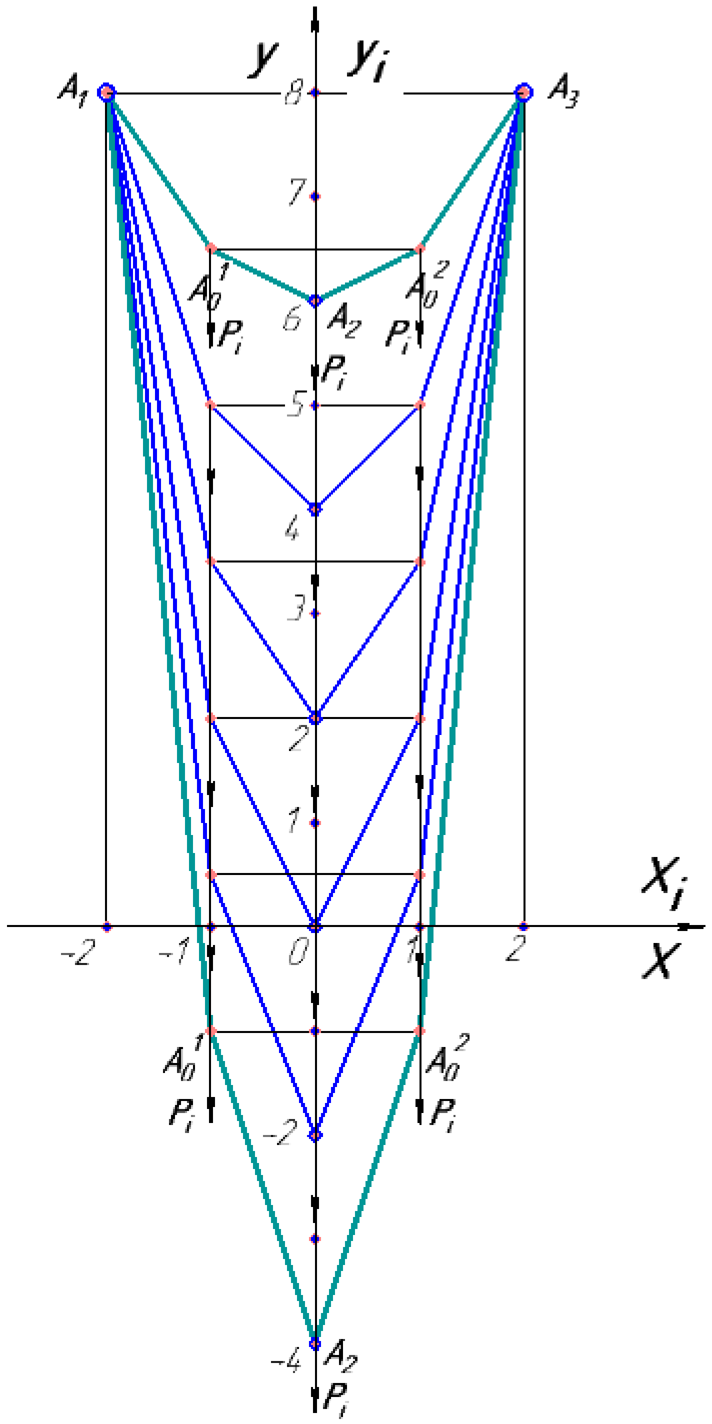

Fig. 1. Formation of discrete models of curves based on uperpositions of two discretely given curves

The solution to this system yields results: $y_{A_2} = 6$, $y_{A_0^1} = 6{,}5$, $y_{A_0^2} = 6{,}5$.

Coefficients of superposition of given points A_1 , A_2 , A_3 calculated by the formulas [4]:

$$k_1 = \frac{(x_0 - x_3)(y_2 - y_3) - (x_2 - x_3)(y_0 - y_3)}{(x_1 - x_3)(y_2 - y_3) - (x_2 - x_3)(y_1 - y_3)},$$

$$k_2 = \frac{(x_1-x_3)(y_0-y_3)-(x_0-x_3)(y_1-y_3)}{(x_1-x_3)(y_2-y_3)-(x_2-x_3)(y_1-y_3)},$$

will matter: $k_1 = 0{,}375$, $k_2 = 0{,}75$, $k_3 = -0{,}125$.

The discrete values of the second curve are formed according to the initial data $y_{A_1} = 8$, $y_{A_3} = 8$, $P_i = -6$.

The system of equations will look:

$$\begin{cases} 8 - 2y_{A_0^1} + y_{A_2} = 6; \\ y_{A_0^1} - 2y_{A_2} + y_{A_0^2} = 6; \\ y_{A_2} - 2y_{A_0^2} + 8 = 6. \end{cases}$$

The solution to this system yields results: $y_{A_0^1} = -1$; $y_{A_2} = -4$; $y_{A_0^2} = -1$.

Coefficients of superposition of given points A_1 , A_2 , A_3 to determine the desired points A_0^1 i A_0^2 , will matter: $k_1 = 0{,}375$, $k_2 = 0{,}75$, $k_3 = -0{,}125$.

The value of the external shaping load and ordinates of the nodal points of the desired discrete models of the curves as a superposition of two pre-formed static-geometric methods of discrete curve models (Fig. 1) will be determined by the formulas:

$$P_i = k_1 P_i^1 + k_2 P_i^2 ,$$

$$y_i = k_1 y_i^1 + k_2 y_i^2 ,$$

where P_i^1 – the magnitude of the uniformly distributed external shaping load applied to the nodes of the simulated first curve, and P_i^2 – the second one; y_i^1 – the ordinate of the i-th node of the first curve, and y_i^2 – the second curve.

The value of the external shaping load and ordinates of the nodal points of the desired discrete curve models with a uniform step along the y axis for the central node from 6 to -4 will be determined by the formulas:

$$P_i = k_1 P_i^6 + k_2 P_i^{-4} ,$$

$$y_i = k_1 y_i^6 + k_2 y_i^{-4} .$$

Given the uniform step of changing the load size from 1 to 6:

$6 - 1 = 5$; $\frac{1}{5} = 0{,}2$, get it $k_1, k_2 = 0{,}2;\ 0{,}4;\ 0{,}6;\ 0{,}8.$

For example, the magnitude of the load $P_i^{A_2}$ and ordinates of nodal points $y_i^{A_2}$ shaped curves will be determined by the formulas:

$$P_i^{A_2=4} = 0{,}2 \cdot (-6) + 0{,}8 \cdot (-1) = -1{,}2 + (-0), 8 = -2 ,$$

$$P_i^{A_2=2} = 0{,}4 \cdot (-6) + 0{,}6 \cdot (-1) = -2{,}4 + (-0{,}6) = -3,$$

$$P_i^{A_2=0} = 0{,}6 \cdot (-6) + 0{,}4 \cdot (-1) = -3{,}6 + (-0{,}4) = -4 ,$$

$$P_i^{A_2=-2} = 0{,}8 \cdot (-6) + 0{,}2 \cdot (-1) = -4{,}8 + (-0{,}2) = -5\ ,$$
$$y_{P_i=-2}^{A_2} = 0{,}2 \cdot (-4) + 0{,}8 \cdot 6 = -0{,}8 + 4{,}8 = 4\ ,$$
$$y_{P_i=-3}^{A_2} = 0{,}4 \cdot (-4) + 0{,}6 \cdot 6 = -1{,}6 + 3{,}6 = 2\ ,$$
$$y_{P_i=-4}^{A_2} = 0{,}6 \cdot (-4) + 0{,}4 \cdot 6 = -2{,}4 + 2{,}4 = 0\ ,$$
$$y_{P_i=-5}^{A_2} = 0{,}8 \cdot (-4) + 0{,}2 \cdot 6 = -3{,}2 + 1{,}2 = -2\ .$$

The magnitude of the load and application of nodal points of the desired discrete curve models are presented in Table 1.

Table 1

The value of the external shaping load

and the ordinates of the nodal points of the desired discrete curve models

	$k_1 = 0{,}8$ $k_2 = 0{,}2$	$k_1 = 0{,}6$ $k_2 = 0{,}4$	$k_1 = 0{,}4$ $k_2 = 0{,}6$	$k_1 = 0{,}2$ $k_2 = 0{,}8$	
$P_i^{A_2=6} = -1$	$P_i^{A_2=4} = -2$	$P_i^{A_2=2} = -3$	$P_i^{A_2=0} = -4$	$P_i^{A_2=-2} = -5$	$P_i^{A_2=-4} = -6$
$y_{P_i=-1}^{A_2} = 6$	$y_{P_i=-2}^{A_2} = 4$	$y_{P_i=-3}^{A_2} = 2$	$y_{P_i=-4}^{A_2} = 0$	$y_{P_i=-5}^{A_2} = -2$	$y_{P_i=-6}^{A_2} = -4$
$y_{P_i=-1}^{A_0^1} = 6{,}5$	$y_{P_i=-2}^{A_0^1} = 5$	$y_{P_i=-3}^{A_0^1} = 3{,}5$	$y_{P_i=-4}^{A_0^1} = 2$	$y_{P_i=-5}^{A_0^1} = 0{,}5$	$y_{P_i=-6}^{A_0^1} = -1$
$y_{P_i=-1}^{A_0^2} = 6{,}5$	$y_{P_i=-2}^{A_0^2} = 5$	$y_{P_i=-3}^{A_0^2} = 3{,}5$	$y_{P_i=-4}^{A_0^2} = 2$	$y_{P_i=-5}^{A_0^2} = 0{,}5$	$y_{P_i=-6}^{A_0^2} = -1$

Results of computed superposition coefficients according to the formulas (1) of coordinates of given points A_1, A_2, A_3 to determine the coordinates of unknown points A_0^1 i A_0^2 shaped discrete curve models with a uniform step $h=1$ along the Ox axis, shown in Fig. 1 are shown in Table 2.

$$\begin{cases} x_0 = k_1 x_1 + k_2 x_2 + (1 - k_1 - k_2) x_3 \\ y_0 = k_1 y + k_2 y_2 + (1 - k_1 - k_2) y_3 \end{cases} \qquad (1)$$

Table 2

Value of superposition coefficients

	A_1	A_0^1	A_2	A_0^2	A_3
P_i=-1					
x_0	-2	-1	0	1	2
y_0	8	6,5	6	6,5	8
k_1		*0,375*		*-0,125*	
k_2		*0,75*		*0,75*	
$k_3=1- k_1- k_2$		*-0,125*		*0,375*	
P_i=-2					
x_0	-2	-1	0	1	2
y_0	8	5	4	5	8
k_1		*0,375*		*-0,125*	
k_2		*0,75*		*0,75*	
$k_3=1- k_1- k_2$		*-0,125*		*0,375*	
P_i=-3					
x_0	-2	-1	0	1	2
y_0	8	3,5	2	3,5	8
k_1		*0,375*		*-0,125*	
k_2		*0,75*		*0,75*	
$k_3=1- k_1- k_2$		*-0,125*		*0,375*	
P_i=-4					
x_0	-2	-1	0	1	2
y_0	8	2	0	2	8
k_1		*0,375*		*-0,125*	
k_2		*0,75*		*0,75*	
$k_3=1- k_1- k_2$		*-0,125*		*0,375*	
P_i=-5					
x_0	-2	-1	0	1	2
y_0	8	0,5	-2	0,5	8
k_1		*0,375*		*-0,125*	
k_2		*0,75*		*0,75*	
$k_3=1- k_1- k_2$		*-0,125*		*0,375*	
P_i=-6					
x_0	-2	-1	0	1	2
y_0	8	-1	-4	-1	8
k_1		*0,375*		*-0,125*	
k_2		*0,75*		*0,75*	
$k_3=1- k_1- k_2$		*-0,125*		*0,375*	

According to the results of Table 2, we can conclude that the magnitudes of the superposition coefficients of the given boundary conditions (coordinates of the initial and last nodes) and, for example, the central node for determining the coordinates of the desired nodes will be the same for all discrete models of curves that are formed on the same same boundary conditions.

Prove this conclusion by writing the system of equations [1] for the above-defined discrete curve models in general form:

$$\begin{cases} 8 - 2y_{A_0^1} + y_{A_2} = P_i; \\ y_{A_0^1} - 2y_{A_2} + y_{A_0^2} = P_i; \\ y_{A_2} - 2y_{A_0^2} + 8 = P_i. \end{cases}$$

The solution to this system yields results:

$$y_{A_2} = \frac{y_{A_1}+y_{A_3}-4P_i}{4}, \; y_{A_0^1} = \frac{3y_{A_1}+y_{A_3}-6P_i}{2}, \; y_{A_0^2} = \frac{y_{A_1}+3y_{A_3}-6P_i}{4} \; .$$

Hence, the coordinates of the nodal points of the given and simulated discrete curves will look:

$A_2(0; \frac{y_{A_1}+y_{A_3}-4P_i}{2})$, $A_0^1(-1; \frac{3y_{A_1}+y_{A_3}-6P_i}{4})$, $A_0^2(1; \frac{y_{A_1}+3y_{A_3}-6P_i}{4})$, $A_1(-2; 8)$, $A_3(2; 8)$.

Substituting into the system of equations [4]

$$\begin{cases} x_0 - x_3 = k_1(x_1 - x_3) + k_1(x_2 - x_3); \\ y_0 - y_3 = k_1(y_1 - y_3) + k_1(y_2 - y_3). \end{cases}$$

point coordinatesA_2 , A_0^1 i A_0^2 , get it:

$$\begin{cases} k_1(-2-2) + k_2(0-2) = -1-2 \\ k_1(8-8) + k_2\left(\frac{y_{A_1}+y_{A_3}-4P_i}{2} - 8\right) = \frac{3y_{A_1}+y_{A_3}-6P_i}{4} - 8 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} -4k_1 - 2k_2 = -3; \\ 0 \cdot k_1 + \left(\frac{y_{A_1}+y_{A_3}-4P_i}{2} - 8\right) \cdot k_2 = \frac{3y_{A_1}+y_{A_3}-6P_i}{4} - 8. \end{cases}$$

The solution to this system yields results:

$$k_1 = \frac{3P_i-8+8}{8P_i-2\cdot8-2\cdot8+32} = \frac{3}{8} = 0{,}375 \; ; \; k_2 = \frac{6P_i-8-3\cdot8+32}{8P_i-2\cdot8-2\cdot8+32} = \frac{3}{4} = 0{,}75 \; .$$

Substituting the coordinates of points into the system of equations A_2 and A_0^2 , get it:

$$\begin{cases} k_1(-2-2) + k_2(0-2) = -1-2 \\ k_1(8-8) + k_2\left(\frac{y_{A_1}+y_{A_3}-4P_i}{2} - 8\right) = \frac{y_{A_1}+3y_{A_3}-6P_i}{4} - 8 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} -4k_1 - 2k_2 = -3 \\ 0 \cdot k_1 + \left(\frac{y_{A_1}+y_{A_3}-4P_i}{2} - 8\right) \cdot k_2 = \frac{y_{A_1}+3y_{A_3}-6P_i}{4} - 8 \end{cases}$$

The solution to this system yields results:

$$k_1 = \frac{8-P_i-8}{8P_i-2\cdot8-2\cdot8+32} = -\frac{1}{8} = -0{,}125 \; ; \; k_2 = \frac{6P_i-3\cdot8-8+32}{8P_i-2\cdot8-2\cdot8+32} = \frac{3}{4} = 0{,}75 \; .$$

Conclusions. Thus, one discrete model of the curve formed by the static-geometric method can form a superposition method by any number of balanced discrete models of curves with an arbitrary number of nodal points under the same boundary conditions and different values of the external shaping load without the assembly and solution of large systems of linear equations, which, in turn, allows us to discretely model curves of different shapes and solve the problems of discrete interpolation on a plane.

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ФОРМИРОВАНИЕ ОДНОМЕРНОГО ДИСКРЕТНОГО ГЕОМЕТРИЧЕСКОГО ОБРАЗА СУПЕРПОЗИЦИЯМИ ТОЧЕЧНЫХ МНОЖЕСТВ НА ОСНОВЕ ДВУХ И ОДНОГО ЗАДАННЫХ ОБРАЗОВ

Воронцов О.В., Тулупова Л.А., Воронцова И.В.

В статье рассмотрен способ дискретного моделирования кривых линий на основе суперпозиций двух кривых, сформированных статико-геометрическим методом и на основе суперпозиций узловых точек одной кривой сформированной статико-геометрическим методом. Предложенный способ моделирования позволяет формировать уравновешены дискретные структуры, сформированные на заданных контурных узлах, а также проходящих через заданные узловые точки без составления и решения систем уравнений.

Ключевые слова: одномерные геометрические образы, дискретные модели кривых, суперпозиции точечных множеств, коэффициенты суперпозиции, статико-геометрический метод.

ФОРМУВАННЯ ОДНОВИМІРНОГО ДИСКРЕТНОГО ГЕОМЕТРИЧНОГО ОБРАЗУ СУПЕРПОЗИЦІЯМИ ТОЧКОВИХ МНОЖИН НА ОСНОВІ ДВОХ ТА ОДНОГО ЗАДАНИХ ОБРАЗІВ

Воронцов О.В., Тулупова Л.О., Воронцова І.В.

У статті розглянуто спосіб дискретного моделювання кривих ліній на основі суперпозицій двох кривих, що сформовані статико-геометричним методом та на основі суперпозицій вузлових точок однієї кривої сформованої статико-геометричним методом. Запропонований спосіб моделювання дозволяє формувати врівноважені дискретні структури, сформовані на заданих контурних вузлах, а також, що проходять через задані вузлові точки без складання і розв'язання систем рівнянь.

Ключові слова: одновимірні геометричні образи, дискретні моделі кривих, суперпозиції точкових множин, коефіцієнти суперпозиції, статико-геометричний метод.