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GEOMETRIC MODELING OF THE CONDITIONS OF NONINTERSECTION OF ELIPS

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The paper considers approaches to geometric modeling of nonintersection conditions of two ellipses.

Keywords: ellipse, geometric modeling, conditions of nonintersection.

Formulation of the problem. The tasks of packaging and cutting (Cutting & Packing), in particular the tasks of optimal packaging of ellipses, also called optimal placement, are the subject of the study of computational geometry, and the methods of their solution are the direction of the theory of operations research. This class of tasks has a wide range of scientific and practical applications in powder metallurgy, the mining industry for modeling the motion of bulk substances, the analysis of structures of liquids and glass, the problems of logistics for the modeling of optimal packages of cargoes having an elliptical cylinder, in the tasks of evacuation of people from buildings during modeling the individual-current movement of people whose projections are approximated by ellipses.

A wide range of scientific and practical applications, some of which are described above, requires the development of effective algorithms based on the application of methods for optimizing the placement of a large number of ellipses. In connection with this, there is a need to develop effective, in terms of complexity, approaches to geometric simulation of non-crossing conditions of ellipses that allow continuous broadcasting, rotation.

Analysis of recent research and publications. As an effective means of geometric simulation of the non-crossing of a pair of ellipses, taking into account allowable distances, functions are used in the class of phi-functions proposed in the papers by Stoiana Yu.G [1,2]. In works [3.4], the construction of non-crossing conditions of ellipses uses the approximation of ellipses in the form of the union of base objects [3] or approximation by the arcs of circles [4]. An overview of publications on this subject makes it possible to conclude that only in [5] is a method for solving the problem of packing true ellipse (without approximations) that involves rotation, using the modern NLP solvers available in GAMS. This article gives a fairly complete overview of the literature on the problems of packaging of ellipses. In [5], a global solution for a small number of ellipses was obtained, however, when N> 14, the authors failed to obtain an acceptable solution. In this regard, the authors offer a heuristic polylithic algorithm for placing more ellipses (up to 100) in a rectangular area of fixed width and variable length. The problem of optimal packing of ellipses, allowing for continuous rotation, is considered in [6]. Quasi-phi-functions are used for the analytical description of the main placement constraints [2]. The approach outlined in [6] allows us to represent the problem of optimal packing of ellipses taking into account allowable distances in the form of a nonlinear programming problem and obtain locally optimal solutions for N <120 (N is the number of objects of placement).

Therefore, there was a need to develop effective approaches to geometric simulation of non-crossing conditions of ellipses, which would allow solving practical problems of greater dimensionality.

Formulating the goals of the article. The purpose of the research is to develop effective algorithms for simulating the conditions of interaction between the ellipses (non-crossing, touch, crossing).

Main part. Consider the ellipses $E_i(u_{E_i})$ and $E_j(u_{E_j})$, which are given in their own coordinate systems. Let the centers O_i, O_j (poles) ellipses $E_i(u_{E_i})$, $E_j(u_{E_j})$ are at points (x_{E_i}, y_{E_i}), (x_{E_j}, y_{E_j}), and the ellipses are turned to the corners $\theta_{E_i}, \theta_{E_j}$ in accordance. Ellipse $E_i(u_{E_i})$ and $E_j(u_{E_j})$ given by large half-forces a_{E_i}, a_{E_j} and small halves b_{E_i}, b_{E_j} in accordance. Between the ellipses E_i and E_j restrictions may be set at the minimum permissible distances r_{ij}, but between the ellipse E_i and the boundary of the area Ω – restrictions on the minimum permissible distances r_i. It is necessary to carry out a geometric simulation of non-crossing conditions of ellipses with the study of their properties for the development of effective algorithms for their placement simulation.

By definition, phi-function for objects $E_i(u_{E_i})$ and $E_j(u_{E_j})$ [1] called continuous continuous function defined everywhere $\Phi^{E_iE_j}(u_{E_i}, u_{E_j}): R^6 \to R^1$, for which the following important property is performed: if $\Phi^{E_iE_j}(u_{E_i}, u_{E_j}) \geq 0$, then $\operatorname{int} E_i(u_{E_i}) \cap \operatorname{int} E_j(u_{E_j}) = \varnothing$.

Consider the method of geometric modeling of the cross-sections of the touch surface of two oriented and non-oriented ellipses. The overall structure of this method is as follows.

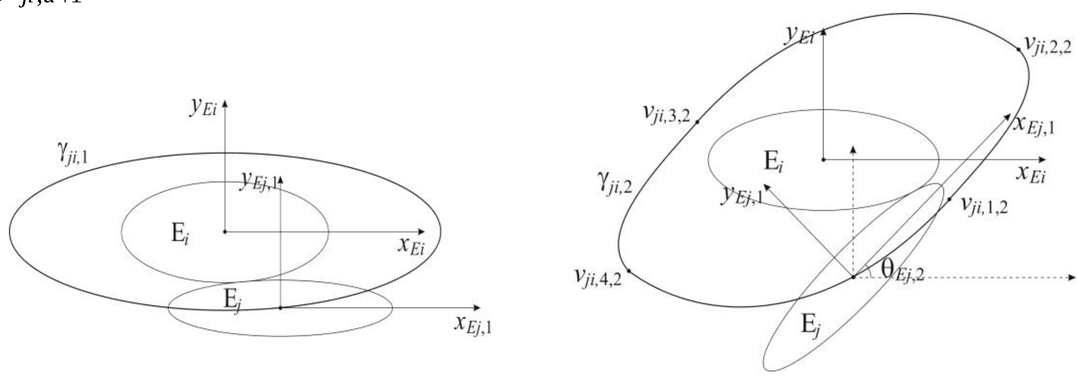
If the centers of the ellipses $E_i(u_{E_i})$, $E_j(u_{E_j})$ are at points $(0,0)$, (x_{E_j}, y_{E_j}), ellipses are oriented, i.e. $\theta_{E_i} = const, \theta_{E_j} = const$ and $r_{ij} = 0$, then $\Phi^{E_iE_j}(u_{E_i}, u_{E_j})$ will be given in space R^2 and represent a section of the surface of the touch of two non-oriented ellipses $E_i(x_{E_i}, y_{E_i}, \theta_{E_i})$ and

$E_j(x_{E_j}, y_{E_j}, \theta_{E_j})$.

Consider construction of the cross sections of the surface of the touch of two ellipses: orientated $E_i(0,0,0)$ and non-oriented $E_j(x_{E_j}, y_{E_j}, \theta_{E_j})$. For this placement options $E_i(0,0,0)$ are fixed, and the other object remains mobile and performs broadcasting along the boundary of the ellipse $E_i(0,0,0)$ so that $O_j \in FrE_i(u_{E_i})$, де $FrE_i(u_{E_i})$ - the border $E_i(u_{E_i})$ (Fig.1). The sample parameter job is executed n_d angle of rotation θ_{E_j} own coordinate system of a moving object. The value of the sampling parameter determines the number of cross sections of the contact surface of the two given ellipses. For everyone $\theta_{E_{j,}d+1} = d \cdot \dfrac{2\pi}{n_d}$, $d = 0, ..., n_d - 1, n_d > 0$, there is a construction of the section of the surface of the object touch $E_j(x_{E_j}, y_{E_j}, \theta_{E_j})$ and $E_i(0,0,0)$, each section being a closed loop.

In Fig. 1 shows an example of cross section construction $\gamma_{ji,1}$ and $\gamma_{ji,2}$ surface of the touch of non-oriented objects $E_j(x_{E_j}, y_{E_j}, \theta_{E_j})$ and $E_i(0,0,0)$ for the corresponding values of the angles of rotation $\theta_{E_j,1} = 0$ and $\theta_{E_j,2} = \dfrac{\pi}{4}$ local coordinate system of a moving object $E_j(x_{E_j}, y_{E_j}, \theta_{E_j})$ and immovable $E_i(0,0,0)$. Similarly, the construction of other sections for the angles of rotation is carried out $\theta_{E_{j,}d+1} = d \cdot \dfrac{2\pi}{n_d}$, $d = 0, ..., n_d - 1$ local coordinate system of a moving object.

Finally, the formation of a plurality of cross sections is carried out $\gamma_{ji,d+1}$,



a) b)

Fig. 1. Construction of the sections of the surface of the object touch

$$E_j(x_{E_j}, y_{E_j}, \theta_{E_j}) \text{ та } E_i(0,0,0)$$

$d = 0, ..., n_d - 1$, touch surfaces of two non-oriented ellipses. Yes, in Fig. 2 shows a plurality of cross sections of the object's surface $E_j(x_{E_j}, y_{E_j}, \theta_{E_j})$ and $E_i(0,0,0)$ near $n_d = 8$.

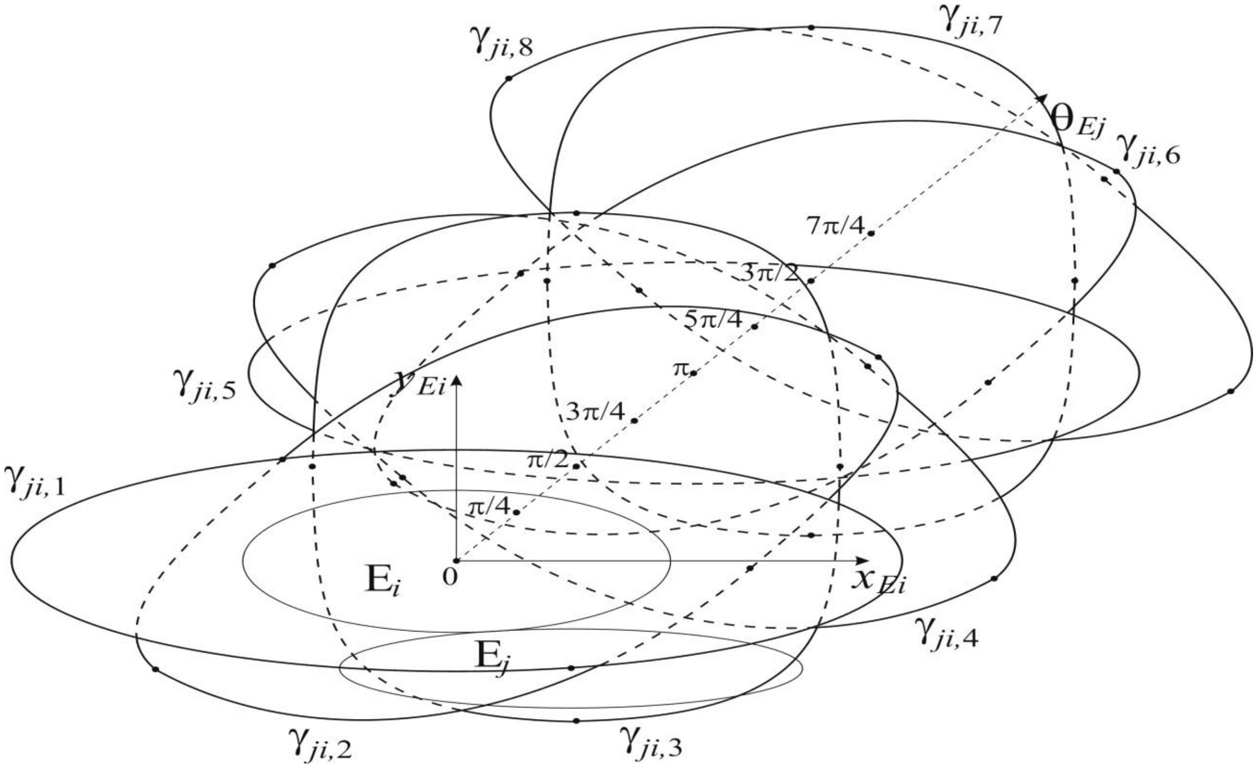


Fig. 2. Multiple cross sections $\gamma_{ji,d+1}$, $d = 0, ..., n_d - 1$ touch surfaces of two unoriented ellipses for $n_d = 8$

This set of cross-sections is a geometric interpretation of the non-crossing conditions of non-oriented ellipses whose rotation is discrete.

Consider the case when the angles change continuously.

The following statement was proved in [7].

Statement 1. If the convex objects (ellipses) do not intersect, then there is such a straight line $L_{ij}^{\perp}$, which passes through the center of the coordinate system in such a way that projections of objects on this straight line do not overlap.

Let the ellipses $E_i(u_{E_i})$ and $E_j(u_{E_j})$ have no common internal points (Fig. 3).

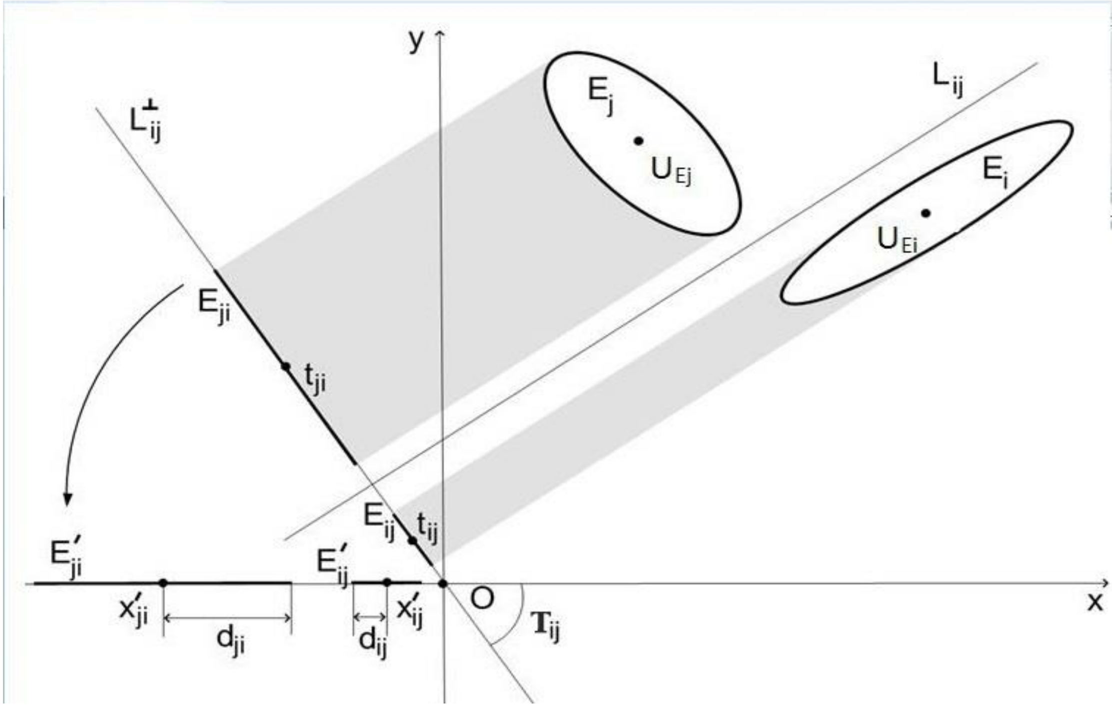


Fig. 3. Geometric illustration for building a phi-function $\Phi'^{E_iE_j}$

Let there be a straight line L_{ij} (dividing straight line), which divides the plane into two half-planes in such a way that objects $E_i(u_{E_i})$, $E_j(u_{E_j})$ lie in different half-planes. Consequently, projections of sets $E_i(u_{E_i})$, $E_j(u_{E_j})$ to any straight, perpendicular L_{ij}, do not intersect (do not have common internal points in R^1). Let's denote through $L_{ij}^{\perp}$ straight line which is perpendicular L_{ij} and passes through the center of the coordinate system, T_{ij} – the angle between the front L_{ij} and the axis *Ox*.

Turning straight $L_{ij}^{\perp}$ along with projections of ellipses E_{ij} and E_{ji} (with centers at points t_{ij} and t_{ji} in accordance) around the point O on the corner $(-T_{ij}$),get projections of ellipses E'_{ij} and E'_{ji} with centers in points x'_{ij} and x'_{ji} .

Thus, as a result of geometric simulation, it is shown that the condition of non-crossing of ellipses $E_i(x_{E_i}, y_{E_i}, \theta_{E_i})$ and $E_j(x_{E_j}, y_{E_j}, \theta_{E_j})$ equivalent condition: $x'_{ij} - x'_{ji} \geq d_{ij} + d_{ji}$, and a quasi-phi-function $\Phi'^{E_iE_j}$ can be written in the form:

$$\Phi'^{E_iE_j} = x'_{ij} - x'_{ji} - d_{ij} - d_{ji},$$

where $x'_{ij} = x_{ij}\cos T_{ij} - y_{ij}\sin T_{ij}$, $x'_{ji} = x_{ji}\cos T_{ij} - y_{ji}\sin T_{ij}$, T_{ij} – the angle between the front $L_{ij}^{\perp}$, which is perpendicular to the dividing line L_{ij}, and the axis *Ox*,

$$d_{ij} = \sqrt{b_i^2 + (a_i^2 - b_i^2)\cos^2(\theta_{E_i} - T_{ij})}, d_{ji} = \sqrt{b_j^2 + (a_j^2 - b_j^2)\cos^2(\theta_{E_j} - T_{ij})}.$$

Property 1. Point on the section of the surface of the object touch $E_j(x_{E_j}, y_{E_j}, \theta_{E_j})$ and $E_i(0,0,0)$, which is obtained by translating the pole of a moving object along the boundary of a fixed motion with a certain value of the angle of rotation $\theta_{E_j,d}$, corresponds to two projections of ellipses in line (according to statement 1), which touch when $r_{ij}=0$, and are at a distance r_{ij} in the presence of the distance between the ellipses.

Property 2. When changing the parameters $(x_{E_i}, y_{E_i}, \theta_{E_i})$ and $(x_{E_j}, y_{E_j}, \theta_{E_j})$ ellipses $E_i(u_{E_i})$, $E_j(u_{E_j})$, changes as the arrangement of pairs of segments on the axis *Ox*, and their size, moreover $d_{ij} \in [b_{E_i}, a_{E_i}]$, a $d_{ji} \in [b_{E_j}, a_{E_j}]$.

Property 3. The set of points of the surface of the surface of the touch of non-oriented objects $E_j(x_{E_j}, y_{E_j}, \theta_{E_j})$ and $E_j(x_{E_j}, y_{E_j}, \theta_{E_j})$, which corresponds to a certain value of the angle of rotation $\theta_{E_j,d}$ – is a continuous subset $M_{E_j,d}$, corresponding to the statement 1, pairs of segments on the axis *Ox*.

Property 4. The set of points of the cross sections of the surface of the touch of non-oriented objects $E_j(x_{E_j}, y_{E_j}, \theta_{E_j})$ and $E_j(x_{E_j}, y_{E_j}, \theta_{E_j})$, which corresponds to the angles of rotation $\theta_{E_j,d+1} = d \cdot \frac{2\pi}{n_d}$, $d = 0, ..., n_d - 1$, The local coordinate system of a moving object is a discrete set of subsets $M_{E_j,d}, d = 0, ..., n_d - 1$.

Property 5. A set of points of a surface of a touch of non-oriented objects $E_j(x_{E_j}, y_{E_j}, \theta_{E_j})$ and $E_j(x_{E_j}, y_{E_j}, \theta_{E_j})$ – This is an infinite set of pairs of chords on the axis *Ox*.

Conclusions. Further research will focus on the development of effective algorithms for geometric modeling of non-oriented ellipses in given areas based on the proposed methods for constructing conditions for their non-crossing.

Literature

1. Стоян Ю.Г. Полный класс Ф-функций для базових объектов / [Ю.Г. Стоян, Т.Е. Романова, Н.И. Чернов, А.В. Панкратов] // Доп. НАН України. – 2010. – № 12. – С. 25 – 30.

2. Стоян Ю.Г. Квази-phi-функции для математического моделирования отношений геометрических объектов / [Ю.Г. Стоян, А.В. Панкратов, Т.Е. Романова, Н.И. Чернов] // Доп. НАН України. – 2014. – Т 9. – С. 49 – 54.
3. Суббота И.А. Задача оптимальной упаковки эллипсов: математические модели и методы решения: дис. … канд. техн. наук: 01.05.02. Математичне моделювання та обчислювальні методи / И.А. Суббота. – Харьков, 2014. – 120 с.
4. Панкратов А.В. Phi-функции для эллипсов, аппроксимированных дугами окружностей / А.В. Панкратов // Радиоэлектроника и информатика. 2015. – 2(69). С. 6 – 9.
5. Kallrath J. Cutting Ellipses from Area-Minimizing Rectangles [Text] / J. Kallrath, S. Rebennack // JournalofGlobalOptimization. – 2013. – Vol. 59 (2-3). – P. 405 – 437. doi:10.1007/s10898-013-0125-3.
6. Панкратов А. В. / Оптимальная упаковка эллипсов с учетом допустимих расстояний [Текст] / А. В. Панкратов, Т. Е. Романова, И. А. Суббота // Журнал обчислювальної математики. – 2014. – Т.1 – С. 27 – 42.
7. Данилин А.Н. Упаковка эллипсов в прямоугольник минимальных размеров / [А.Н. Данилин, В.В. Комяк, В.М.Комяк, А.В.Панкратов] // УСиМ. – К.,2016. –№5. – С.5 – 9.

ГЕОМЕТРИЧЕСКОЕ МОДЕЛИРОВАНИЕ УСЛОВИЙ НЕПЕРЕСЕЧЕНИЯ ЭЛЛИПСОВ

Комяк В.М., Соболь А.Н., Данилин А.Н.

В статье рассматриваются способы геометрического моделирования условий непересечения двух эллипсов.

Ключевые слова: эллипс, геометрическое моделирование, условия непересечения.

ГЕОМЕТРИЧНЕ МОДЕЛЮВАННЯ УМОВ НЕПЕРЕТИНАННЯ ЕЛІПСІВ

Комяк В.М., Соболь О.М., Данилин О.М.

У роботі розглядаються способи геометричного моделювання умов неперетинання двох еліпсів.

Ключевые слова: еліпс, геометричне моделювання, умови неперетинання.