УДК 514.18

MAPLE-MODEL OF A PARTICLE MOVEMENT BY A ROUGHNESS PLANE, WHICH MAKES SLIDING OSCILLATIONS IN SPACE

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Trajectory-kinematic properties of the motion of a particle along a rough plane are presented, and translational oscillations are made along the parabola at the vertical plane.

Key words: rough plane, vibrational displacement, particle motion, trajectory, velocity.

Formulation of the problem. In many technological processes there is a motion of a particle on a rough plane, which in turn carries out a given displacement in space. For example, in the cultivating machines of the grain hopper there is a grains movement along a sloping rough plane that performs forward oscillations in a vertical direction. The difficulty of studying the motion of a particle in a moving rough plane depends on their position, the diversity of the laws of motion of the plane, and the initial conditions for the particle flutter in the plane. The solution to this problem is due to the development of a computer model that covers the various laws of rough plane motion and provides an interactive research mode.

Analysis of recent research and publications. In the classical papers [2, 3] the analytical foundations of the motion of a particle on the rough surfaces of the working organs of the s.-g. cars Using the accompanying triangular trajectory to describe the motion of a particle on a rough surface is disclosed in [4].

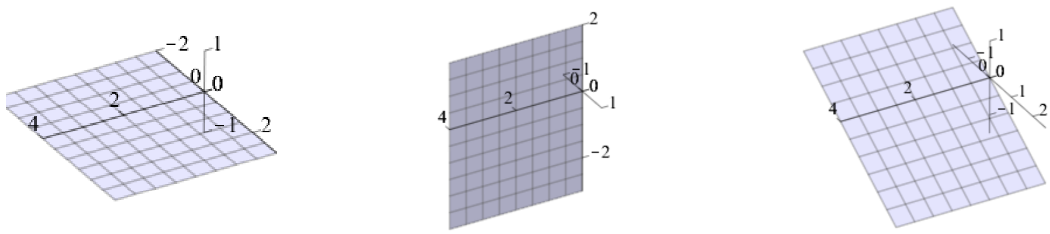
Formulating the goals of the article. To develop analytical and software tools for the Maple [1] symbolic algebra environment, a computer model for studying the motion of a particle on a rough plane, which carries out certain forward translational vibrations in space.

Main part. The developed model of motion of a particle in a moving rough plane consists of 3 blocks: 1) the task and analysis of initial conditions; 2) formation of the law of motion; 3) visualization of the results of the study.

There are three characteristic positions of the plane in three-dimensional space - vertical, horizontal and inclined (Fig. 1). Parametric equation of plane with axis Ox cartesian coordinate system $Oxyz$ is:

$$\boldsymbol{R}(u,v) = \boldsymbol{R}[u,\ v\cos(\xi)\,, v\ \sin(\xi)], \tag{1}$$

where ξ – angle of inclination of the plane $\boldsymbol{R}(u,v)$ to the plane Oxz.



a) b) c)

Fig.1 Characteristic plane positions in three-dimensional space

Move the plane $\boldsymbol{R}(u,v)$ in space can occur under the most diverse laws. We restrict the translational movement of the plane $\boldsymbol{R}(u,v)$, at which any straight in the plane $\boldsymbol{R}(u,v)$ when it moves in space remains parallel to itself. For all these cases, a portable trajectory $\boldsymbol{H}(u,v,t)$ the plane $\boldsymbol{R}(u,v)$ in the Cartesian coordinate system $Oxyz$ is the sum of the coordinate vector of the point of the plane $\boldsymbol{R}(u,v)$ and vector-functions of parallel transfer $\boldsymbol{M}[x(t),y(t),z(t)]$:

$$\boldsymbol{H}[x(u,v,t),y(u,v,t),z(u,v,t)] = \boldsymbol{R}[u,\ v\,cos(\xi)\,,\ v\,sin(\xi)] + \boldsymbol{M}[x(t),y(t),z(t)], \qquad (2)$$

Where $t = [0;t_n]$ – time (independent parameter);

$\boldsymbol{M}[x(t),y(t),z(t)]$ – guiding the curve in the coordinate system $Oxyz$.

The vector equation of motion of a particle in a rough plane, which performs translational displacements in space, has the form:

$$m\,\boldsymbol{w} = mg\,\boldsymbol{G} - f\left(\pm F_g\,cos\big(\widehat{\boldsymbol{N},\boldsymbol{G}}\big) \pm F_C\,cos\big(\widehat{\boldsymbol{N},\boldsymbol{n}}\big)\right)\boldsymbol{\tau_\rho}, \qquad (3)$$

where m – particle mass;

$\boldsymbol{w}$ - vector of absolute acceleration of a particle;

$\boldsymbol{\tau_\rho} = \frac{\boldsymbol{V_\rho}}{|\boldsymbol{V_\rho}|}$ - unit vector relative particle velocity;

$\boldsymbol{N}$ - the vector is normal along the trajectory in the relative motion of the particle;

$\boldsymbol{G} = [0,0,-1]$ - unit gravity vector;

$F_g = mg$ - gravity, $g = 9{,}81$ *м/с²*;

$F_C = m\,V^2\,k$ - centrifugal force, where V - absolute particle velocity, and k - curve of the absolute particle trajectory.

In the projections on the Orty $\boldsymbol{u} \equiv \boldsymbol{R}'_u$ and $\boldsymbol{v} \equiv \boldsymbol{R}'_v$ triangular $\boldsymbol{OuvN}$ vector equation of motion of a particle in the function of the time parameter t will look like (4):

$$\begin{cases} Ou := \ m\,W\,cos\big(\widehat{\boldsymbol{R}'_u,\boldsymbol{w}}\big) \ = F_g\ cos\big(\widehat{\boldsymbol{R}'_u,\boldsymbol{G}}\big) - f\ F_N\,cos\big(\widehat{\boldsymbol{R}'_u,\boldsymbol{\tau}}\big) \\ Ov := \ m\,W\,cos\big(\widehat{\boldsymbol{R}'_v,\boldsymbol{w}}\big) = F_g\ cos\big(\widehat{\boldsymbol{R}'_v,\boldsymbol{G}}\big) - f\ F_N\,cos\big(\widehat{\boldsymbol{R}'_v,\boldsymbol{\tau}}\big) \end{cases}. \qquad (4)$$

The above analytical support forms the basis of the developed application to the system of computer algebra Maple [1] studying the motion of a particle on a rough plane $\boldsymbol{R}$, which carries forward movements by any direction $\boldsymbol{M}[x(t), y(t), z(t)]$. In the Maple environment, the law of motion of a particle (4) is automatically formed, the equations of which are so cumbersome that they are not able to produce them here.

Example. Let the horizontal plane $\boldsymbol{R}[u, v, 0]$ performs reciprocating displacements in a vertical plane in a parabolic form:

$$\boldsymbol{M}(t) = [a\,sin(vt+\theta),\ 0,\ b\,sin^2(vt+\theta)\,], \qquad (5)$$

where do we get its mobile trajectory $\boldsymbol{H}[x(u, v, t), y(u, v, t), z(u, v, t)]$:

$$\boldsymbol{H} = \boldsymbol{R}[u, v, 0] + \boldsymbol{M}[a\,sin(vt+\theta),\ 0,\ b\,sin^2(vt+\theta)\,] = \boldsymbol{H}[u + a\,sin(vt+\theta)\,, v,\ b\,sin^2(vt+\theta)]. \qquad (6)$$

where v - angular velocity, rad / s;

θ - angle of the initial position of the plane, rad.

In Fig. 2, a five-point horizontal plane is constructed $\boldsymbol{R}$, all points of which move along the parabola (5). The plane begins to move from the bottom point to the top in the direction of the axis Ox, and then returns.

Trajectory-kinematic properties of the abandoned particle in a moving horizontal plane $\boldsymbol{R}$ depend on seven variables:

1. a, b – parameters of the shape of the points of the plane of the trajectory $\boldsymbol{R}$;
2. v, θ – angular velocity of the plane $\boldsymbol{R}$ and its initial position;
3. α_o, V_o – the angle of the throwing of the particle in the plane and its initial velocity;
4. f – coefficient of external friction of a particle.

We will show the effect on the motion of a particle of only one variable parameter - angular velocity v. In Fig. 2, b-f constructed absolute $\boldsymbol{r}(t)$, relative $\boldsymbol{\rho}(t)$ trajectory, graphs of absolute $V(t)$ and relative $V_\rho(t)$ its speed and normal reaction $F_N(t)$ depending on the angle $\alpha_o = -90°, -45°, 0°, 45°$ casting of a particle, coefficient of friction $f = 0.3$, initial speed $V_o = 6$м/с, parameters $a = 2, b = 1$м and angular velocity $v = 1$ с$^{-1}$. You can see the relative graph $V_\rho(t)$ speed (Fig. 2, e) that all particles stop. The very first through $t \approx 1.3c$ stop the particle that was thrown in the opposite direction ($\alpha_o = -90°$) plane movement.

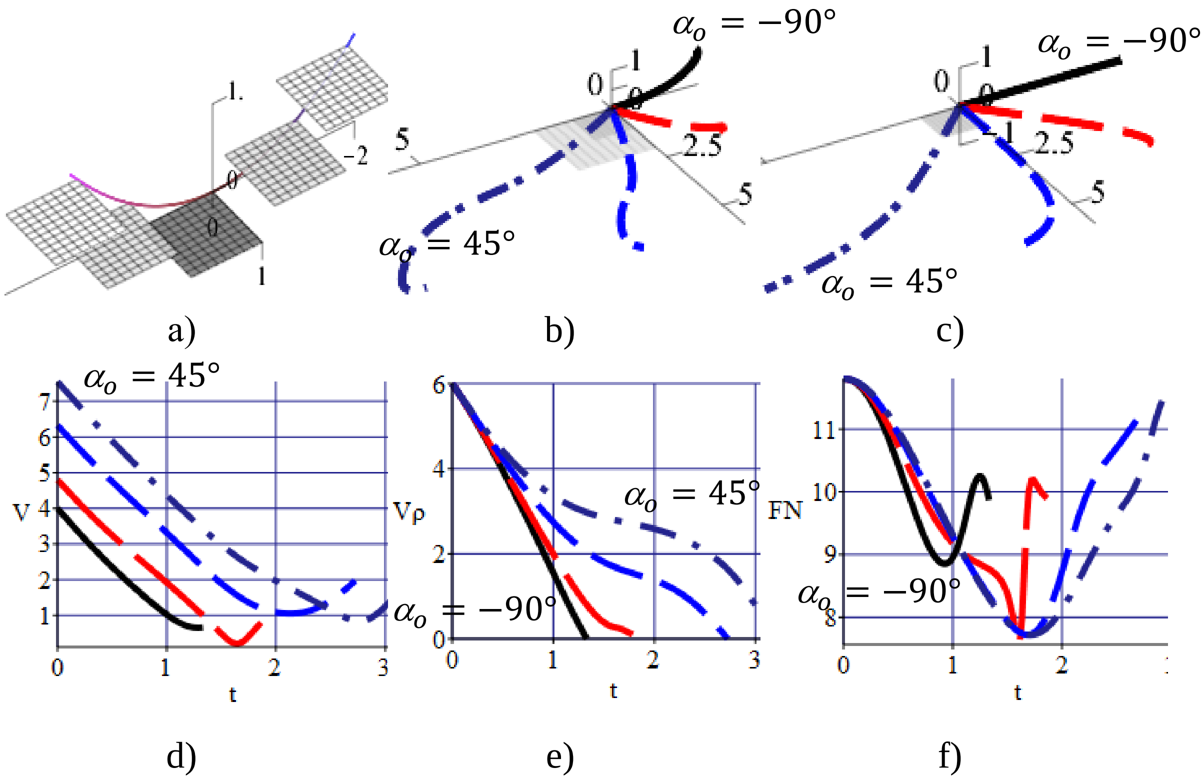


Fig.2 Absolute $\boldsymbol{r}(t)$, relative $\boldsymbol{\rho}(t)$ trajectory, graphs of absolute $V(t)$, relative $V_\rho(t)$ speed, normal reaction $F_N(t)$ particle at $\upsilon = 1$

We increase the angular velocity of the plane to the value $\upsilon = 2$ c^{-1} (Fig.3). The particles are thrown at an angle $\alpha_o = -45°, 0°, 45°$ never stop in the oscillatory plane. At $t > 6c$ graphs of absolute $V(t)$ and relative $V_\rho(t)$ the velocities of the particles zigzag-like coincide (Fig. 3, d, e), which means the transition to stationary mode. Relative trajectories $\boldsymbol{\rho}(t)$ for such a regime there are straight lines (Fig. 3, b), which are parallel to the direction of the parabola (5). Absolute trajectories $\boldsymbol{r}(t)$ after the transition process, there will be flat curve lines that are also parallel to the plane Oxz (Fig.3,a). Graphs of normal reaction $F_N(t) > 0$ and centrifugal force $F_C(t)$ (Fig.3,d,c) is a congruent curve. According to these graphs it can be argued that the smallest particle reaction is at the highest point of the trajectory - it is there that the particle can break away from the oscillatory plane. Note that only a particle is thrown at an angle $\alpha_o = -90°$ stop in the plane after a period of time $t \approx 1.9c$ and there will be no more movement in the plane - its absolute trajectory after the stop of the congruent parabola (5).

Increase the angular velocity to the value $\upsilon = 2.2$ c^{-1} will lead to the separation of all particles from the plane in a period of time $t \approx 0.7c$.

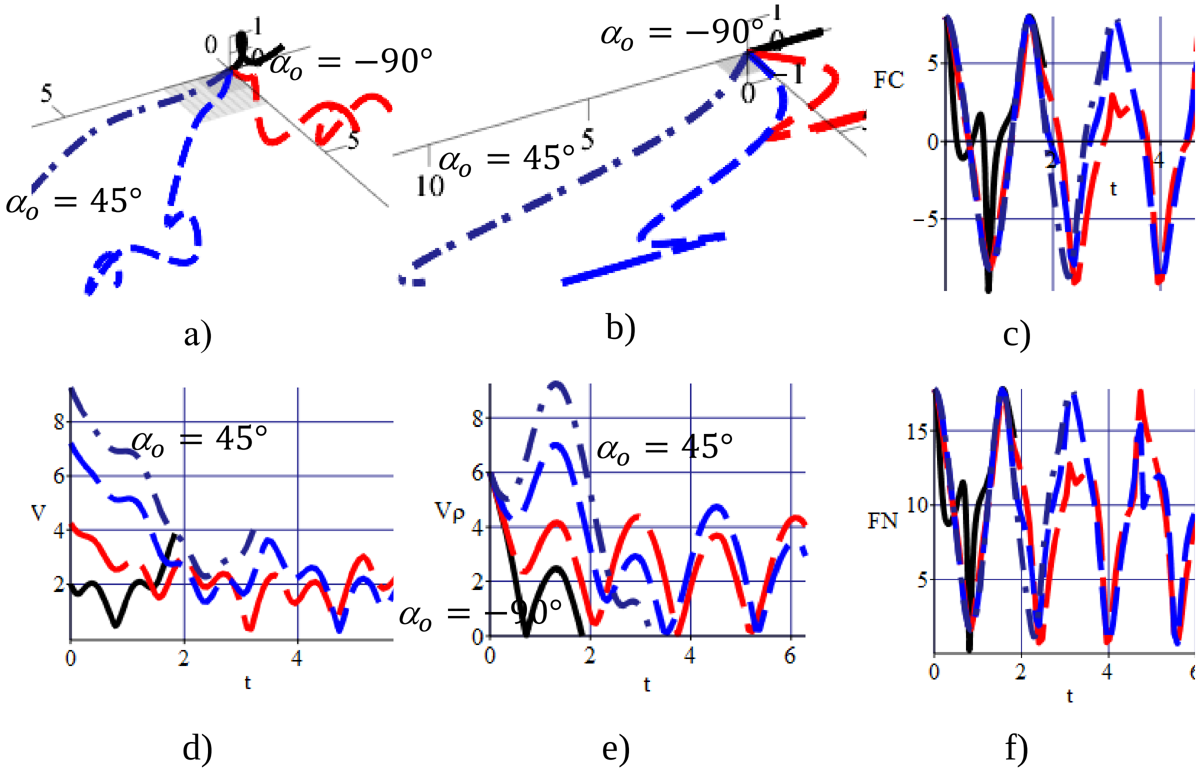


Fig. 3 Trajectory-kinematic properties of the abandoned particle at $\upsilon = 2$

Conclusions. The given example of studying the motion of a particle on a rough horizontal plane, which performs vibrational displacements in a parabola, reveals the multifactorial nature of the process, which without the development of similar applications to systems of computer mathematics to perform a complete study is not possible. One of the possible laws of motion of a rough horizontal plane, all of which in a vibrational motion describing a parabola in a vertical plane, is disclosed. The regularities of the motion of a particle in a plane, depending on the magnitude of its angular velocity, are found, namely, when the particles stop in the plane, or they will constantly move in it, or they will break away from it.

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MAPLE-МОДЕЛЬ ДВИЖЕНИЯ ЧАСТИЦЫ ПО ШЕРЕХОВАТОЙ ПЛОСКОСТИ, КОТОРАЯ СОВЕРШАЕТ ПОСТУПАТЕЛЬНЫЕ КОЛЕБАНИЯ В ПРОСТРАНСТВЕ

Несвидомин В.Н., Бабка В.Н., Несвидомин А.В.

Разработана maple-модель исследования движения частицы по шероховатой плоскости, которая осуществляет поступательные колебания. Приведены траекторно-кинематические свойства движения частицы по горизонтальной шероховатой плоскости, которая осуществляет поступательные колебания в вертикальной плоскости по параболе.

Ключевые слова: шероховатая плоскость, колебательное перемещение, движение частицы, траектория, скорость.

MAPLE-МОДЕЛЬ РУХУ ЧАСТИНКИ ПО ШОРСТКІЙ ПЛОЩИНІ, ЯКА ЗДІЙСНЮЄ ПОСТУПАЛЬНІ КОЛИВАННЯ В ПРОСТОРІ

Несвідомін В.М., Бабка В.М., Несвідомін А.В.

Розроблено maple–модель дослідження руху частинки по шорсткій площині, яка здійснює поступальні коливання. Наведено траєкторно-кінематичні властивості руху частинки у горизонтальній площині, що коливається по параболі у вертикальній площині.

Ключові слова: шорстка площина, коливальне переміщення, рух частинки, диференціальні рівняння, траєкторія, швидкість.