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DETERMINATION OF PARAMETERS OF NOT CHAOTIC TRAJECTORIES OF FLUCTUATIONS FREIGHT OF THE PENDULUM WITH THE MOBILE SUSPENSION

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The graphical computer method of modeling of fluctuations of freight of pendulums with mobile points of a suspension for the purpose of the choice of the parameters providing not chaotic nature of their trajectories is developed.

Keywords: pendulum with movable suspension point, Kapitsa pendulum, trajectory of the pendulum load.

Formulation of the problem. Mathematical pendulums with a movable pivot point are convenient models for testing methods for studying oscillatory processes [1-3]. Interestingly, geometrical forms of trajectories of movement along the plane (center) of the cargo cause. After all, they illustrate the solutions of the corresponding differential equations, which by analogy can be used in similar tasks. For example, with the help of a pendulum with a vibrational point in the suspension, Academician P.L. Kapitsa explained the effect of a high-frequency generator (nigotronoma) used in nuclear physics.

Analysis of recent research and publications. There is a large number of publications devoted to mathematical pendulums with a movable point of the suspension, among which the work is dedicated to the Kapitza pendulums [1-3]. They deal with various manifestations of the phenomenon of this kind of pendulums. The most interesting of these is manifested in the fact that the point of absolutely unstable equilibrium for a mathematical pendulum may prove to be the point of a stable equilibrium for the Kapitza pendulum. The problems of parametric resonance are considered, when the lower equilibrium position is not stable, and the amplitude of small deviations of the pendulum increases over time. Also interesting are the effects when, at a large amplitude of forced oscillations, chaotic modes can exist in the system.

These studies should be expanded by graphically illustrating the result of solving the equations describing the dynamics of pendulum oscillations with a movable pivot point. Namely - to visualize the trajectories of cargo fluctuations in order to detect among them non-chaotic trajectories.

Formulating the goals of the article. Develop a graphical computer method for selecting the values of parameters for obtaining non-chaotic trajectories of pendulum cargo fluctuations with a moving point of a suspension.

Main part. First, let us consider the case of the oscillation of a mathematical pendulum, whose hinged point moves along the horizontal *Ox* axis. To describe the dynamics of motion, we use [3] the differential equation:

$$L\left(\frac{d^2}{dt^2}v(t)\right)+\left(\frac{d^2}{dt^2}f(t)\right)\cos(v(t))+g\sin(v(t))=0\,. \qquad (1)$$

The following notation is taken in formula (1): *v (t)* is the function of changing the angle of the deflection of the pendulum; *L* - length of the pendulum; *f (t)* - the law of changing the position of the pendulum pivot point along the axis *0x; g* = 9.81.

Solving equation (1) will be a numerical Runge-Kutti method with initial conditions: a) v (0) = v0; b) v′(0) = dv0. To determine the values of the parameters v0 and dv0, which would provide a non-chaotic trajectory of the pendulum load, apply projective focus [4]. For this, a numerical method with selected initial conditions and taking into account the function *f(t)* we solve the equation (1) and construct an image of the integral curve in the phase space {*v, dv, t*} depending on the value of the "controlling" parameter - length *L*. At random values v0 and dv0 in the phase space a "confused" integral curve is formed (Fig. 1a). We design it on the phase plane {v, *dv*}, where we also observe the corresponding "confused" phase trajectory. For a certain critical meaning *L = L0* the character of the phase trajectory will change on a qualitative level - it will turn into a "natural" curve. At the phase plane, the optical effect of "sharpening" the confusion of phase trajectories will be observed (Fig. 1b). This process of finding the critical values of parameters is called projection focus.

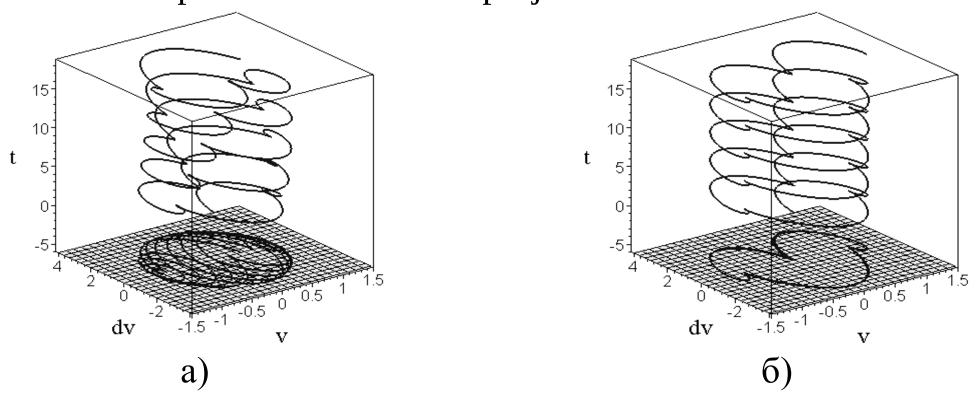
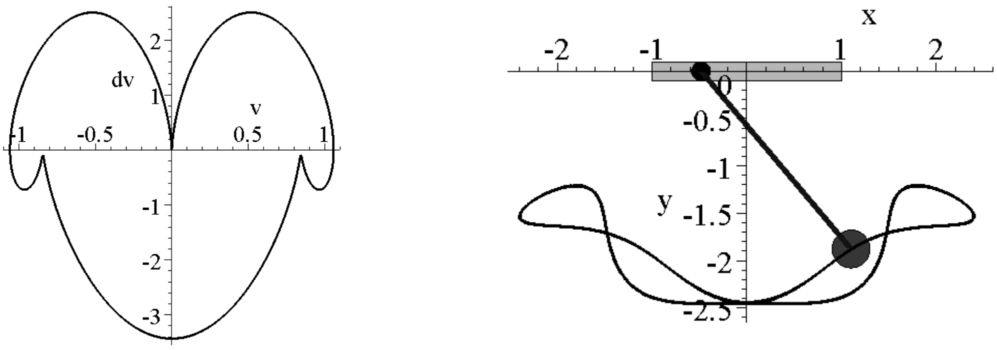


Fig. 1. Phase trajectories as projections of integral curves:
a) for an arbitrary value of the "controlling" parameter L;
b) for the critical value of the "controlling" parameter L0

Considering the critical value of the parameter L0 during the solution of the differential equation (1) will lead to the coordinates of the points on the plane {x, y}, which must be located on a non-chaotic trajectory.

Here is an example of solving the system of equations (1) with the following conditions: v0 =0; dv0 = 0; *f(t)* = sin(7*t*)/2. If you change the parameter *L*, for example: within 2 ≤ *L* ≤ 3 with a step *h* =0,2 we obtain a set of integral curves, one of which is shown in Fig. 2a The critical value of the parameter is obtained as a result of projection focusing at the value *L*0 = 2,456, which corresponds to rice. 1b and rice 2a In fig. 2b shows an example of a geometric modeling of the oscillation of the pendulum and the construction of a non-chaotic trajectory. In fig. 3 shows other discovered variants of non-chaotic trajectories.



а б

Fig. 2. An example of a pendulum oscillation modeling with parameters: v0 = 0; dv0 = 0; L = 2,456; *f(t)* = sin(7*t*)/2; a) the phase trajectory; b) a frame of the animation scheme of the oscillation of the pendulum

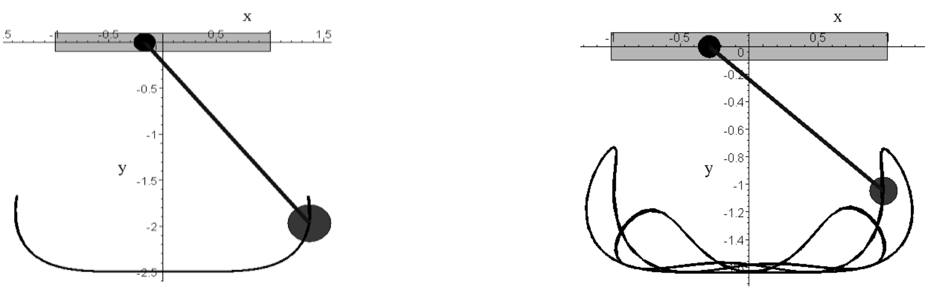


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Fig. 3. An example of a pendulum oscillation modeling with parameters: v0 = 0; dv0 = 0; *f(t)* = sin(5*t*)/2; at a) L = 2,496333; b) L = 1,64

Then let us consider the case of the oscillation of a mathematical pendulum whose point of the suspension moves along the vertical axis Oy. The theory of such a vibrational system in 1951 was proposed by Academician P.L. Kapitsa [1]. To describe the dynamics of the motion of a pendulum, we use [3] the differential equation:

$$L\left(\frac{d^2}{dt^2}v(t)\right)+\left(g+Aw^2\cos\left(wt\right)\right)\sin\left(v(t)\right)=0. \qquad (2)$$

In the formula (2) the following notation is adopted: v*(t)* – фfunction of changing the magnitude of the angle of the pendulum deflection; *L* –

length of the pendulum; cos*(wt)* – the law of changing the position of the pendulum pivot point along the axis *0y*; *A* – amplitude of oscillations; *w* – frequency of oscillations; g = 9,8.

Solving equation (2) will be a numerical Runge-Kutti method with initial conditions: v (0) = v0; v′(0) = dv0. To determine the values of the parameters *A* and *w*, which would provide a non-chaotic trajectory of the cargo movement of the pendulum, apply the method of projection focusing.

For this we find the solution of equation (2) for *L* = 0.1; *w* = 150 and with the initial conditions v0 = π/20; dv0 = 0. Using the projection focus method, we obtain the critical values of the amplitude of the oscillations A, which provide non-chaotic fluctuations of the load.

In fig. 4. Examples of geometric modeling of the trajectories of the cargo of the Kapitza pendulum for the obtained values of the amplitude of oscillations are shown (the frames of the animation scheme of the oscillation process are shown). In fig. 4, g for comparison is an example of chaotic fluctuations of cargo. In fig. 5 shows the phase trajectories, corresponding to the cases of fluctuations in rice. 4

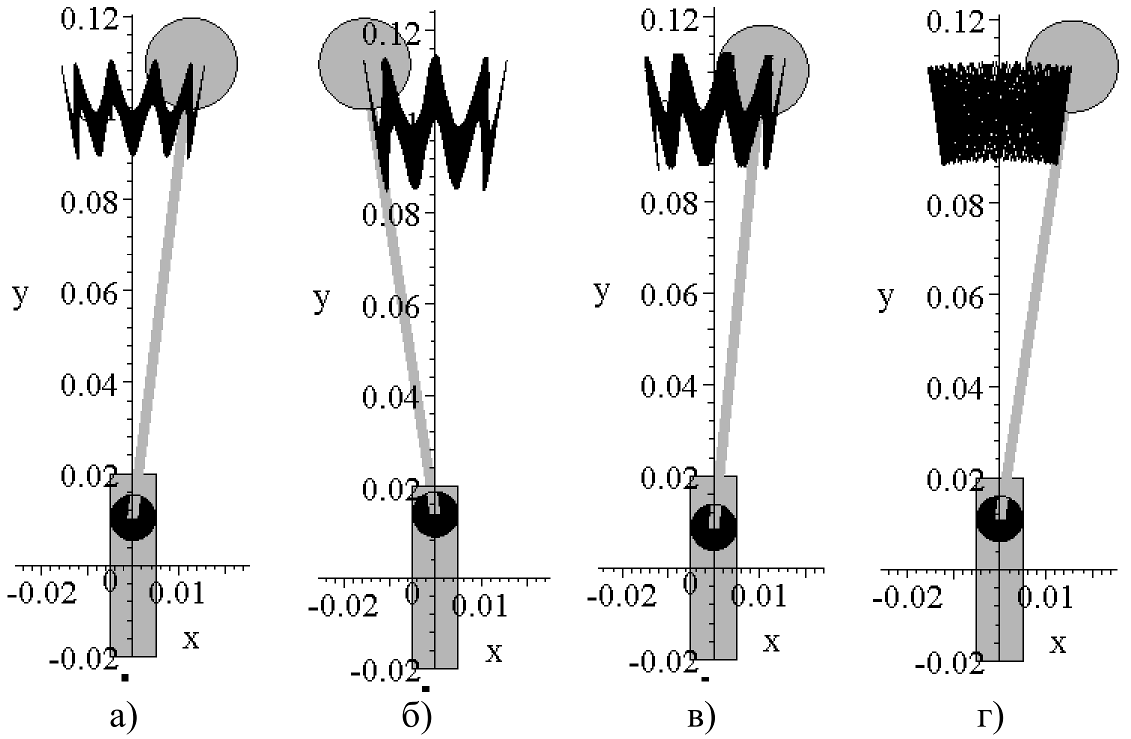
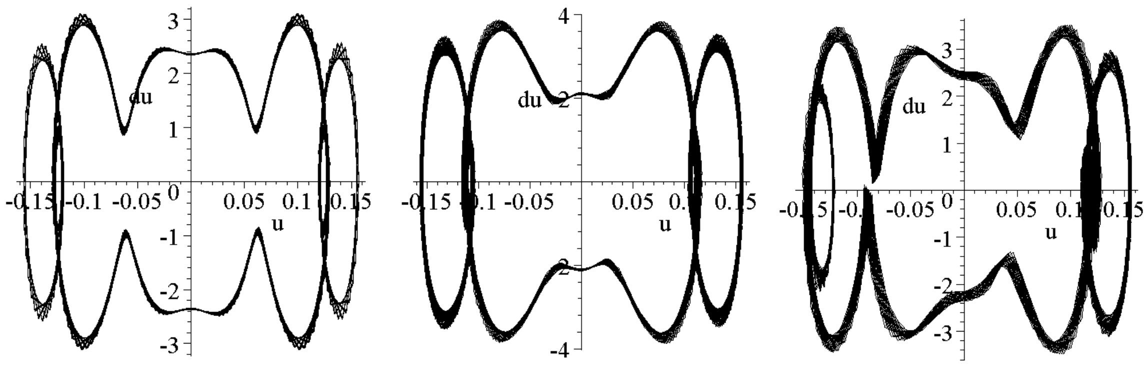


Fig. 4. Geometric simulation of trajectories
cargo of Kapitza Swing
for: a) A = 0,0105; b) A = 0,01469; c) A = 0.01245; d) A = 0.011.



а) б) в)

Fig. 5. Phase trajectories for:
a) A = 0,0105; b) A = 0,01469; c) A = 0.01245

Conclusions. A graphical computer model for fluctuations in the load of pendulums with moving points of the suspension is developed for the purpose of choosing the parameters that provide the non-chaotic technological character of their trajectories. The method is based on the approximate solution of Lagrange differential equations of the second kind, the definition of the projection of the resulting integral curve on the phase plane, and the calculation of the proposed method of projection focusing of the critical values of one of the parameters.

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ВЫЧИСЛЕНИЕ ПАРАМЕТРОВ НЕХАОТИЧЕСКИХ ТРАЕКТОРИЙ КОЛЕБАНИЙ ГРУЗА МАЯТНИКА С ПОДВИЖНЫМ ПОДВЕСОМ

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Приведен способ выбора значений параметров для получения нехаотических траекторий колебаний груза маятников с подвижными точками подвеса.

Ключевые слова: маятник с подвижной точкой подвеса, маятник Капицы, траектория перемещения груза маятника.

ОБЧИСЛЕННЯ ПАРАМЕТРІВ НЕХАОТИЧНИХ ТРАЄКТОРІЙ КОЛИВАНЬ ВАНТАЖУ МАЯТНИКА З РУХОМИМ ПІДВІСОМ

Семків О.М. , Руденко С.Ю.

Наведено спосіб вибору значень параметрів для одержання нехаотичних траєкторій коливань вантажу маятників з рухомою точкою підвісу.

Ключові слова: маятник із рухомою точкою підвісу, маятник Капиці, траєкторія переміщення вантажу маятника.