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THREE-DIMENSIONAL COMPOSITION MATRICES AND THEIR APPLICATIONS FOR CREATION OF COMPOSITIONAL GEOMETRIC MODELS OF VOLUME OBJECTS OF ANY ARBITRARY FORM

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In the research the geometrical way of creation of models of dynamics in space of discretely presented separate states of process on the basis of use of methods of compositional geometry is offered.

The definition of basic states, three-dimensional composite matrices is introduced, the rules of designation of indexing of elements of three-dimensional composite matrices (compomatrices) are offered.

It is pointed out that a three-dimensional composite cannot be presented in the form of a single table, so it is proposed to provide them in the form of a set of tables in the areas of parameterization of the geometric figure for which this three-dimensional computer matric is compiled.

Examples of their general and detailed presentation are given.

It is reminded that in composite geometric modeling (CGM) each initial geometric figure (GF), before solving the problem, must be unified, ie lead to a form suitable for its use in composite geometric modeling.

The geometric component of the unified GF is presented in the form of point matric matrices in parametric directions.

The parametric component of the unified GF is presented in the form of parametric compomatrices.

It is emphasized that all calculation operations are carried out through the use of three-dimensional coordinate matrices (calculated), which are compiled according to the scheme of the corresponding point compomatrices.

It is pointed out that the initially formed three-dimensional computer matric is parametric, almost always, non-harmonized, in the sum of all its elements is not equal to one.

An algorithm for harmonizing a parametric three-dimensional computer matric is provided.

The sequence of operations in the compomatric form concerning obtaining a compomatrices of three-dimensional for a three-dimensional *geometric figure of arbitrary form is given.*

Keywords: three-dimensional compomatrices, composite model, harmonized point polynomial, geometric method of interpolation, basis state, types of compomatrices, designation of compomatrices, compomatrices point, compomatrices parametric, compomatrices coordinate.

Statement of problem. Unification of each geometric figure (GF) is by dividing it into two components: geometric and parametric.

The geometric component of the unified GF is presented in the form of point matrices in parametric directions.

The parametric component of the unified GF is presented in the form of parametric compomatrices. An algorithm for the formation of parametric three-dimensional computer matrices is provided.

It is emphasized that all calculation operations are carried out through the use of three-dimensional coordinate matrices (calculated), which are compiled according to the scheme of the corresponding point matrices.

Emphasis is placed on the fact that in this study a compositional model of a segment of a geometric body consisting of three points in each of the parametric directions is built. It is also pointed out that the proposed algorithm will be true for creating models of segments of three-dimensional geometric shapes that hold more points in each of the three directions that indicate the size of this segment.

Research method. In this work, the research is carried out by the methods of compositional geometry (CG) [13] with powerful tools for the formation of point polynomials that perform the initial conditions and compositional matrices.

CG is based on the point calculus of Balyuba-Naidysh (point BN-calculus). [1]

Geometric methods of interpolation with one-, two- and three-parameter point polynomials are used in CG methods.

Geometric method of interpolation (GMI).

Option 1. It is provided by point polynomials, which have their components of the product of each of the basis points for the corresponding, specially formed, characteristic functions, and is that each of these characteristic functions is equal to one only at the basis point, which is its multiplier and through which , for the corresponding value of the parameter t, is a point polynomial, and at other basis points, this characteristic function is zero. That is, with the geometric method of interpolation in each interpolation node, a point polynomial has only one term equal to the corresponding basis point, thus ensuring global interpolation of all source basis points.

Option 2. Provided with characteristic functions that are formed based on the geometric conditions laid down in the original geometric composition, and are components of point polynomials, which in the interpolation nodes are zero or one, due to which global interpolation of starting points geometrically. GSI does not need to compose and solve systems of linear equations to find the coefficients that provide the initial geometric conditions, because in point polynomials such coefficients do not exist at all.

A single-parameter unharmonized point polynomial is a parametric curve consisting of the sum of the products of integer rational functions and basis points, and rational functions (characteristic functions) are given as the product of the differences between the parameter values for basis points and the current parameter $0 \le t \le 1$, which are composed for each of the basis points separately and have the same degree (*n-1*) for all its (point polynomial) terms.

Harmonization of a parametric compomatrices is a change in the values of its parameter-elements by dividing each of the elements of this parametric compomatrices by the sum of all its elements.

Recent research and publication analysis. Referring to [9], we agree that dynamics models are much more complex than statics models. This is due to the fact that static models describe the transformation of the input value to the output, and dynamics models describe the transformation of the input time function to the output time function.

The basis and the simplest form of dynamics models are differential equations [9], the application of which requires consideration of whether or not time is included as their component, explicitly or implicitly.

However, the application of differential equations to create models in the space of states is a rather cumbersome process [9], which requires special mathematical training and some experience in their application. This requirement remains even with the application of methods of simplification of differential equations by transforming them into ordinary algebraic equations by creating a differentiation operator.

The peculiarity of composite geometric modeling (CGM) is that any initial geometric figure (GF) before immersion in CGM should be divided into geometric and parametric components, each of which is described using appropriate composite matrices. However, in these works [4, 1, 7, 5, 6], the construction of composite geometric models is considered only for one- and two-parameter GF using one- and two-dimensional composite matrices of point, parametric and coordinate.

Based on the analysis, the problem of geometric methods of constructing dynamics models, which would be mathematically formalized by the methods of compositional geometry, is relevant.

Setting article objectives. Develop rules for the formation of three-dimensional composite matrices of point, parametric, coordinate and computer matrices of a geometric figure, in general. Develop rules for the notation of all mentioned compositional matrices. Using three-dimensional computer matrices to create a method of forming harmonized three-parameter point polynomials.

Main part. At present, the basis and tools for creating models of dynamics in the state space are differential equations [9], for the compilation of which it is necessary to take into account many initial requirements that are not always easy to algorithmize. At the same time, the application of differential equations to create models in the state space is a rather cumbersome process [9]. The use of differential equations for modeling requires special mathematical training and some experience in their application. In addition, analytical methods of using differential equations are deprived of visual visualization of the solution. This paper proposes a geometric way to create models of dynamics in the space of states based on the use of methods of compositional geometry.

Consider the simplest case where the process is described by only three basis states, in each of which nine basis points are defined (Fig. 1).

In the given example (Fig. 1), the values of the parameters for each of the three states are given by nine points, which for unloading Figs. 1 are indicated only by indexes consisting of three digits, and the capital letter is the same for all 27 points.

The first digit in the index indicates a change in the base point numbers in the positive direction of the parameter U.

The second digit is to change, in the direction of increase, the numbers of base points in the positive direction of the parameter V.

The third digit is to increase the number of base points in the positive direction of the parameter W.

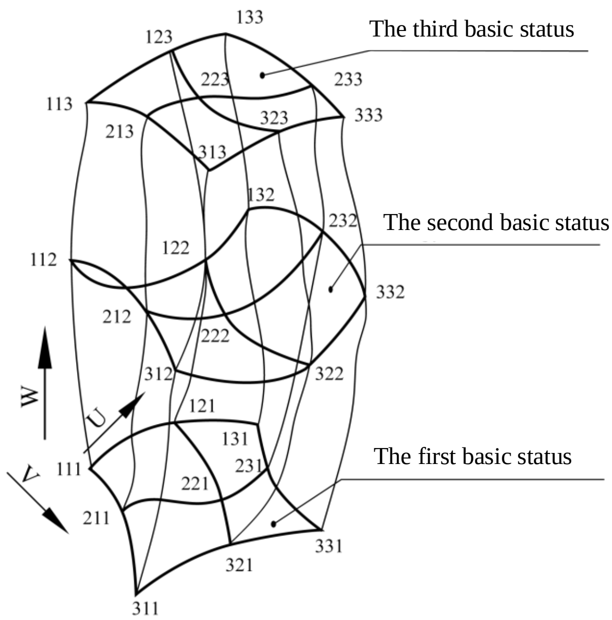

Fig. 1. Initial geometric figure for the model of dynamics in the state space

If any of the numbers consists of two or more digits, we suggest that to highlight this number, underline it. For example, 42 36; 2124; 21 32 43.

To create a geometric figure (GF) (Fig. 1), a geometric composition is considered, among the points of which are selected those that determine the first, second and third basis states. The verbal component of the unified GF determines that the formed base states are, at some point, cross sections of the process being modeled, and that using them it is necessary to create a continuous model of the body of this process, which is determined by three parameters U, V, W. formalize by creating three-dimensional composite matrices, which will be denoted in double square or parentheses, a capital Latin letter with an inscription at the bottom indicating the size of this three-dimensional composite matrices – $[[\underset{l\times m\times n}{A}]]$. In expanded form, any three-dimensional composite matric cannot be presented in the form of a single table corresponding to the three directions of process parameterization. In this case, three-dimensional composite matrices (compomatrices) and point – $\underset{l\times m\times n}{[[A_T]]}$ and parametric – $\underset{l\times m\times n}{[[A_n]]}$ are represented by three tables. In order to understand in the direction of which of the parameters of the composite matrices, at the top of the bracket we will write the notation of this parameter:

$$\underset{l\times m\times n}{\overset{u}{[[A_T]]}};\quad \underset{l\times m\times n}{\overset{u}{[[A_n]]}};\quad \underset{l\times m\times n}{\overset{v}{[[A_T]]}};\quad \underset{l\times m\times n}{\overset{v}{[[A_n]]}};\quad \underset{l\times m\times n}{\overset{w}{[[A_T]]}};\quad \underset{l\times m\times n}{\overset{w}{[[A_n]]}} \qquad (1)$$

We open three-dimensional point compomatrices (1) for an example of a three-dimensional process of arbitrary shape, which is shown in Fig. 1:

$$\underset{l\times m\times n}{\overset{u}{[[A_T]]}}=\underset{l\times m\times n}{\overset{u}{[[A_{ijk}]]}}=\overset{u}{\left[\left[\begin{matrix} A_{111} & A_{121} & A_{131}\\ A_{211} & A_{221} & A_{231}\\ A_{311} & A_{321} & A_{331}\\ A_{112} & A_{122} & A_{132}\\ A_{212} & A_{222} & A_{232}\\ A_{312} & A_{322} & A_{332}\\ A_{113} & A_{123} & A_{133}\\ A_{213} & A_{223} & A_{233}\\ A_{313} & A_{323} & A_{333} \end{matrix}\right]\right]};\ i:j:k=\overline{1{,}3}; \qquad (2)$$

$$\underset{l\times m\times n}{\overset{v}{[[A_T]]}}=\underset{l\times m\times n}{\overset{v}{[[A_{ijk}]]}}=\overset{v}{\left[\left[\begin{matrix} A_{111} & A_{211} & A_{311}\\ A_{121} & A_{221} & A_{321}\\ A_{131} & A_{231} & A_{331}\\ A_{112} & A_{212} & A_{312}\\ A_{122} & A_{222} & A_{322}\\ A_{132} & A_{232} & A_{332}\\ A_{113} & A_{213} & A_{313}\\ A_{123} & A_{223} & A_{323}\\ A_{133} & A_{233} & A_{333} \end{matrix}\right]\right]};\ i:j:k=\overline{1{,}3}; \qquad (3)$$

$$\underset{l\times m\times n}{\overset{w}{[[A_T]]}}=\underset{l\times m\times n}{\overset{w}{[[A_{ijk}]]}}=\overset{w}{\left[\left[\begin{matrix} A_{111} & A_{112} & A_{113} \\ A_{211} & A_{212} & A_{213} \\ A_{311} & A_{312} & A_{313} \\ A_{121} & A_{122} & A_{133} \\ A_{221} & A_{222} & A_{223} \\ A_{231} & A_{232} & A_{233} \\ A_{131} & A_{132} & A_{133} \\ A_{231} & A_{232} & A_{233} \\ A_{331} & A_{332} & A_{333} \end{matrix}\right]\right]};\ i:j:k=\overline{1,3}. \qquad (4)$$

Three-dimensional compomatrices (2), (3), (4) form a set of composite point:

$$\underset{l\times m\times n}{[[A_T]]}\Rightarrow\left\{\begin{matrix} \overset{u}{[[A_T]]} \\ \overset{v}{[[A_T]]} \\ \overset{w}{[[A_T]]} \end{matrix}\right], \qquad (5)$$

using which simplifies the possibility of forming a point, a polynomial that continuously determines the position of any current point not only on the surface but also in the middle of the three-dimensional three-parameter GF, which is shown in Fig.1.

It is clear that three-dimensional point compomatrices (2), (3), (4) are schemes for creating a model. All calculation operations according to the created three-dimensional process model should be performed according to calculated (coordinate) three-dimensional compomatrices, which are similar to point (2), (3), (4) with the corresponding coordinate of the parameter space for basis points. The fact is that the GF points (Fig. 1) in compositional modeling can be determined by *k* coordinates of the *k*-space of parameters $(K=\overline{1,k})$. Then the set (5) in the coordinate form of the k-space of parameters will have the following record:

$$[[A(K)]]\Rightarrow\left\{\begin{matrix} \overset{u}{[[A_{ijk}(K)]]} \\ \overset{v}{[[A_{ijk}(K)]]} \\ \overset{w}{[[A_{ijk}(K)]]} \end{matrix}\right],\ K=\overline{1,k};\ i:j:k=\overline{1,3}. \qquad (6)$$

Here in the (6) compomatrices the coordinate forms according to the assembly scheme are identical to the point compomatrices (2), (3), (4), (5). We will reveal three-dimensional parametric matrices from (1), which are combinational, formed by performing certain algebraic operations in order to determine the parameters - characteristic functions based on the geometric indicators of the original geometric composition. Thus, point compomatrices represent the geometric component of a unified GF and are a geometric composition, and parametric compomatrices represent the parametric component of the same unified GF, and are combinational, namely those in which any change in one element - must make appropriate changes to all its others. elements. However, we leave the term "composite matric" for the parametric compomatrices due to the fact that operations with it are subject to the rules of action on compomatrices. Characteristic functions are integer rational functions in parametric form for parameter values which are usually denoted $P_{ijk}(u);q_{ijk}(v);r_{ijk}(w)$. To reduce entries, we will not specify the parameters in parentheses. We will identify them by the letters that denote the most characteristic functions, namely:

$$P_{ijk}(u)=P_{ijk};q_{ijk}(v)=q_{ijk};r_{ijk}(w)=r_{ijk}. \quad (7)$$

Conclusions. The information presented in this study contributes to the further development of a new theory of composite matrices. The use of computer matrices in geometric modeling allows the shortest possible description of analytical solutions performed by geometric methods. Compomatrices are the basis for constructing point polynomials, the use of which in compositional geometry allows you to create geometric models of lines, surfaces, and three-dimensional objects of arbitrary shape under predetermined conditions. In this case, the use of point polynomials allows interpolation of lines with multiple points and even those that degenerate into a point; perform interpolation of surfaces with irregular grids holding triangular cells; create point three-parameter equations of point polynomials, which are used to determine, on the segment of a three-dimensional geometric figure, the position of any current point not only on the surface but also inside the segment.

This feature is extremely important for use on 3D printers, which will allow you to print products with the necessary cavities inside, as well as to model the trajectory of the manipulators.

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ТРИРОЗМІРНІ КОМПОЗИЦІЙНІ МАТРИЦІ ТА ЇХ ЗАСТОСУВАННЯ ДЛЯ СТВОРЕННЯ КОМПОЗИЦІЙНИХ ГЕОМЕТРИЧНИХ МОДЕЛЕЙ ОБ'ЄМНИХ ОБ'ЄКТІВ ДОВІЛЬНОЇ ФОРМИ

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У дослідженні запропонованого геометричний спосіб створення моделей динаміки у просторі дискретно поданих окремих станів процесу на базі використання методів композиційної геометрії.

Вводиться означення базисних станів, тририозмірних композиційних матриць, пропонуються правила позначення індексації елементів тририозмірних композиційних матриць (компоматриць).

Вказується на те, що тририозмірну композиційну неможливо подати у вигляді однієї таблиці, тому запропоновано надавати їх у вигляді сукупності таблиць за напрямками параметризації геометричної фігури, для якої складається ця тририозмірна компоматриця.

Наведено приклади загального та розгорнутого їх подання.

Нагадується, що у композиційному геометричному моделюванні (КГМ) кожну вихідну геометричну фігуру (ГФ), перед розв'язанням задачі, необхідно уніфікувати, тобто привести до вигляду, придатного для її використання у композиційному геометричному моделюванні.

Геометрична складова уніфікованої ГФ подається у вигляді компоматриць точкових за параметричних напрямків.

Параметрична складова уніфікованої ГФ подається у вигляді компоматриць параметричних.

Наголошується, що усі розрахункові операції здійснюються через використання тририозмірних компоматриць координатних (розрахункових), які складаються за схемою відповідних компоматриць точкових.

Вказується на те, що початково сформована тривимірна компоматриця параметрична, майже завжди, є негармонізованою, тобто сума всіх її елементів не дорівнює одиниці.

Надається алгоритм гармонізації параметричної тривимірної компоматриці.

Надається послідовність операцій у компоматричній формі щодо здобуття компоматриці тривимірної для об'ємної геометричної фігури довільної форми.

Ключові слова: тривимірна компоматриця, композиційна модель, гармонізований точковий поліном, геометричний спосіб інтерполяції, базисний стан, види компоматриць, позначення компоматриць, компоматриці точкові, компоматриці параметричні, компоматриці координатні.

ТРЁХРАЗМЕРНЫЕ КОМПОЗИЦИОННЫЕ МАТРИЦЫ И ИХ ПРИМЕНЕНИЕ ДЛЯ СОЗДАНИЯ КОМПОЗИЦИОННЫХ ГЕОМЕТРИЧЕСКИХ МОДЕЛЕЙ ОБЪЕМНЫХ ОБЪЕКТОВ ПРОИЗВОЛЬНОЙ ФОРМЫ

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В исследовании предложено геометрический способ создания моделей динамики в пространстве дискретно представленных отдельных состояний процесса на базе использования методов композиционной геометрии.

Вводится определение базисных состояний, трёхразмерных композиционных матриц, предлагаются правила обозначения индексации элементов трёхразмерных композиционных матриц (компоматриц).

Указывается на то, что трёхразмерную композиционную невозможно представить в виде одной таблицы, поэтому предложено предоставлять их в виде совокупности таблиц по направлениям параметризации геометрической фигуры, для которой составляется эта трёхразмерная компоматрица.

Приведены примеры общего и развернутого их представления.

Напоминается, что в композиционном геометрическом моделировании (КГМ) каждую исходную геометрическую фигуру (ГФ), перед решением задачи, необходимо унифицировать, то есть привести к виду, пригодного для ее использования в композиционном геометрическом моделировании.

Геометрическая составляющая унифицированной ГФ подается в виде компоматриц точечных при параметрических направлениях.

Параметрическая составляющая унифицированной ГФ подается в виде компоматриц параметрических.

Отмечается, что все расчетные операции осуществляются через *использование трёхразмерных компоматриц координатных (расчетных), которые составляются по схеме соответствующих компоматриц точечных.*

Указывается на то, что изначально сформирована трёхразмерная компоматрица параметрическая почти всегда является негармонизированной, то есть сумма всех ее элементов не равна единице.

Предоставляется алгоритм гармонизации параметрической трёхразмерной компоматрицы.

Предоставляется последовательность операций в компоматричной форме по получению компоматрици трёхразмерной для объемной геометрической фигуры произвольной формы.

Ключевые слова: трёхразмерная компоматрица, композиционная модель, гармонизированный точечный полином, геометрический способ интерполяции, базисное состояние, виды компоматриц, обозначения компоматриц, компоматрицы точечные, компоматрицы параметрические, компоматрицы координатные.

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