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**MODELING SURFACES ACCORDING TO SPECIFIED CONDITIONS  
BASED ON AN ARRAY OF POINTS**

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*The development of technology of design of computer models of surfaces defined by points array, and programs of their processing on the machines with computer numerical control (CNC). The main requirement to the surface of products which interact with the environment, is to ensure a given character of their flow. Functional quality of the surface is ensure by its geometric characteristics. Laminar nature of flow of surfaces can be provided by monotonous change of curvature values, torsion, radiuses of adjoining spheres along the curves that are part of the determinant of the surface. The initial points set and geometrical characteristics of the curve are the initial data for the formation of linear elements of carcass of surface. The methods of formation of plane and spatial curves with regular change of characteristics on the basis of an arbitrary points set is developed. At each site, which is bounded by successive points the area within which are all the curves corresponding to the conditions of the problem is determined. The resulting curve, which is represented by a new point series consisting of an arbitrarily large number of points, it is considered formed when the area of its possible location is less than a predetermined value. Practical application of the proposed technology is demonstrated by the example of designing of surfaces inter scapular channel of impeller of the turbocharger and the working surface of the disk cutter for loosening the soil. The developed technology for the design of surfaces of complex technical products includes the creation of geometric models of surfaces and control programs for their processing on CNC machines. From the initial array of points allocated subset - point sets, on the basis of which the linear elements of the carcass of surface are formed. A computer model of the surface is created on the basis of discretely carcass the line represented by the families of longitudinal and transversal curves. The control program is created in the automated mode using the CAM-packets based on three-dimensional computer model which is formed in a package of three-dimensional parametric modeling*

*SolidWorks.*

*Keywords: dynamic surface; carcass of surface; linear element of model; discretely represented curve; regular change of curvature and torsion*

**Statement of problem.** At the current stage of development of engineering it is actual the task of surface modeling on the basis of array of points. Coordinates of the points obtained by measurements on physical samples or calculated according to the conditions operation of the product.

The process of surface modeling includes the steps:

- discrete line carcass of surface, linear elements of which are presented by point set, is formed on the basis of the initial points array;
- continuous contours that interpolate sets of points are formed;
- the model of surface of product is created on the basis of the line carcass;
- computer model is used as input data for the creation of a control program for processing the product of the machine with a computer numerical control (CNC).

Given the nature of ensuring flow is main requirement to the surface of products that interact with the environment [7,9,11]. Functional qualities of surface are ensured by its geometrical characteristics. Laminar flow of surfaces can be provided by monotone change of curvature values, torsion, radiuses of adjoining spheres along the lines belonging to the surface.

**Recent research and publication analysis.** The existing methods [10,12] of interpolation (method of piecewise-smooth approximations, Bezier curves, B-spline) do not provide a given law of change of characteristics along the curve.

The methods of forming of curves with the regular change of differential-geometric characteristics are proposed in [2,3,6,8]. The initial data for the formation of the curve is the points set and its geometric characteristics. This curve will be called discrete represented curve (DRC).

The starting points set is divided into plots that is possible be interpolated by DRC with constant move and along which the radiuses of curvature and adjoining spheres change monotonically. Such areas will be called monotone curves.

Consider the points set which located on the monotonous curve  $l$ . Every four consecutive points define the sphere –  $Sp_i(i-1, i, i+1, i+2)$  and two circles –  $Cir_i(i-1, i, i+1)$  and  $Cir_{i+1}(i, i+1, i+2)$  (fig. 1).

On the fig. 1 curve  $l$  is located so that the view of the observer is directed along a line  $(i, i+1)$ . Contours  $Sp_{i-1}, Sp_i, Sp_{i+1}$  – circles of maximum radius located on a plane  $P_i$  which extends perpendicular to the chord  $[i, i+1]$  in its middle.  $Cir_i$  and  $Cir_{i+1}$  projected to the notochord of contour  $Sp_i$ . If the distance is between two points is infinitely small, they define adjoining sphere and two adjoining circles. At increase in distance between successive points belonging to  $l$ , the circle and the sphere which are determined by these two points will cross the curve.  $Sf_i$  cross  $l$  in points  $i-1, i, i+1, i+2$ . The parts of curve

... $i-1$ ,  $i-i+1$ ,  $i+2$ ... is located outside  $Sf_i$ , parts  $i-1-i$  and  $i+1-i+2$  – inside of it. This implies that consecutive  $Sf_{i-1}$ ,  $Sf_i$ ,  $Sf_{i+1}$  limit area ( $\delta_i$ ) inside of which is located plot  $i-i+1$  of curve  $l$ . Fig. 1 shows a section  $\delta_i$  by plane  $P_i$ .

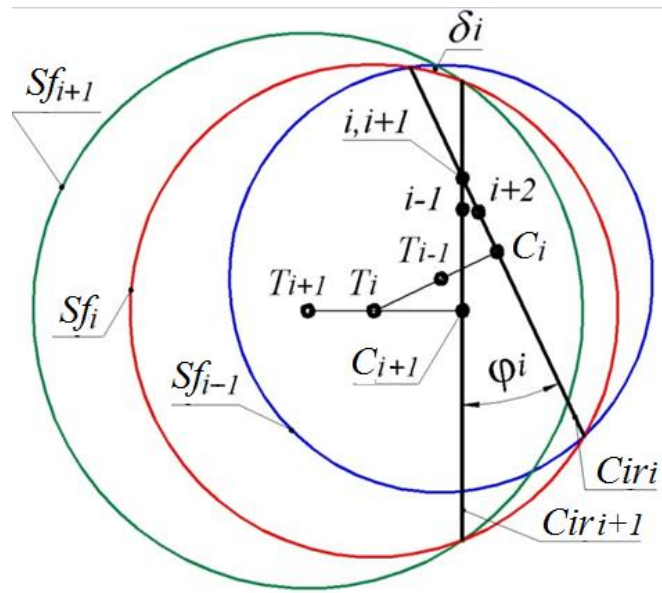


Fig. 1

Monotonous plots of DRC are formed by destination of points of thickening within the area of possible solutions  $\delta_i$ .

The same areas determined at the other plots constitute the area of possible location of the DRC. All curves interpolating points set which characteristics are match the characteristics of  $l$ , are located inside this area.

In the case of forming a plane DRC “the monotone curve” will be called the plot a smooth curve along which the values of radiuses of curvature monotonically increasing or decreasing.

The curve is formed within the chain of basic triangles (BT), which are limited to the tangent extending through the adjacent points and the chord joining these points. Following the appointment of thickening point ( $i_{sg}$ ) and the tangent at this point ( $t_{sg}$ ) inside the initial BT obtained two new triangles (fig. 2).

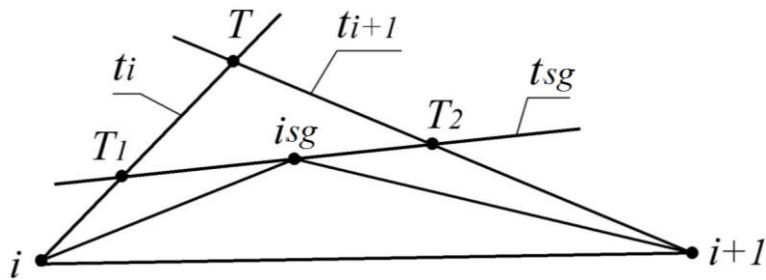


Fig. 2

The position of tangents is assigned so that the parameters of consecutive BT ensure the opportunity of interpolation points set by monotone curve [6].

Determining the area of possible location of the curve allows to evaluate

the maximum absolute error which is formed by the DRC contour. DRC is considered formed when the area of its possible location is less than a given value. Thereafter, the contour is forming by plots of continuous curves.

On the basis of the proposed methods is developed software that allows to create the DRC consisting of an arbitrarily large number of points. [4] The resulting points set is interpolating by B-spline or contour of the arcs of the second order curves, which is located within the area of possible location of the monotonic curves.

**Setting article objectives.** The development of technology of design of computer models of surfaces defined by points array, and programs of their processing on the machines with computer numerical control (CNC).

**Main part.** The practical application of the developed methods [2,3,6,8] consider at an example of designing of surfaces interscapular channel of turbocharger impeller and the working surface of the disk cutter for loosening the soil.

Interscapular channel of impeller of the turbocharger is limited by surface of the hub, the cover and the adjacent blades.

The initial data for the simulation of the working surface of the blade is ordered array of points that belong to the family of horizontal planes.

The family of flat DRC is formed on the basis of the initial point array (fig. 3). Points sets defining curves with monotone change the radiuses of curvature are obtained as a result of condensations of initial DRC. The maximum absolute error of the formation of monotonic curves is  $10^{-4}$ .

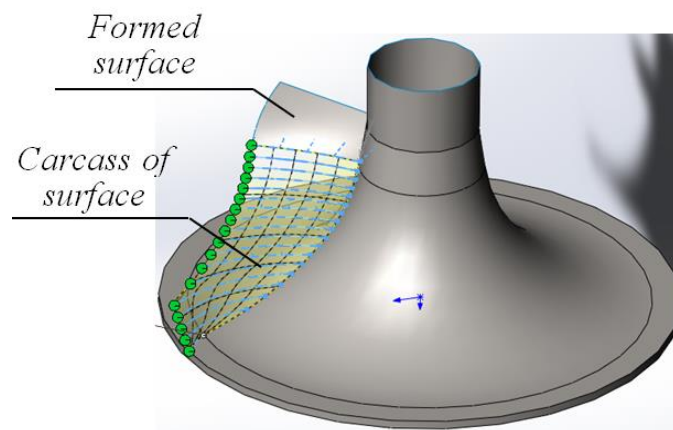


Fig. 3

The guide lines of carcass are formed on the points sets which are arranged on the hub and edge of blade. On the basis of the initial points set formed spatial DRC right move along which the radiuses of the tangent spheres and circles increase monotonically.

The resulting DRC are interpolated by inhomogeneous aperiodic cubic B-spline [4]. After that the elements of carcass imported into the three-dimensional parametric modeling package SolidWorks and the model of working surface of the blade is formed by means of standard functions SolidWorks.

The developed method of designing of impellers adopted for the introduction of state of emergency on "Tavriya Turbo Plus" and LLC "Melitopol factory turbochargers" (Melitopol) and is used in the manufacture of turbo-type impellers YaMZ TRC, TCR-11N1, 11N2-TCR, TCR-11N3.

The tool for loosening the soil is a cutter consisting of a disk with three soil tillage elements (knives) (Fig. 4). During operation the knives move translational-rotational.

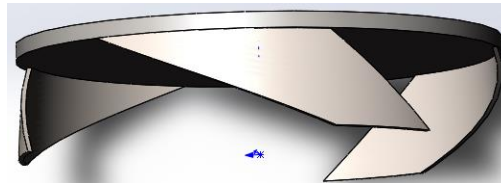


Fig. 4

The model of surface of the blade is formed on the basis of carcass, consisting of a rail line and 4 horizontal generators. As a guide line is used edge of knife – a cylindrical helix.

Generating lines of model created on the basis of the DRC, the starting points of which are defined on the basis of the path of movement of the cutting edge. The minimum deviation of the path of movement of points that define the horizontal section of the knife from the trajectory of point of this section (located at the cutting edge) is provided. This contributes to the minimum soil compaction during processing [1,5,13]. On the basis of the initial points set flat monotone DRC which are approximated by B-spline with a given accuracy of are formed.

Developed technology for forming of computer model of surface which predetermined by the ordered points set comprises the following steps.

1. From the initial array of point are allocated points sets representing the flat or spatial curves - linear elements of the carcass of surface. These DRC divided into sections on the basis of which it is possible to form monotonic curves.

2. Points sets consisting of an arbitrarily large number of points and that is possible be interpolated curve with given characteristics are formed on the basis of the initial DRC using the software developed in computer algebra system Maple.

3. The contours consist of plots of continuous curves with given characteristics are formed in the three-dimensional parametric modeling system SolidWorks on basis of the received DRC.

4. Computer model of the product is formed on the basis on the carcass using standard SolidWorks functions. The control program for the CNC machines is created on the basis of the model using CAM package functions.

**Conclusions.** The technology for creating with given accuracy of computer models of surfaces which represented by array of points is developed.

Functional qualities of surfaces provide defined geometrical characteristics of linear elements of model.

The use of the developed technology makes it possible to reduce the time of creating a computer model of the products and software for processing on CNC machines, to improve the functional properties of surfaces that restrict the product.

As a result of the research the technology of designing of complex surfaces of technical products is designed. The includes the creation of geometric models of surfaces and control programs for their processing on CNC machines.

From the initial array of points allocated subset - point sets, on the basis of which the linear elements of the carcass of surface are formed. A computer model of the surface is created on the basis of discretely carcass the line represented by the families of longitudinal and transversal curves. The control program is created in the automated mode using the CAM-packets based on three-dimensional computer model which is formed in a package of three-dimensional parametric modeling SolidWorks.

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## **МОДЕЛЮВАННЯ ПОВЕРХОНЬ ЗА ЗАДАНИМИ УМОВАМИ НА ОСНОВІ МАСИВУ ТОЧОК**

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*Розробка технології проектування комп'ютерних моделей поверхонь, заданих масивом точок, та програм їх обробки на верстатах з числовим програмним управлінням (ЧПУ). Основна вимога до поверхонь виробів, які взаємодіють із середовищем, - забезпечення заданого характеру їх обтікання. Функціональні якості поверхні забезпечуються її*

геометричними характеристиками. Ламінарний характер обтікання поверхонь можна забезпечити за рахунок монотонної зміни значень кривини, скруту, радіусів стичних сфер уздовж ліній, що входять у визначник поверхні. Вихідними даними для формування лінійних елементів каркасу поверхні є вихідний точковий ряд і геометричні властивості кривої. Розроблено методи формування плоских і просторових кривих із закономірною зміною характеристик на основі довільного точкового ряду. На кожній ділянці, обмеженій послідовними точками, визначається область, усередині якої розташовані всі криві лінії, що відповідають умовам задачі. Отримана крива, яка представлена новим точковим рядом, що складається з будь-якої кількості точок, вважається сформованою, якщо область її можливого розташування не перевищує заданої величини. Практичне застосування запропонованої технології продемонстровано на прикладі проектування поверхонь міжлопаткового каналу робочого колеса турбокомпресора та робочої поверхні дискової фрези для розпушування ґрунту. Розроблена технологія проектування поверхонь складних технічних виробів включає створення геометричних моделей поверхонь та керуючих програм для їх обробки на верстатах з ЧПУ. Із вихідного масиву точок виділяються підмножини – точкові ряди, на основі яких формуються лінійні елементи каркасу поверхні. Комп'ютерна модель поверхні створюється на основі дискретного лінійного каркасу, представленого сімействами твірних та напрямних кривих ліній. Управляюча програма створюється в автоматизованому режимі з використанням САМ-пакетів на основі тривимірної комп'ютерної моделі, сформованої в пакеті тривимірного параметричного моделювання SolidWorks.

Ключові слова: динамічна поверхня; каркас поверхні; лінійний елемент моделі; дискретно представлена крива; закономірна зміна диференціально-геометричних характеристик.

## **МОДЕЛИРОВАНИЕ ПОВЕРХНОСТЕЙ ПО ЗАДАНЫМ УСЛОВИЯМ НА ОСНОВЕ МАССИВА ТОЧЕК**

Холодняк Ю.В., Гавриленко Е.А., Івженко О.В., Пыхтеева И.В.

Разработка технологии проектирования компьютерных моделей поверхностей, заданных массивом точек, и программ их обработки на станках с числовым программным управлением. Основное требование к поверхностям изделий, взаимодействующих со средой, – обеспечение заданного характера их обтекания. Функциональные качества поверхности обеспечиваются ее геометрическими характеристиками. Ламинарный характер обтекания поверхностей можно обеспечить за счет монотонного изменения значений кривизны, кручения, радиусов соприкасающихся сфер вдоль линий, входящих в определитель поверхности. Исходными данными для формирования линейных элементов

каркаса поверхности является исходный точечный ряд и геометрические свойства кривой. Разработаны методы формирования плоских и пространственных кривых с закономерным изменением характеристик на основе произвольного точечного ряда. На каждом участке, ограниченном последовательными точками, определяется область, внутри которой расположены все кривые линии, отвечающие условиям задачи. Полученная кривая, представленная новым точечным рядом, состоящим из сколь угодно большого числа точек, считается сформированной в случае, когда область ее возможного расположения не превышает заданной величины. Практическое применение предложенной технологии продемонстрировано на примере проектирования поверхностей межлопаточного канала рабочего колеса турбокомпрессора и рабочей поверхности дисковой фрезы для рыхления почвы. Разработанная технология проектирования поверхностей сложных технических изделий включает в себя создание геометрических моделей поверхностей и управляющих программ для их обработки на станках с ЧПУ. Из исходного массива точек выделяются подмножества – точечные ряды, на основе которых формируются линейные элементы каркаса поверхности. Компьютерная модель поверхности создается на основе дискретного линейчатого каркаса, представленного семействами образующих и направляющих кривых линий. Управляющая программа создается в автоматизированном режиме с использованием САМ-пакетов на основе трехмерной компьютерной модели, сформированной в пакете трехмерного параметрического моделирования SolidWorks.

*Ключевые слова:* динамическая поверхность; каркас поверхности; линейный элемент модели; дискретно представленная кривая; закономерное изменение дифференциально-геометрических характеристик.

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